Lesson 37

Review of Logarithmic Functions:

- the inverse of an exponential function
  - the function that 'undoes' an exponential function
  - Example: $x = 2^y$ is equivalent to $y = \log_2 x$
- written $f(x) = \log_a x$, where the input x is a power of the base a and the output is the exponent which produces that power
  - the input x is known as the **argument** (power), and must be positive
  - the base is a subscript, but the argument is not
- the domain is the set of all positive real numbers $(0, \infty)$ and the range is the set of all real numbers $(-\infty, \infty)$
  - this is the inverse of an exponential function; ordered pairs' coordinates are 'switched', domain and range sets are switched

Common logarithm:

- $f(x) = \log x$
  - when no base is denoted, it is understood to be base 10
- $\boldsymbol{f(x) = \log x}$ **is equivalent to** $\boldsymbol{f(x) = \log_{10} x}$

Natural logarithm:

- $f(x) = \ln x$
  - this is a logarithm with a base of e
- $\boldsymbol{f(x) = \ln x}$ **is equivalent to** $\boldsymbol{f(x) = log_e x}$
  - $\ln x$ means log base e, so if you write $\ln x$, there is no need to include a base
  - also, do not write x (the input or argument) as the base

It is important to be able to convert from exponential form to logarithmic form, and vice versa. Keep in mind that when converting from one form to another, **the base does not change**. Base a in log form is still base a in exponential form; we simply **switch the input and output**.

$$y = \log_a x \text{ is equivalent to } a^y = x$$
$$4 = \log_3 81 \text{ is equivalent to } 3^4 = 81$$

Being able to change (convert) from one form to another will be crucial when solving exponential and logarithmic equations. Also knowing how to use your TI-30Xa calculator is crucial.

Use your TI-30Xa calculator to find the following. Round to 4 decimal places, if necessary.

A $\log 20.3$

B $\ln 0.5689$

C $10^{3.45}$

D e^4

<u>Example 1:</u> Solve each equation and round your answers to the nearest thousandths.

a. $\log x = 3.712$

b. $\log x = -0.32$

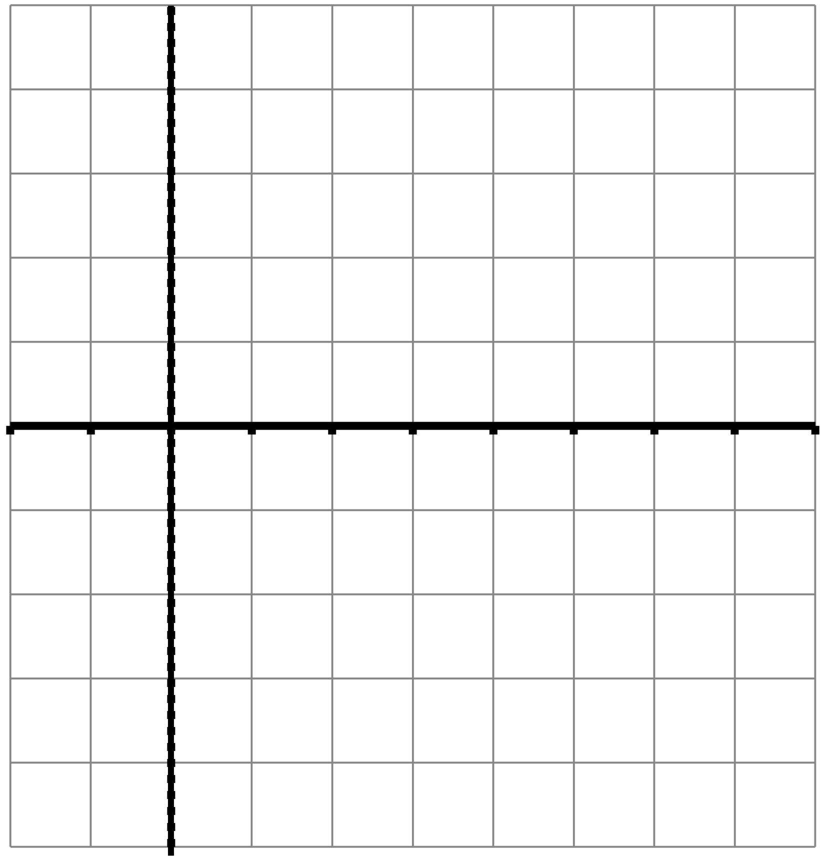
c. $\ln x = -5.152$

d. $\ln x = 0.736$

Example 2: Sketch the graph of $f(x) = \log_2 x$, then find the following:

