MA 153 Lesson 38 Notes

Properties of Exponents:

a. Product Rule: $b^m \cdot b^n =$

b. Quotient Rule: $\frac{b^m}{b^n} =$

c. Power Rule: $(b^m)^n =$

The properties of logarithms parallel the properties of exponents.
Add exponents when multiplying, subtract exponents when dividing, multiply exponents when a power has been raised to another power.

Properties of Logarithms:

a. Product Rule:

$$\log_c(a \cdot b) = \log_c a + \log_c b$$
$$\log_c a + \log_c b = \log_c(a \cdot b)$$

b. Quotient Rule:

$$\log_c\left(\frac{a}{b}\right) = \log_c a - \log_c b$$
$$\log_c a - \log_c b = \log_c\left(\frac{a}{b}\right)$$

c. Power Rule:

$$\log_c(a^p) = p \cdot \log_c a$$
$$p \cdot \log_c a = \log_c(a^p)$$

Keep in mind that logarithms are exponents; this should make it easier to make the connection between the properties of exponents and the properties of logarithms.

(The proofs for these properties of logarithms are found on page 324 of the textbook. If interested, you may examine those proofs. If is more important that you are able to use the properties of logarithms than to be able to prove the properties.)

Also, notice that the properties of logarithms work in both directions. Logarithms of products, quotients, and powers can be expanded; sums, differences, and multiplies of logarithms can be condensed (assuming the bases are the same).

Example 1: Evaluate each expression (find the number value). Be aware of order of operations.

a. $\log_5\left(\frac{0.2}{125}\right)$

b. $\log_3\sqrt{27}$

c. $\log_6 3 + \log_6 12$

d. $\log_2 80 - \log_2 5$

e. $\log_2 12 - \log_2 15 + \log_2 40$

f. $2 \cdot \log_4 8$

g. $\log_2(4+4)$ $\xleftrightarrow{\text{compare}}$ h. $\log_2 4 + \log_2 4$

i. $\log_2(32-16)$ $\xleftrightarrow{\text{compare}}$ j. $\log_2 32 - \log_2 16$

<u>There is NO Sum Rule for Logarithms.</u>
<u>There is NO Difference Rule for Logarithms.</u>

Example 2: Express each in terms of logarithms of $w, x, y,$ or z (expand). **Keep in mind order of operations.**

a. $\log\left(\frac{x^3 w}{y^2 z^4}\right)$

b. $\ln\left(\frac{\sqrt[3]{z}}{x\sqrt{y}}\right)$

c. $\ln\left(x \cdot \sqrt[3]{\frac{y^4}{z^5}}\right)$

d. $\log\sqrt{\frac{x^2-1}{x^2+1}}$

Example 3: Write each expression as one logarithm (condense). **Keep in mind order of operations.**

a. $3\ln x - \frac{1}{2}\ln 2y + 4\ln\frac{z}{3}$

b. $5\log_3 x - \frac{1}{2}\log_3(9x^2) - 3\log_3\frac{5}{x}$

c. $\ln y^3 + \frac{1}{3}\ln(x^3y^6) - 5\ln y$

d. $\log\left(\frac{x^2}{y^3}\right) + 4\log y - 6\log\sqrt{xy}$