NAME:______________________________________

MA 23200
Sample Final Exam

PUID:______________________________________

INSTRUCTIONS

- There are 25 problems on 14 pages.
- Record all your answers on the answer sheet provided. The answer sheet is the only thing that will be graded.
- No books or notes are allowed.
- You may use a one-line scientific calculator. No other electronic device is allowed. Be sure to turn off your cellphone.
- Show all your work on the exam. If you need more space, use the backs of the pages.
- The last page is a formula sheet. You may detach this page from the exam for easy reference, but you must hand it with your exam booklet.

1. Evaluate

$$\int \frac{e^{3x}}{7-e^{3x}}dx.$$

A. $\frac{1}{3(7-e^3x)^2}+C$

B. $\frac{-\frac{1}{3}e^{3x}}{7x-\frac{1}{3}e^{3x}}+C$

C. $\frac{\frac{1}{3}e^{3x}}{7x-\frac{1}{3}e^{3x}}+C$

D. $-\frac{1}{3}\ln\left|7-e^{3x}\right|+C$

E. $\frac{1}{3}\ln\left|7-e^{3x}\right|+C$

2. Evaluate

$$\int_1^4 \frac{\ln x}{\sqrt{x}}dx.$$

A. $2\ln 4+1$

B. $2\ln 4-1$

C. $\frac{1}{2}(\ln 4)^2$

D. $4\ln 4+4$

E. $4\ln 4-4$

3. Use the Trapezoid Rule with $n = 4$ trapezoids to estimate

$$\int_1^3 \frac{x}{x+1}\,dx.$$

Round your answer to three decimal places.

A. 0.808
B. 1.490
C. 1.303
D. 2.606
E. 2.981

4. Find the volume of the solid generated by rotating about the x-axis the region bounded by the curves $y = x + \frac{1}{x}$, $x = 1$ and $x = 4$ and the x-axis.

A. 21.75π
B. 26.25π
C. 27.75π
D. $(7.5 - \ln 4)\pi$
E. $(7.5 + \ln 4)\pi$

5. A volume has cross-sections which are rectangles with length $x+3$ and width $\sqrt{x}$ for $0 \leq x \leq 4$. Find the volume of this solid.

   A. 24
   B. $\frac{144}{5}$
   C. 48
   D. 98
   E. $\frac{320}{3}$

6. Evaluate

$$\int_0^\infty xe^{-\frac{1}{2}x}dx.$$

   A. 0
   B. $\frac{1}{4}$
   C. 2
   D. 4
   E. This integral diverges.

7.

$$f(x,y) = \frac{y}{x^2+y^2}.$$

Compute $f_x(3,-4)$.

A. $\frac{24}{625}$

B. $\frac{12}{625}$

C. 0

D. $-\frac{12}{625}$

E. $-\frac{24}{625}$

8.

$$f(x,y) = e^{x\cos y}.$$

Compute $f_{yy}(x,y)$.

A. $(x^2\cos^2 y - x\sin y)e^{x\cos y}$

B. $(x\sin y\cos y - \sin y)e^{x\cos y}$

C. $x^2\cos y\sin y e^{x\cos y}$

D. $-x\sin y e^{x\cos y}$

E. $(x^2\sin^2 y - x\cos y)e^{x\cos y}$

9. The function $f(x, y)$ has partial derivatives

$$f_x(x,y) = 3y - x - 7, \qquad f_y(x,y) = 3y^2 - 6y + 3x + 3$$

and critical points

$$(-1, 2), \qquad (-16, -3)\,.$$

(There are no other critical points.) Which statement best describes these critical points?

A. Both points are saddle points.
B. $(-1, 2)$ is a saddle point and $(-16, -3)$ is a relative maximum.
C. $(-1, 2)$ is a saddle point and $(-16, -3)$ is a relative minimum.
D. $(-1, 2)$ is a relative minimum and $(-16, -3)$ is a relative maximum.
E. $(-1, 2)$ is a relative maximum and $(-16, -3)$ is a relative minimum.