Hint: Convert to exponential form. $\boldsymbol{x = 2^y}$

Inputs $(0, \infty)$	**Outputs** $(-\infty, \infty)$
x	$f(x) = \log_2 x$
$x \to 0$	$y \to$
	-4
	-3
	-2
	-1
	0
	1
	2
	3
	4
$x \to \infty$	$y \to$



Domain:

Range:

$f(x) = 0$ when $x =$

x-intercepts:

$f(x) > 0$:

$f(0) =$

$f(x) < 0$:

y-intercepts:

Increasing:

Decreasing:

$f(x)$ is even/odd/neither: $f(-1) =$ $f(1) =$

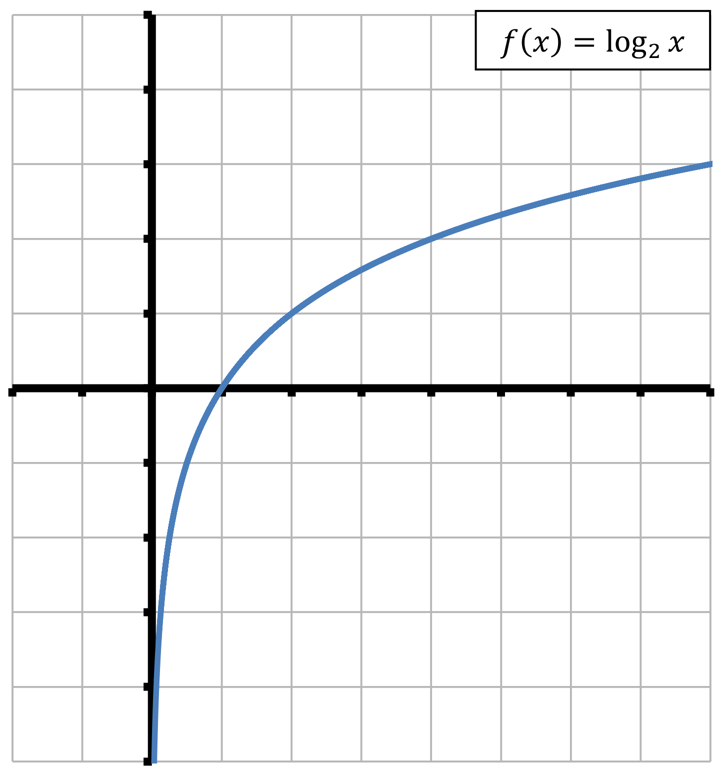
Asymptotes:

Vertical Asymptote:

- a vertical line ($x = \#$) that the graph of a function approaches, but never touches or crosses, when the inputs approach an undefined value ($x \to \#$, where # is a value that is not part of the domain)
  - in the case of $f(x) = \log_2 x$, as the inputs get closer and closer to zero ($x \to 0$), the outputs get smaller and smaller ($f(x) \to -\infty$), so the graph has a vertical asymptote at $x = 0$
  - the graph of every logarithmic function will have a vertical asymptote ($x = \#$)
- in upcoming examples (and in WebAssign), vertical asymptotes will not be dotted lines when they lie on the y-axis, but only when they are to the left or right of the y-axis

Example 3: Re-write each of the following functions in terms of $f(x) = \log_2 x$ ($graphed\ below\ right$), then match the transformation with the appropriate graph.
(Transformed Graphs are on the page 7.)

a. $g(x) = -\log_2 x$



b. $h(x) = \log_2(-x)$

c. $j(x) = \log_2(x - 2)$

d. $k(x) = (\log_2 x) - 2$

e. $m(x) = 2\log_2 x$

f. $n(x) = \log_2(2x)$

Think: How have some points in the graph of function f on the previous page been changed?

