MA 15300 Lesson 1 Notes

I REAL NUMBERS

Natural Numbers: 1, 2, 3, …. (Natural Numbers are called the Counting Numbers in some textbooks.)

Whole Numbers = Natural Numbers + Zero: {0, 1, 2, 3,…}

Integers: All whole numbers plus their opposites (Negatives)
$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

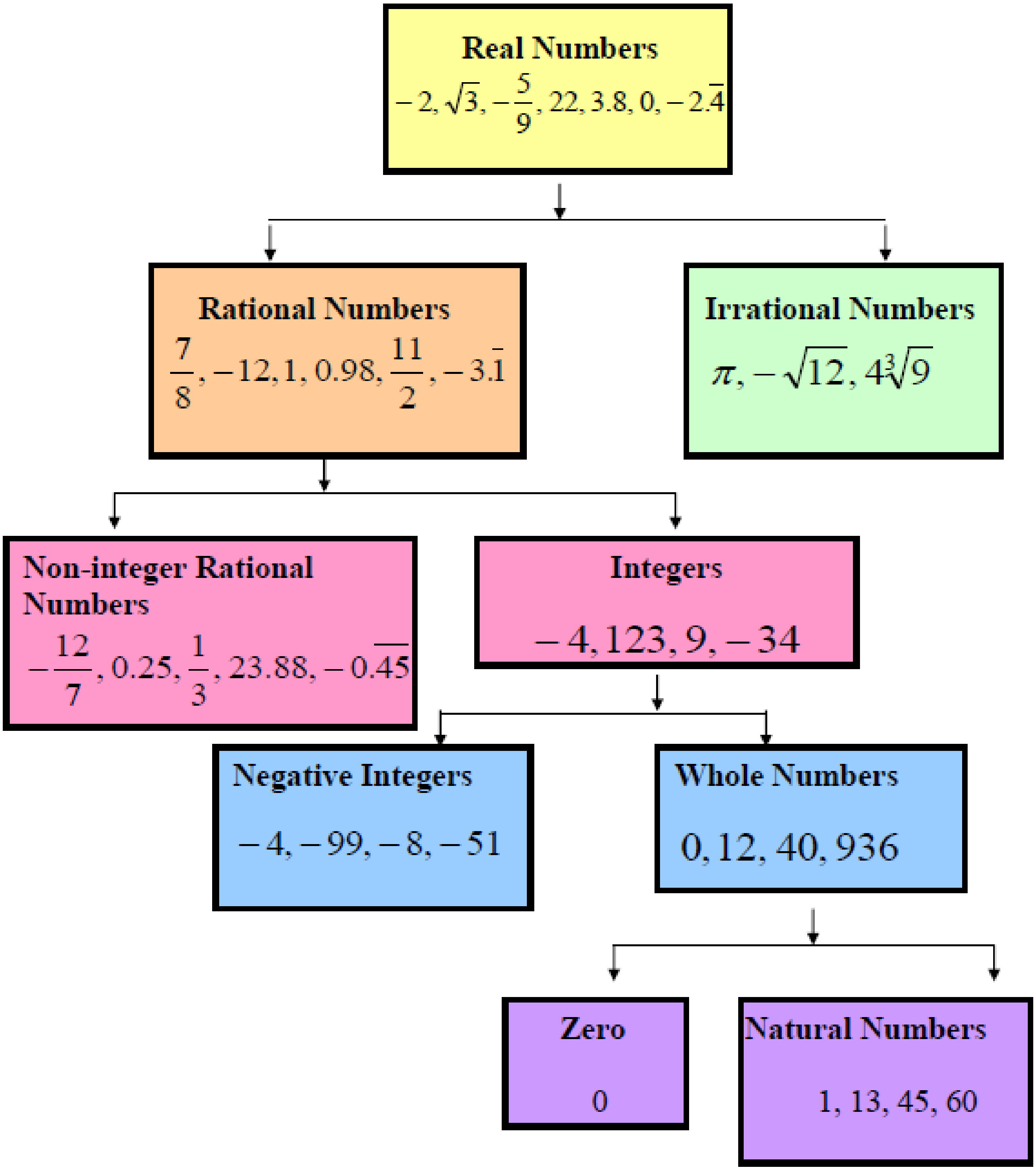
Rational Numbers: Any number that can be expressed as an integer divided by any non-zero integer (in other words, expressed as fractions)
Alternative definition: Any number that can be expressed as a terminating or repeating decimal.

Irrational Numbers: Any number that **cannot** be expressed as a ratio of two integers. Alternative definition: Any number that **cannot** be expressed as a terminating or repeating decimal.

Real Numbers: Any number that is rational or irrational.

(See page 2 for relationships between these sets of numbers.)

Relationship between Sets of Numbers and Examples



II Other classifications of numbers

A Even or Odd

An **even number** has a factor of 2 (is divisible by 2). An **odd number** does not have a factor of 2 (is not divisible by 2).

B Prime or Composite

A **prime number** is an integer **larger than 1** whose **only** factors are 1 and itself.
A **composite number** is an integer **larger than 1** that has factors other than 1 and itself.