10. Sally sells seashells down by the seashore. She has found that if, in the morning, she spends x hours looking for seashells, and y hours polishing and decorating the seashells, she will sell

$$S = 4xy + 4y - 4x^2 - 2y^2 + 196$$

seashells that day. How many hours should Sally spend looking for seashells, and how many polishing, in order to sell the highest number possible of shells that day?

A. 1 hour looking and 3 hours polishing.
B. 3 hours looking and 1 hour polishing.
C. 1 hour looking and 1 hour polishing.
D. 1 hour looking and 2 hours polishing.
E. 2 hours looking and 1 hour polishing.

11. Evaluate

$$\int_0^1 \int_0^{x^2} (y + 2x^2)^5 dydx.$$

A. $\frac{27}{2}$

B. $\frac{665}{78}$

C. $\frac{665}{54}$

D. $\frac{243}{26}$

E. $\frac{745}{6}$

12. Suppose

$$f''(x) = \frac{1}{\sqrt{x}} + x$$

with

$$f(1) = 1 \quad \text{and} \quad f'(1) = 0.$$

Find $f(4)$.

A. $\frac{35}{3}$

B. $\frac{40}{3}$

C. $\frac{125}{6}$

D. 25

E. $\frac{65}{2}$

13. Suppose

$$y' - 4y = e^{6x}$$

$$y(0) = 3.$$

Compute $y(1)$.

A. $\frac{1}{2}e^6 + \frac{5}{2}e^4$
B. $e^6 + 2e^4$
C. $e^6 + 12$
D. $\frac{1}{11}e^6 + \frac{32}{11}e^{-4}$
E. $\frac{26}{9} - \frac{5}{9}e^6$

14. Find the general solution to the differential equation

$$xy' + 6y = x^2.$$

A. $y = \frac{x^3}{3} + \frac{x^3}{9\ln x} + \frac{C}{\ln x}$
B. $y = \frac{x^3}{3} - \frac{x^3}{9\ln x} + \frac{C}{\ln x}$
C. $y = \frac{1}{3x^4} + \frac{C}{x^6}$
D. $y = \frac{x^2}{8} + \frac{C}{x^6}$
E. $y = \frac{x^3}{9} + \frac{C}{x^6}$

15. The **autonomous differential equation**,

$$y' = y^4 - 4y^3 - 3y^2 + 18y$$

has exactly three equilibrium solutions:

$$y = -2, \quad y = 0 \quad \text{and} \quad y = 3.$$

Which statement best describes the stability of these equilibrium solutions?

A. $y = -2$ is asymptotically stable, $y = 0$ is unstable and $y = 3$ is semi-stable.
B. $y = -2$ is asymptotically stable, $y = 0$ is semi-stable and $y = 3$ is unstable.
C. $y = -2$ is semi-stable, $y = 0$ is unstable and $y = 3$ is asymptotically stable.
D. $y = -2$ is unstable, $y = 0$ is semi-stable and $y = 3$ is asymptotically stable.
E. $y = -2$ is unstable, $y = 0$ is asymptotically stable and $y = 3$ is semi-stable.

16. Find the general solution to the differential equation

$$y' = x^2y^2.$$

A. $Ce^{-\frac{1}{3}x^3}$
B. $\pm\sqrt{Ce^{\frac{1}{3}x^3}}$
C. $\frac{3}{C-x^3}$
D. $-\frac{3}{x^3} + C$
E. $\ln\left|C - \frac{x^3}{3}\right|$

17. Suppose

$$y' = x + y^2$$
$$y(0) = 1.$$

Use Euler's method with $\Delta x = 0.5$ to approximate $y(1)$.

A. 1.5
B. 1.75
C. 2.5
D. 2.875
E. 3.78125

18. A certain vine grows in such a way so that its length satisfies the differential equation

$$L' = 3\sqrt{L}$$

where L is the length of the vine, in feet, t years from now. If a vine is initially measured at 4 feet long, how long will the vine be in 10 years?