3(a) $g(x) = -\log_2 x$

$(8,3) \rightarrow$
$(4,2) \rightarrow$
$(2,1) \rightarrow$
$(1,0) \rightarrow$
$(\frac{1}{2},-1) \rightarrow$

3(b) $h(x) = \log_2(-x)$

$(8,3) \rightarrow$
$(4,2) \rightarrow$
$(2,1) \rightarrow$
$(1,0) \rightarrow$
$(\frac{1}{2},-1) \rightarrow$

3(c) $j(x) = \log_2(x-2)$

$(8,3) \rightarrow$
$(4,2) \rightarrow$
$(2,1) \rightarrow$
$(1,0) \rightarrow$
$(\frac{1}{2},-1) \rightarrow$

3(d) $k(x) = (\log_2 x) - 2$

$(8,3) \rightarrow$
$(4,2) \rightarrow$
$(2,1) \rightarrow$
$(1,0) \rightarrow$
$(\frac{1}{2},-1) \rightarrow$

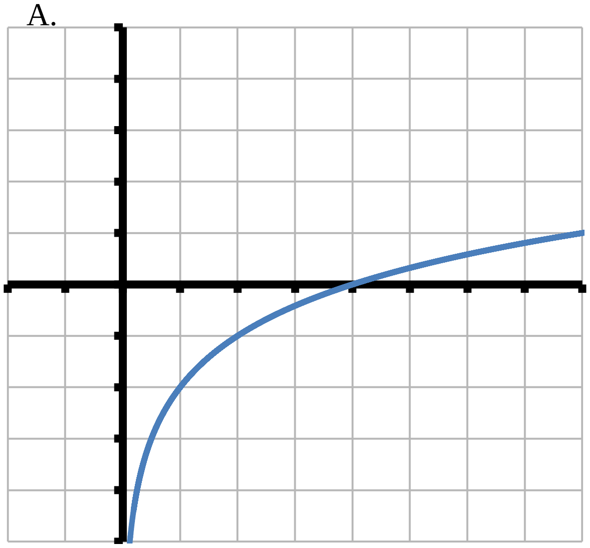
3(e) $m(x) = 2\log_2 x$

$(8,3) \rightarrow$
$(4,2) \rightarrow$
$(2,1) \rightarrow$
$(1,0) \rightarrow$
$(\frac{1}{2},-1) \rightarrow$

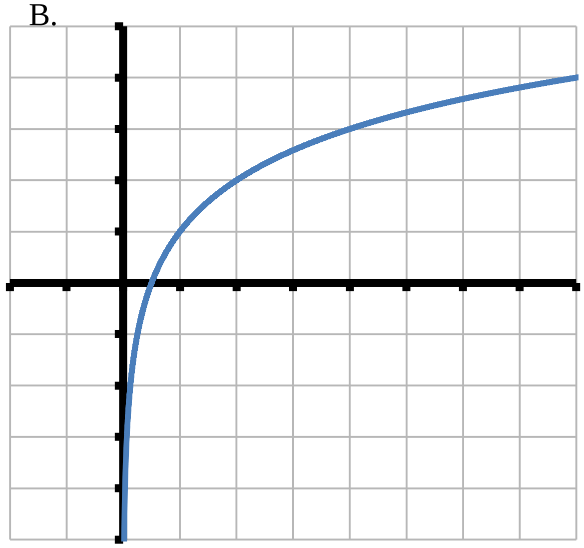
3(f) $n(x) = \log_2(2x)$

$(8,3) \rightarrow$
$(4,2) \rightarrow$
$(2,1) \rightarrow$
$(1,0) \rightarrow$
$(\frac{1}{2},-1) \rightarrow$

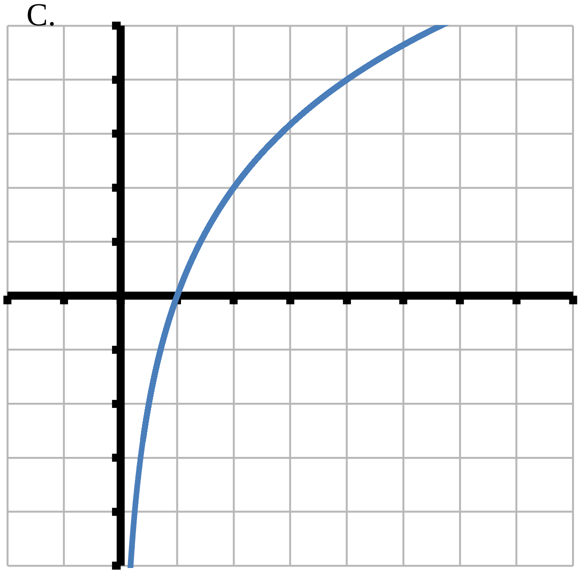
A.



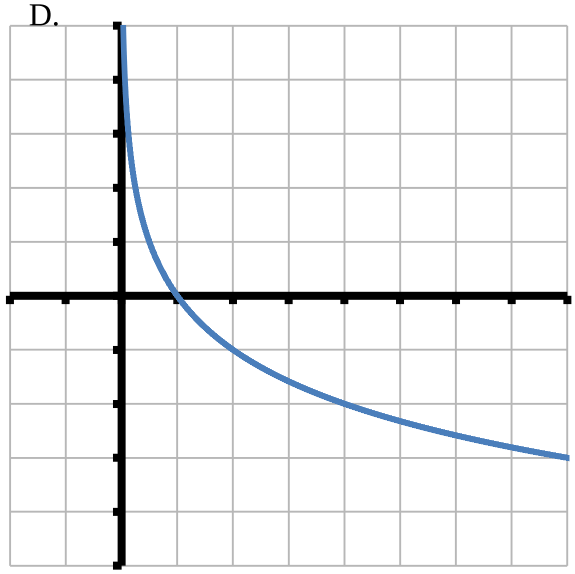
B.



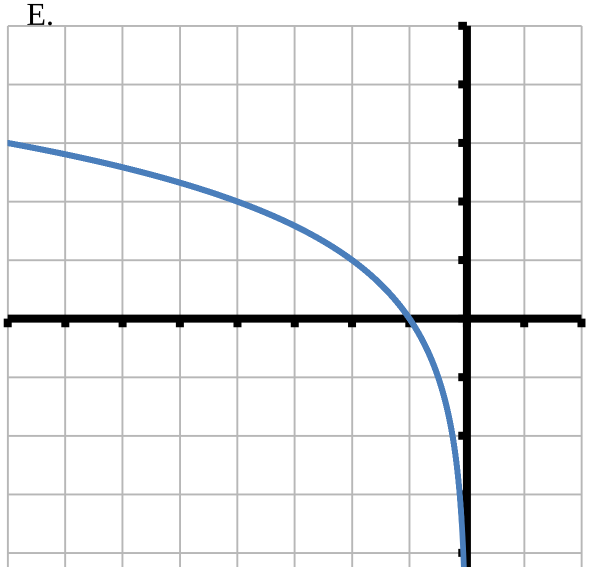
C.



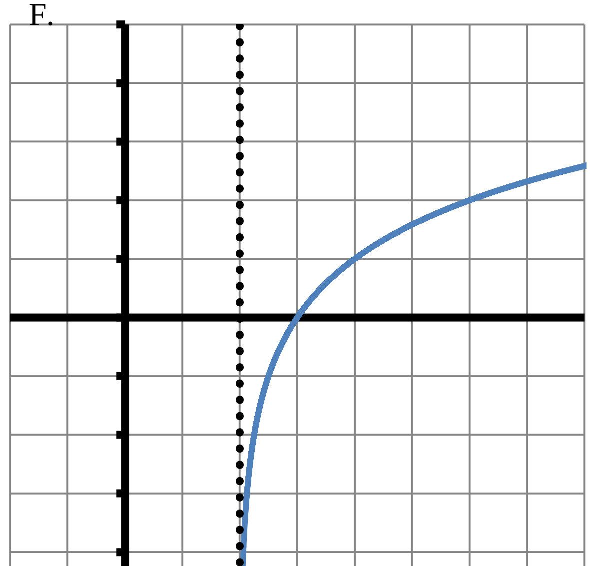
D.



