Lesson 39

Properties of Logarithms: **(Review)**

a. Product Rule:

$$\log_c(a \cdot b) = \log_c a + \log_c b$$
$$\log_c a + \log_c b = \log_c(a \cdot b)$$

Order is not important when multiplying or adding, so changing the order of the factors or terms does not change the answer.

b. Quotient Rule:

$$\log_c\left(\frac{a}{b}\right) = \log_c a - \log_c b$$
$$\log_c a - \log_c b = \log_c\left(\frac{a}{b}\right)$$

Order is important when dividing and subtracting; the numerator is always associated with the first log term listed (or the term listed first is always associated with the numerator).

c. Power Rule:

$$\log_c(a^p) = p \cdot \log_c a$$
$$p \cdot \log_c a = \log_c(a^p)$$

Keep in mind that the properties of logarithms work in both directions. Logarithms of products, quotients, and powers can be expanded; sums, differences, and multiplies of logarithms can be condensed (assuming the bases are the same).

There is NO Rule for a Logarithm of a SUM.

There is NO Rule for a Logarithm of a DIFFERENCE.

<u>Methods for solving logarithmic equations:</u>

1. If a logarithm equals a logarithm, and the bases are the same, set the arguments equal to each other and solve.
   - Example: $\log(3-5x)=\log(x^2-3)$

2. If a logarithm equals a number, convert from logarithmic form to exponential form and solve.
   - Example: $\ln(x^2-6x)=1$
     Both do check (make a positive argument)

3. If a logarithm equals a logarithm, and at least one log is raised to a power, set one side of the equation to zero and solve by factoring.
   - Example: $\ln x^2=(\ln x)^2$
     Hint: We must solve by using factoring.

Possible solutions to logarithmic equations must ALWAYS be checked to verify that they lead to positive arguments (positive inputs).

If an equation has more than one logarithm on either side, use the Properties of Logarithms to **simplify as much as possible before solving**.

REGARDLESS OF WHICH METHOD YOU USE TO SOLVE, YOU MUST CHECK YOUR SOLUTIONS TO VERIFY THEY DO NOT RESULT IN A NEGATIVE ARGUMENT.

Example 1: Solve the equations and **CHECK YOUR SOLUTIONS**.

a. $\log_5 10 - \log_5 50 = \log_5 0.5 + \log_5(4x + 6)$

b. $2 = \log_3 x - \log_3(x - 1)$

c. $\ln\sqrt{x} = \sqrt{\ln x}$

Hint: Factoring will again be used after both sides are ‘squared’.

d. $\log(5 - 6x) + \log(-x) = \log 5 + \log(0.2)$

e. $\ln(x - 2) + \ln x = 1$

Hint: Remember to check!

f. $\ln(x+2) - \ln x = \ln e$

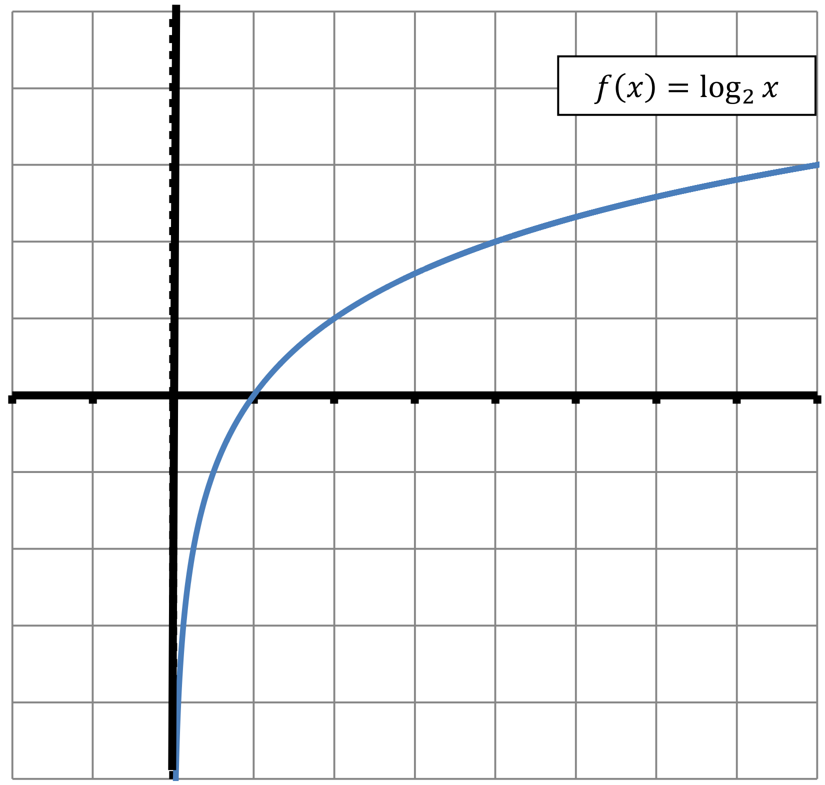
g. $3\log x - \log x = 4\log\sqrt{10}$

h. $(\ln x)^4 = \ln x^8$

i. $\log_2(x+3) = \log_2(x-3) + \log_3 9 + 4^{\log_4 3}$

Example 2: Given below is the graph of the function $f(x) = \log_2 x$. Simplify the following functions (*a* – *c*) using the properties of logarithms, and then identify the correct graph of each function on page 7 using transformations.