III Properties of Real Numbers

Addition and Multiplication are **commutative (order does not matter)**.

a and b are any two real numbers.

$$a+b=b+a$$

$$a\cdot b=b\cdot a$$

Addition and Multiplication are **associative (grouping does not matter).**

a, b, and c are any real numbers

$$a+(b+c)=(a+b)+c$$

$$a\cdot(bc)=(ab)\cdot c$$

Zero is the **additive identity** and 1 is the **multiplicative identity**.

a is any real number

$$a+0=a$$

$$a\cdot 1=a$$

Distributive Property:

$$a(b+c)=ab+ac$$

$$ab+ac=a(b+c)$$

IV Absolute Value of a real number is its <u>distance</u> from zero. (Distance is always assumed to be positive.)

Let a be any real number

$$|a|=\begin{cases} a & \text{if} \quad a>0 \\ 0 & \text{if} \quad a=0 \\ -a & \text{if} \quad a<0 \end{cases}$$

Example 1:

Find the 'exact' absolute value of each.

$|5-\pi|=$

$|2-\sqrt{11}|=$

V Exponential Notation: $x^n = \underbrace{x \cdot x \cdot x \ldots . x}_{n \text{ times}}$ (n times, if n = 1, 2, 3, 4....)

1. Read as 'x to the nth power'; x is the base and n is the exponent.
2. If there is no exponent denoted, it is understood to be a power of 1.
3. If no coefficient is denoted, it is also understood to be 1.

Example 2: Evaluate each.

Evaluate means 'find the numeric value'.

$a)\quad 4^5 =$

$b)\quad (-5)^2 =$

$c)\quad -3^4 =$

VI Order of Operations: "Please Excuse My Dear Aunt Sally" may be used to help you remember the order of operations.

Please: **First** complete Parentheses or other grouping symbols (working inside to outside).

Excuse: **Secondly**, evaluate any exponent expression (power).

My Dear: **Next,** complete multiplication and/or division operations **left to right.**

Aunt Sally: **Lastly,** complete addition and/or subtraction operations **left to right.**

Example 2:

$$\frac{2^3 + 3(4) - \sqrt{9+7}}{3 - 4(2) + 4} =$$

Evaluate:

Ex. 3:

Given: $x < 0,\ y > 0$, and $z < 0$, determine the sign of...

a) $yz\left|y^2 z\right|$

b) $\dfrac{xy\left|x^2 + yz\right|}{z}$

c) $\dfrac{x\left|xyz\right|}{y}$

d) $\dfrac{x^2 yz}{\left|z\right|}$

There are two techniques that could be used to determine the sign of expressions like those at the left.

1) Use reasoning with a 'diagram'.

2) Select specific positive or negative numbers for the variables and evaluate.

VII Properties of Exponents

Example 2: Simplify the following by writing the bases the number of times indicated by the exponents. Then determine if there is another method that could be used.

Example 4:

$a)\quad x^2 \cdot x^3 =$

$b)\quad \frac{x^5}{x^2} =$

$c)\quad (x^2)^3 =$

$d)\quad (3x^4y^3)^2 =$

$e)\quad \left(\frac{4x^3}{5y^4}\right)^2 =$

SUMMARY

1. **Product Rule for Exponents**: $b^m \cdot b^n = b^{m+n}$ $(9^6)(9^7)(9) = 9^{14}$

   When common bases are multiplied, the exponents are added.

2. **Quotient Rule for Exponents:** $\frac{b^m}{b^n} = b^{m-n}$ $\frac{r^8}{r^3} = r^5$

   When common bases are divided, the exponents are subtracted (exponent in numerator minus exponent in denominator).

3. **Power Rule for Exponents:** $(b^m)^n = b^{mn}$ $(4^3)^4 = 4^{12}$

   When a base is raised to a power and then raised to another power, the exponents are multiplied.

4. **Product to a Power Rule:** $(ab)^n = a^n b^n$ $(3xy^2)^3 = 27x^3y^6$

When a product is raised to a power, the exponent is applied to each factor.

5. **Quotient to a Power Rule:** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\left(\frac{-4}{xy^3}\right)^4 = \frac{256}{x^4y^{12}}$

When a quotient is raised to a power, the exponent is applied to each factor in the numerator and in the denominator.

6. **Zero Exponent Rule:** $b^0 = 1$ $2^0 mn^0 = m$

Examine: Simplify $\frac{5^3}{5^3}$ by using arithmetic and by using the quotient rule.

Examine: Simplify $\frac{x^0}{x^n}$ by using the zero exponent rule and the quotient rule.

7. **Negative Exponent Rules:**

$b^{-n} = \frac{1}{b^n}$ $2^{-3} = \frac{1}{2^3}$

$\frac{1}{b^{-n}} = b^n$ $\frac{x}{y^{-4}} = xy^4$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $\left(\frac{x}{3}\right)^{-2} = \left(\frac{3}{x}\right)^2 = \frac{9}{x^2}$

A base to a negative power is its reciprocal to the positive power.

Example 5:

$a)\quad (-3)^2$

$b)\quad -3^2$

$c)\quad (-3)^{-2}$

$d)\quad -3^{-2}$

Example 6: Simplify each. (Use the rules for exponents.)

$a)\quad \left(8x^4y^{-3}\right)\left(\frac{1}{2}x^{-5}y^2\right)=$

$b)\quad \left(3y^3\right)^4\left(4y^2\right)^{-3}=$

$c)\quad \left(\frac{1}{5}x^4y^{-3}\right)^{-2}=$

$d)\quad \left(4a^2b\right)^3\left(\frac{-a^3}{2b}\right)^2=$