Lesson 33

The two methods we have used to solve systems are substitution and elimination. Substitution was covered in Lesson 31 and elimination was covered Lesson 32.

Method of Substitution:

1. solve one equation for one variable (it doesn't matter which equation you choose or which variable you choose).
2. substitute the solution from step 1 into the other equation.
3. solve the new equation from step 2.
4. back substitute to solve the equation from step 1.

Method of Elimination:

1. multiply at least one equation by a nonzero constant so the coefficients for one variable will be opposites (same absolute value)
2. add the equations so the variable with the opposite coefficients will be eliminated.
3. take the result from step 2 and solve for the remaining variable.
4. take the solution from step 3 and back substitute to any of the equations to solve for the remaining variable.

What happens if both variables are eliminated and we end up with an equation such as $0 = a$, where a is a nonzero constant? **NO SOLUTION**

What happens if both variables are eliminated and we end up with an equation such as $0 = 0$? **INFINITELY MANY SOLUTIONS**

You can verify whether your solution is correct by plugging the ordered pairs back into the original equations. If the ordered pairs in your solution set make both of the original equations true, they are correct; if not, they are incorrect.

Review: Solve each system of equations below.

1) $\begin{cases} 2x^2 + y = 5 \\ 3x - 4y = 18 \end{cases}$ Use the substitution method.

2) $\begin{cases} 3x + 4y = -7 \\ 2x + 5y = 0 \end{cases}$ Use the elimination method.

3) $\begin{cases} y = 2x - 7 \\ 6x - 3y = 21 \end{cases}$ Use either method.

Steps for solving applications:

1. assign variables to represent the unknown quantities
2. set-up equations using the variables from step 1
3. for problems involving motion, it might be helpful to set-up a table using the equation $d = r \cdot t$
4. pay attention to units (are they reasonable, do they match (mph and minutes?), are they positive)

Example 1: Set-up a system of equations and solve using any method.

a. When a popular band played at Elliott Hall, the box office receipts totaled $120,000. Student tickets cost $10 and non-student tickets cost $25. If 6,000 people attended the concert, how many were students and how many were non-students?

Let x = the number of students attending the concert

Let y = the number of non-students attending the concert

Write an Equation about number of people attending:

Write an Equation about the revenue from ticket sales:

b. A large solar heating panel requires 120 gallons of a fluid that is 30% antifreeze. The fluid comes in either a 50% solution or a 20% solution. How many gallons of each should be used to prepare the 120-gallon solution?

1^{st}		2^{nd}		MIX
50% x gal.	+		=	30% 120 gal.

Let x = number of gallons of the 50% antifreeze solution

y= number of gallons of the 20% antifreeze solution (fill info above)

$\begin{cases} \text{equation about number of gallons} \\ \text{equation about amount of antifreeze} \end{cases} \longrightarrow$

c. British sterling silver is a copper-silver alloy that is 7.5% copper by weight. How many grams of pure copper and how many grams of British sterling silver should be used to prepare 200 grams of a copper-silver alloy that is 10% copper by weight? Round answers to the nearest hundredth of a gram.

Pure Copper		Sterling Silver		MIX
	+		=	

x =

y = Fill in diagram above.

First equation about the number of grams:

Second equation about the amount of copper:

d. After a boulder is pushed down a hill, its speed $s(t)$ in feet per second at time t is given by the formula $s(t) = v + at$, where v is the initial velocity and a is the acceleration. If $s(2) = 16$ and $s(5) = 25$, find the initial velocity and initial acceleration.

e. A short airplane trip between two cities took 30 *minutes* when traveling with the wind. The return trip took 45 *minutes* when traveling against the wind. If the speed of the plane with no wind is 320 mph, find the speed of the wind and the distance between the cities (*pay attention to units*).

w = wind speed *d* = distance between cities

	$\boldsymbol{d}$ $(\boldsymbol{mi})$ $=$	$\boldsymbol{r}$ $(\boldsymbol{mph})$ $\cdot$	$\boldsymbol{t}$ $(\boldsymbol{hr})$
Original Flight			
Return Flight			

f. With the current, a person can canoe 24 miles in 4 hours. Against the same current, the same person can canoe 18 miles in 6 hours. Find the person's average rate in still water and the average rate of the current.

R = rate in still water c = current rate

	d	$= r$	$\cdot\ t$
With Current			
Against Current			

g. (This is the same problem as example 4 from the lesson 8 notes. At that time, we solved the problem using one variable only.)

An electrician and his apprentice make \$55 and \$20 an hour, respectively. If they bill a customer \$735 for a job, and the apprentice worked 3 hours fewer than the electrician, how many hours did each work?

h = number of hours the electrician worked
k = number of hours the apprentice worked

Write two equations using h and k. Hint: One equation will show the relationship between the number of hours; the other equation will involve the billing.

h. (This is example 10 from the lesson 8 notes.)

A snowplow leaves a town at 5:00 AM heading west at 20 miles per hour. At 5:15 AM, a second snowplow leaves the same town heading east at 30 miles per hour. If the snowplows continue heading in opposite directions and the CB radios in the trucks have a maximum range of 50 miles, <u>how long</u> has each driver been on the road when they are no longer able to communicate by radio?

	Distance (mi.)	Rate (mph)	Time (hr.)
Snowplow 5:00 AM			
Snowplow 5:15 AM			

Picture:

i. A Vermont Railways freight train, loaded with logs, leaves Boston heading toward Washington DC, at a speed of 60 kilometer/hour. Two hours later, an Amtrak Metroliner Leaves Boston, bound for Washington DC, on a parallel track at 90 kilometer/hour. At what points (distance) and what time for each will the Metroliner 'catch up' to the freight train?

	Distance (km)	Rate (kph)	Time (hr)
Vermont Railways freight train			
Amtrak Metroliner			

Picture: