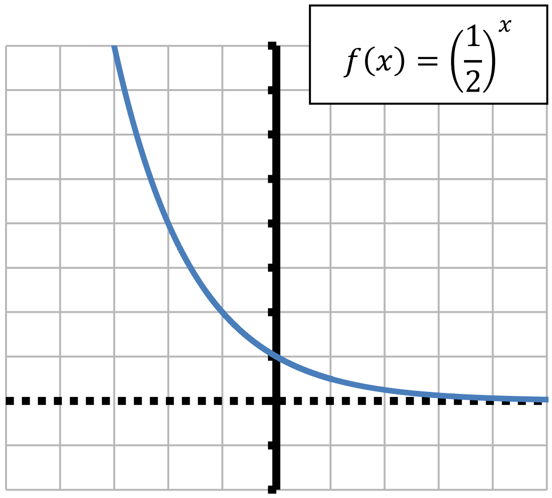
Lesson 36

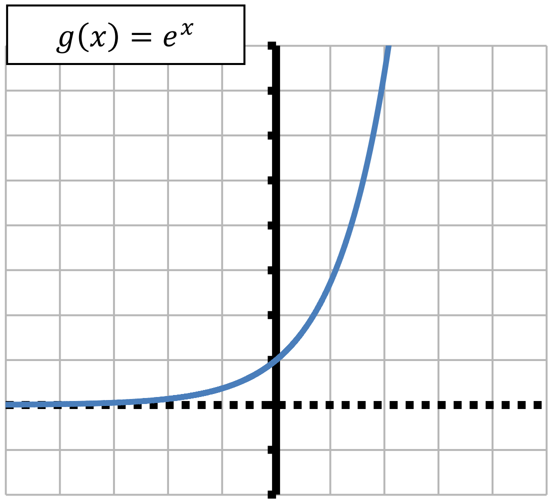
Review of Exponential Functions:

- $\boldsymbol{f(x) = a^x}$, where the exponent x is a variable and base a is a constant
- the base a must be a positive number other than 1
  - $(a > 0 \text{ and } a \neq 1)$
- the domain is the set of all real numbers $(-\infty, \infty)$ and the range is the set of positive real numbers $(0, \infty)$
  - a positive base taken to any power results in a positive output
- the function has no zeros
  - the only way an exponential function can have zeros is if it is shifted down
- the graph is either always increasing or always decreasing
  - exponential functions are one-to-one
- when graphed, all exponential functions will have a horizontal asymptote
  - I recommend graphing exponential functions using an input/output table and using 'transformations', if necessary

Natural Exponential Functions:

- $f(x) = e^x$, where the exponent x is a variable and base e is the number 2.71828…
- the base e is **NOT** a variable, it is simply a number (like π or i)





Finding a Logarithmic Function

We have already determined in Lesson 35 that exponential functions are one-to-one. Knowing that an exponential function is one-to-one, it must have an inverse. For exponential functions, the input is an exponent, and the output is a power of the base.

Exponential Function: $y = f(x) = a^x$ with $D = (-\infty, \infty)$ and $R = (0, \infty)$

The inverse of a function switches the input and output; so the inverse of an exponential function would have an input that is the power of a base (the y or $f(x)$ above) and an output that is an exponent (the x above). This is called a **logarithmic function**. In exponential form, the logarithmic function would be written $\boldsymbol{x = a^y}$**.** However, we usually write functions in terms of y, so the notation below has been developed for logarithmic functions.

Logarithmic Functions:

- the inverse of an exponential function
  - the function that undoes an exponential function
- written $\boldsymbol{f(x) = \log_a x}$, where the input x (called the argument) is a power of the base a and the output is the exponent which produces that power
  - the base a is a subscript
- This function has $D = (0, \infty)$ and $R = (-\infty, \infty)$

Example 1: Given the logarithmic function $f(x) = \log_2 x$ (inverse of $y = 2^x$), find the following:

a. $f(8) = \log_2 8$ b. $f(2)$ c. $f\left(\frac{1}{2}\right)$

c. $f(-2)$ e. $f(1)$ f. $f(0)$

f. Domain of f h. Range of f

Important: What are the restrictions on the inputs of logarithmic functions (the domain)?

What are the restrictions on the outputs of logarithmic functions (the range)?

A logarithmic function is the inverse of an exponential function. When changing from a function to its inverse, the inputs and outputs are switched, so:

- the domain of a logarithm function is the range of an exponential function
- the range of a logarithmic function is the domain of an exponential function.

For an exponential function, anything can go in but only positive values can come out; <u>for a logarithmic function, only positive values can go in, but anything can come out.</u>

It is important to keep in mind that since exponential and logarithmic functions are inverses, they undo one another.

$$y = \log_a x \text{ if and only if } a^y = x$$
$$3 = \log_2 8 \text{ if and only if } 2^3 = 8$$

Being able to convert from log form to exponential form and from exponential form to log form will be **crucial** when solving exponential and logarithmic equations. When converting from log form to exponential form (or vice versa), <u>**THE BASE DOES NOT CHANGE**</u>. Base a in log form is still base a in exponential form. We simply switch the inputs and outputs.

<u>**Example 2:**</u> Convert each exponential equation to logarithmic form.

a. $2^5 = 32$

b. $5^{-3} = \frac{1}{125}$

c. $A^c = V$

d. $3^{-2x} = \frac{P}{F}$

<u>**Example 3:**</u> Convert each logarithmic equation to exponential form.

a. $\log_4 64 = 3$

b. $\log_r s = t$

c. $\log_b 219 = \frac{1}{3}$

d. $\log_5 w = 2x - 3$

Keep in mind that whether you're changing from exponential to logarithmic form, or from logarithmic to exponential form, **<u>the base does not change</u>. Simply switch the input and output, and leave the base the same.**

<u>*It is also important to keep in mind that a logarithm is an exponent</u>**. The expression $\log_a x$ simply asks the question "What exponent should base a be raised to, so it results in x?".***

<u>Example 4:</u> Simplify, if possible. (Find the logarithm.)

a. $\log_3 9$ b. $\log_2 8$ c. $\log_3(-3)$

d. $\log_4 4$ e. $\log_4 \frac{1}{4}$ f. $\log_3 \frac{1}{81}$

g. $\log_{13} 1$ h. $\log_{13} 0$ i. $\log_2 2^4$

j. $\log_{19} 19^3$ k. $5^{\log_5 25}$ l. $3^{\log_3 13}$

BASIC PROPERTIES OF LOGARITHMS

1) $\log_a 1 = 0$

2) $\log_a a = 1$

3) $\log_a a^x = x$

4) $a^{\log_a x} = x$

$m.$ $\log_7 1$ $n.$ $\log_9 9$ $o.$ $\log_{11} 11^5$

$p.$ $5^{\log_5 24}$

Since the logarithmic function with base b is the inverse of the exponential function with base b, the logarithmic function must be one-to-one. Since the logarithmic function is one-to-one, the following condition is true for all positive real numbers:

$$\text{if } \log_b w = \log_b z, \text{ then } w = z$$
$$\text{if } \log_2 x = \log_2 3, \text{ then } x = 3$$

(a logarithm is an exponent; since the logs are equal, the exponents are the same)

If two logarithms with the same base are equivalent, the inputs of those logarithms must be equivalent. **Remember: inputs > 0**

Example 5: Solve the equations and **CHECK YOUR SOLUTIONS**. **Arguments must be positive**.

a. $\log_4 x = \log_4(8 - x)$

b. $\log_5(x - 2) = \log_5(3x + 7)$

c. $\log_2(x - 5) = 4$

d. $\log_9 x = \frac{3}{2}$

Common logarithm:

- $f(x) = \log x$
  - when no base is denoted, it is understood to be base 10
- $f(x) = \log x$ is equivalent to $f(x) = \log_{10} x$

Natural logarithm:

- $f(x) = \ln x$
  - this is a logarithm with a base of e
- $f(x) = \ln x$ is equivalent to $f(x) = log_e x$
  - $\ln\square$ means log base e, so if you write $\ln\square$, there is no need to include a base

Example 6: Find the number, if possible (**without a calculator**).

a. $\log 10$

b. $\log 100$

c. $\log 1$

d. $\log 0.1$

e. $\log 0.0001$

f. $\log 10^3$

h. $\ln e$

h. $\ln \frac{1}{e}$

i. $\ln 0$

l. $e^{\ln 13}$

k. $\ln e^{\frac{2}{3}}$

l. $e^{1+\ln 5}$

Example 7: Solve the equations and **CHECK YOUR SOLUTIONS**.
(Remember, arguments must be positive, domain is only positives.)

a. $\log x = \log(3 - x)$

b. $\ln x^2 = \ln(12 - x)$

c. $\ln x = -2$

d. $\log x^2 = 4$