Lesson 31

System of equations:

- two or more equations containing common variables
    - Example: $\begin{cases} x^2 - y = 9 \\ x + y = 21 \end{cases}$
- the solution set of a system are the set ordered pairs that make each equation true
    - the solution set of the **system given above** is $(-6, 27), (5, 16)$
        - substitution of (-6, 27) yields: $36 - 27 = 9$; $-6 + 27 = 21$
        - substitution of (5, 16) yields: $25 - 16 = 9$; $5 + 16 = 21$
- we will only cover systems with two equations and two variables, **so our solutions will always be sets of ordered pairs $(\boldsymbol{x}, \boldsymbol{y})$.**

The solutions of systems of equations are not only the values that make each equation true, they are also the points where the graphs of the equations intersect. If there are no solutions, that means the graphs never intersect. If there are infinitely many solutions, that means the graphs intersect at every point (same graph; we'll see this in Lesson 32).

The two methods we will use to solve systems are substitution and elimination. We will cover substitution in this lesson and elimination in the next.

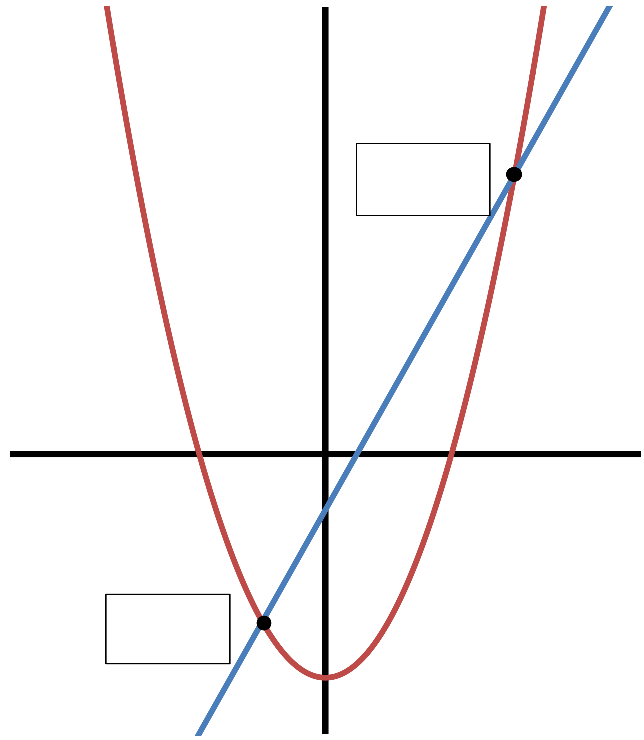
Method of Substitution:

1. solve one equation for one variable
    a. **IT DOESN'T MATTER WHICH EQUATION YOU CHOOSE OR WHICH VARIABLE for which YOU CHOOSE TO SOLVE first.**
2. substitute the solution from step 1 into the **other** equation
3. solve the new equation from step 2
4. back-substitute to solve the equation from step 1for the other variable

*Checking solutions: You can verify whether your solution is correct by plugging the ordered pairs back into the original equations. If the ordered pairs in your solution set make both of the original equations true, they are correct; if not, they are incorrect.

Example 1: Use the method of substitution to find all the **REAL SOLUTIONS** for the following systems. **Keep in mind, it doesn't matter which equation you choose or which variable you choose to solve.**

a. $\begin{cases} x^2 - y = 4 \\ 2x - y = 1 \end{cases}$



b. $\begin{cases} x + y^2 = 1 \\ x + 2y = 1 \end{cases}$

What are the shapes for these two equations?

c. $\begin{cases} y = x^2 \\ 4x + y + 10 = 0 \end{cases}$

What are the shapes for these two equations?

d. $\begin{cases} 2y = x^2 \\ y = 4x^3 \end{cases}$

e. $\begin{cases} xy = 2 \\ 3x - y + 5 = 0 \end{cases}$

f. $\begin{cases} x^2 - y^2 = 4 \\ x^2 + y^2 = 12 \end{cases}$

g. $\begin{cases} xy = 24 \\ 2x^2 - y^2 + 4 = 0 \end{cases}$

Example 2: A cylindrical tube is to be made from a sheet of paper that has an area of $1496\ in^2$. Is it possible to construct a tube that has a volume of $1496\ in^3$? If so, find r and h. (Round to 2 decimal places, if necessary.)

Write a system of equations (with variables r and h) with one equation representing the area of the sheet of paper and the other representing the volume of the tube.