A. 225 ft
B. 229 ft
C. 289 ft
D. 904 ft
E. 1024 ft

19. Which of the following matrices is **singular**?

A. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

B. $\begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 4 \\ -6 & -8 \end{bmatrix}$

D. $\begin{bmatrix} 5 & -1 \\ -1 & -4 \end{bmatrix}$

E. $\begin{bmatrix} 105 & 0 \\ 1 & 3 \end{bmatrix}$

20. Find $\det(A)$, where

$$A = \begin{bmatrix} 7 & 6 & 2 \\ 3 & 4 & 1 \\ -2 & 5 & -1 \end{bmatrix}.$$

A. 59

B. 23

C. 9

D. -11

E. -19

21. Find the inverse of the matrix

$$A = \begin{bmatrix} 4 & -2 \\ -13 & 7 \end{bmatrix}.$$

A. $\begin{bmatrix} \frac{7}{2} & \frac{1}{2} \\ \frac{13}{2} & 1 \end{bmatrix}$

B. $\begin{bmatrix} \frac{7}{2} & 1 \\ \frac{13}{2} & 2 \end{bmatrix}$

C. $\begin{bmatrix} \frac{7}{2} & 1 \\ \frac{1}{2} & 2 \end{bmatrix}$

D. $\begin{bmatrix} 7 & 2 \\ 13 & 4 \end{bmatrix}$

E. $\begin{bmatrix} 14 & 1 \\ 26 & 2 \end{bmatrix}$

22. Find the **eigenvalues** for the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}.$$

A. 3 and 6

B. -2 and 11

C. -11 and 2

D. -3 and 2

E. 2 and 7

23. The matrix

$$A = \begin{bmatrix} -1 & -1 & 1 \\ -4 & 0 & 2 \\ -14 & -6 & 8 \end{bmatrix}$$

has $r = 4$ as one of its **eigenvalues**. Which of the following is an **eigenvector** associated to this matrix and eigenvalue?

A. $\begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$

B. $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}$

E. $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

24. The matrix A has eigenvalues $r_1 = 2$ and $r_2 = 3$, with eigenvectors $v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Compute $A^5 \begin{bmatrix} 4 \\ 5 \end{bmatrix}$.

A. $\begin{bmatrix} -1 \\ 5 \end{bmatrix}$

B. $\begin{bmatrix} 13 \\ 15 \end{bmatrix}$

C. $\begin{bmatrix} 211 \\ 243 \end{bmatrix}$

D. $\begin{bmatrix} 371 \\ 243 \end{bmatrix}$

E. $\begin{bmatrix} 1183 \\ 1215 \end{bmatrix}$

25. Consider the difference equation

$$x_{n+1} = x_n + 6x_{n-1}.$$

If $x_0 = 0$ and $x_1 = 20$, find x_{15}.

A. 57526700

B. 57395628

C. 57264556

D. 19066340

E. 14381675

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Formula Sheet

The Trapezoid Rule The Trapezoid Rule for estimating the integral $\int_a^b f(x)dx$ with n trapezoids is given by

$$T_n = \frac{1}{2}\Delta x \left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

where $\Delta x = \frac{b-a}{n}$, $x_0 = a$, $x_1 = a + \Delta x$, $x_2 = a + 2\Delta x$, $\ldots$, $x_n = a + n\Delta x = b$.

D-Test To find the relative maximum and minimum values of f:

1. Find f_x, f_y, f_{xx}, f_{yy} and f_{xy}.
2. Solve $f_x(x, y) = 0$ and $f_y(x, y) = 0$.
3. Evaluate $D = f_{xx}f_{yy} - [f_{xy}]^2$ at each point (a, b) found in Step 2.
   (a) If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$ then f has a relative maximum at (a, b).
   (b) If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$ then f has a relative minimum at (a, b).
   (c) If $D(a, b) < 0$ then f has a saddle point at (a, b).
   (d) If $D(a, b) = 0$ then the test is inconclusive... you will have to do something else to determine what is happening at that point.

Euler's Method To approximate the solution to $y' = f(x, y)$, $y(x_0) = y_0$ using Euler's method with increments of Δx, we use the formula

$$y_{n+1} = y_n + f(x_n, y_n)\Delta x$$

where $x_{n+1} = x_n + \Delta x$ and $y_n = y(x_n)$.