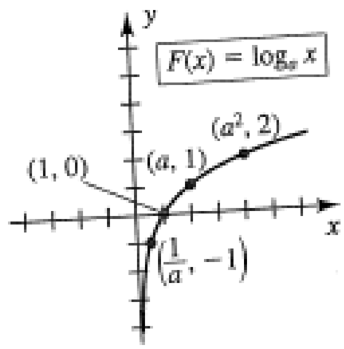
E.



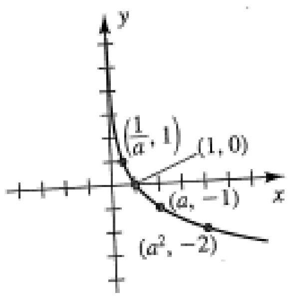
F.



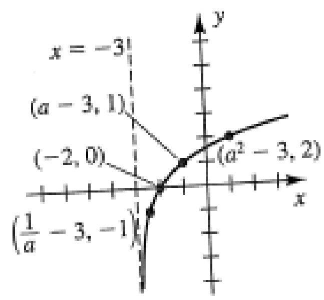
Example 4: Shown below is a graph of a function *F* (the original graph). Express the graphs of functions *f* in terms of *F*.



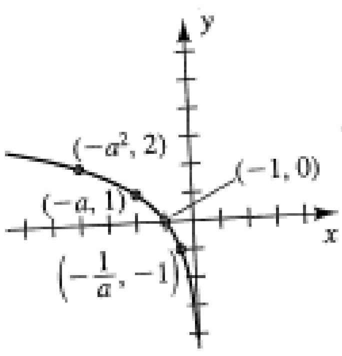
a)



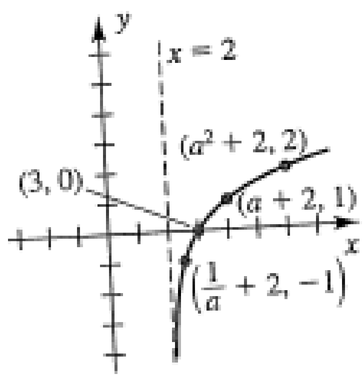
b)



c)



d)



When solving problems like Example 3 on the homework, you can either use transformations (like I did) or you can make new input/output tables for each new function to sketch/identify the graphs. Transformations will most likely be quicker, but it is your choice.

<u>Example 4:</u> Solve each of the following formulas for the specified variable. Keep in mind that logarithms (regardless of base) are functions, so multiplying or dividing a logarithm by any value only changes the output, not the input.

a. $A = Pe^{rt}$; solve for t

First: Isolate the power.

b. $N = n2^{\frac{t}{a}}$; solve for a

c) $Pc = M - Ce^{-kt}$; solve for t

d) $I = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$; solve for L (use the back of the paper)

e) $L = a(1 - be^{-kt})$; solve for k

f) $V = 0.78C(0.85)^{t-1}$; solve for t

<u>Example 5:</u> The bacteria count in a culture is given by the function $n(t) = 5e^{0.4t}$, where the time t is in hours and the population $n(t)$ is **<u>in millions</u>**.

a. What is the initial population?

b. When will the population reach one billion (round to the nearest hundredth of an hour)?

<u>Example 6:</u> Based on present birth and death rates, the population of Kenya is expected to increase according to the function $N(t) = 30.7e^{0.022t}$, with $N(t)$ in millions and $t = 0$ corresponding to 2000.

a. What was the population in 2000?

b. When will the population double?

c. When will the population reach 1.2 billion

<u>Example 7:</u> Parents of a newborn baby are given a gift of $20,000 to invest for their child's college fund. The parents find a fund that pays an average annual interest rate of 4%, compounded continuously.

a. Determine the amount of time needed for the investment to double (**<u>DO NOT APPROXIMATE</u>**).

b. If the initial amount is increased to $30,000, determine the amount of time needed for the investment to double (**<u>DO NOT APPROXIMATE</u>**).

c. Does the initial amount of an investment (the principle) have any effect on the doubling time?