<u>**Inputs**</u> $(0, \infty)$	<u>**Outputs**</u> $(-\infty, \infty)$
x	$f(x) = \log_2 x$
$x \to 0$	$f(x) \to -\infty$
$\frac{1}{8}$	$f\left(\frac{1}{8}\right) = -3$
$\frac{1}{4}$	$f\left(\frac{1}{4}\right) = -2$
$\frac{1}{2}$	$f\left(\frac{1}{2}\right) = -1$
1	$f(1) = 0$
2	$f(2) = 1$
4	$f(4) = 2$
8	$f(2) = 3$
$x \to \infty$	$f(x) \to \infty$

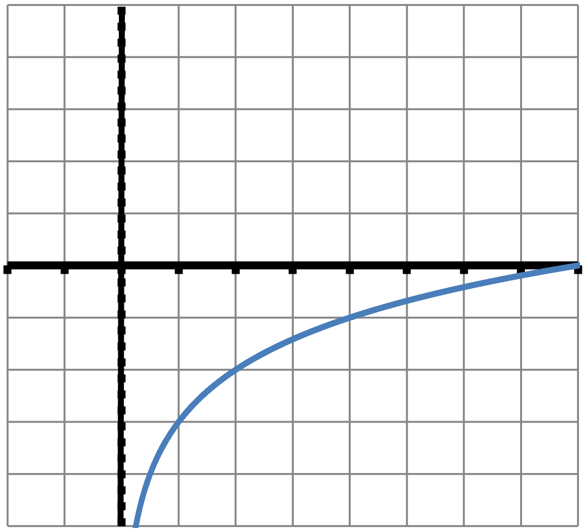


a. $g(x) = \log_2 x^3$

b. $h(x) = \log_2\left(\frac{1}{x}\right)$

c. $j(x) = \log_2(8x)$

A.



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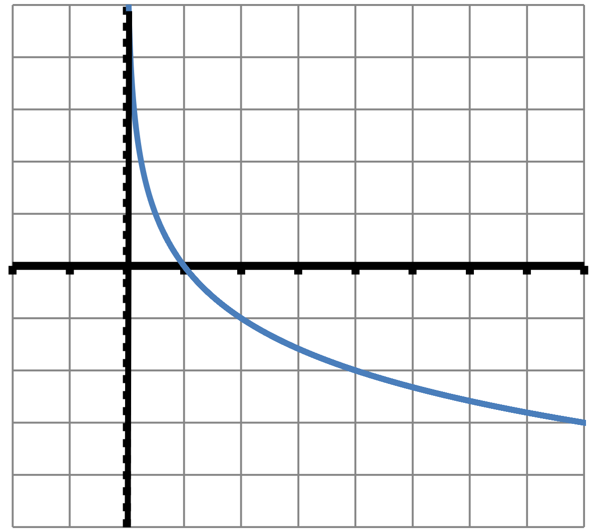

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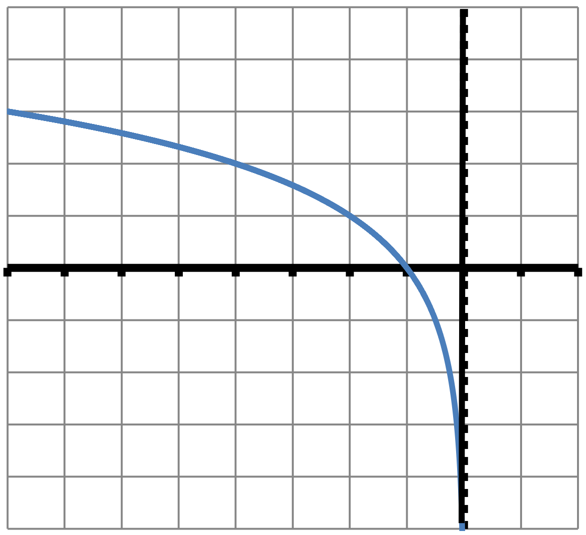

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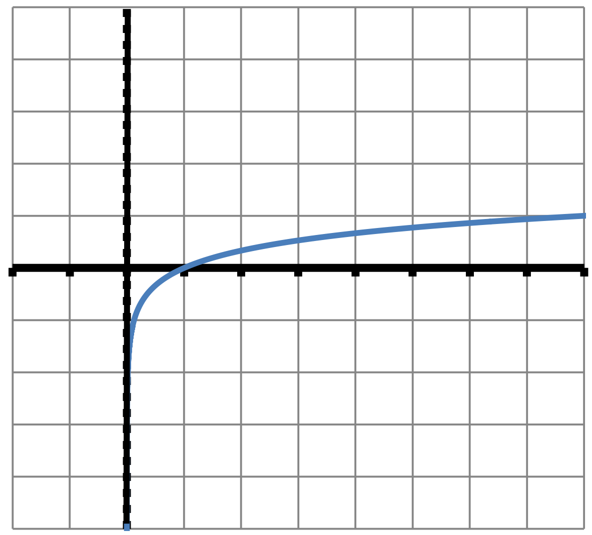

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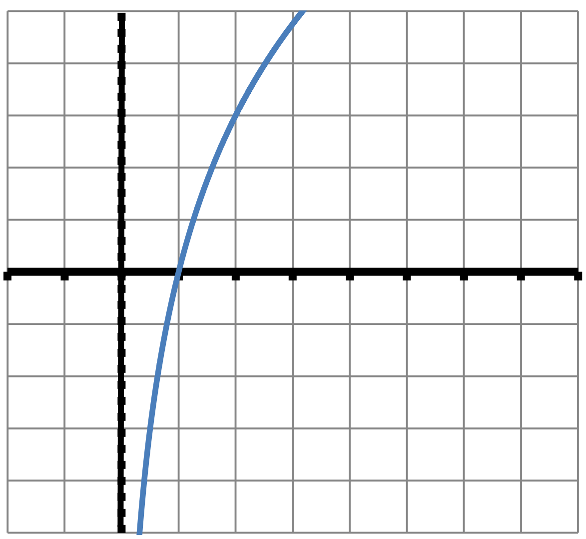

F.