Lesson 40

When solving algebraic equations, the number one rule is:

Whatever you do to one side, you must do to the other.

$$a = b$$
$$a + c = b + c$$

$$a = b$$
$$a \cdot c = b \cdot c$$

$$a = b$$
$$a^m = b^m$$

$$a = b$$
$$\sqrt[n]{a} = \sqrt[n]{b}$$

The same is true for logarithms; if you take the log (of any positive base other than 1) of one side of an equation, you must take the logarithm (of same base) of the other side of the equation.

$$a = b$$

$$\log_c a = \log_c b$$

$$\log a = \log b$$

$$\ln a = \ln b$$

As long as a and b are both positive (both logarithms defined), you can take the logarithm (of same base) of both sides of an equation and the two sides will still be equal. This gives us another method for solving exponential equations. All the Properties of Logarithms from Lessons 38 & 39 still hold true.

<u>Methods for solving exponential equations:</u>

1. If an exponential expression is equal to another exponential expression, and both expressions **<u>CAN</u>** be written in terms of the same base, set the exponents equal to each other and solve.
   - Example: $2^{x-1} = 4^x$

2. If an exponential expression is equal to another exponential expression, and both expressions **<u>CANNOT</u>** be written in terms of the same base, take the logarithm (of any base) of both sides and solve.
   - Example: $2^{x-1} = 3^x$

3. If <u>an exponential expression is equal to a number</u>, convert from exponential form to logarithmic form and solve.
   - Example: $3^{1-2x} = 5$

Keep in mind that the first method (making the bases the same) will not work for every exponential equation (such as the second and third examples given above).

Also, depending on which method you choose, **your answer may be in a different form than the correct option on the Final Exam.**

<u>Problem #46 from the Practice Final Exam:</u>

Solve for x:

$$3^{x-5} = 4$$

I will demonstrate two ways to solve this equation.

A. $x = \log 4 + 5 \log 3$

B. $x = 5 + \log\left(\frac{4}{3}\right)$

C. $x = 5 + \frac{\log 4}{\log 3}$

D. $x = 5 + \log 4$

E. $x = \frac{5+\log 4}{\log 3}$

Notice: The answer choices are given in terms of common logs (base 10). Try to use a method that involves common logs.

Method 1:

Method 2:

<u>Example 1:</u> Solve the following exponential equations using any method. Find the **<u>exact solution</u>** and a **<u>three-decimal-place approximation</u>** of each, if necessary.

a. $5^x = 50^{x+1}$

b. $5^{x+4} = 125^{1-3x}$

c. $7^{\frac{x}{2}} = 5^{1-x}$

d. $e^{1-x} = 5$

e. $8^{2x+2} = 16^{x-3}$

f. $12^{1-x} = 6^{x}$

g. $10^{x-2} = 4$

When solving exponential equations, we do not have to check our answers like we do with logarithmic equations because there are no restrictions on the domain of exponential equations. However, you can still check your answers to verify they are correct.

Example 2: Solve the exponential equation $3^{x} = 729$ using the following methods:

a. convert to logarithmic form to isolate x

b. take the common log of both sides of the equation, then isolate x

c. take the natural log of both sides of the equation, then isolate x

Change of Base Formula:

- formula for re-writing a logarithm as a ratio of logarithms with bases that can be approximated
- the numerator is **always** the logarithm of the original input and the denominator is **always** the logarithm of the original base
    - $\log_{base}(input) = \frac{\log_{new\ base}(input)}{\log_{new\ base}(old\ base)}$
- $\log_a b = \frac{\log_c b}{\log_c a} = \frac{\log b}{\log a} = \frac{\ln b}{\ln a}$
        - Examples: $\log_5 7$ $\log_4 8$

- just like the Properties of Logarithms, it is important to be able to go in both directions with the Change of Base Formula; changing one logarithm into a ratio of logs and converting a ratio of logs into one logarithm
        - Examples: $\log_7 21$ $\frac{\log_7 16}{\log_7 2}$

- keep in mind that the Change of Base Formula is **NOT** the same as the Quotient Rule for Logarithms
    - Example: Simplify using the **Quotient Rule first**, then the Change of Base Formula.
        - Example: $\log_5 72 - \log_5 8$

Example 3: Re-write the following logarithmic expressions using the change of base formula, and approximate to three-decimal places, if necessary.

a. $\log_3 6$

b. $\log_5 0.5$

c. $\frac{\log_7 81}{\log_7 3}$

d. $\frac{\log_3 4}{\log_3 16}$

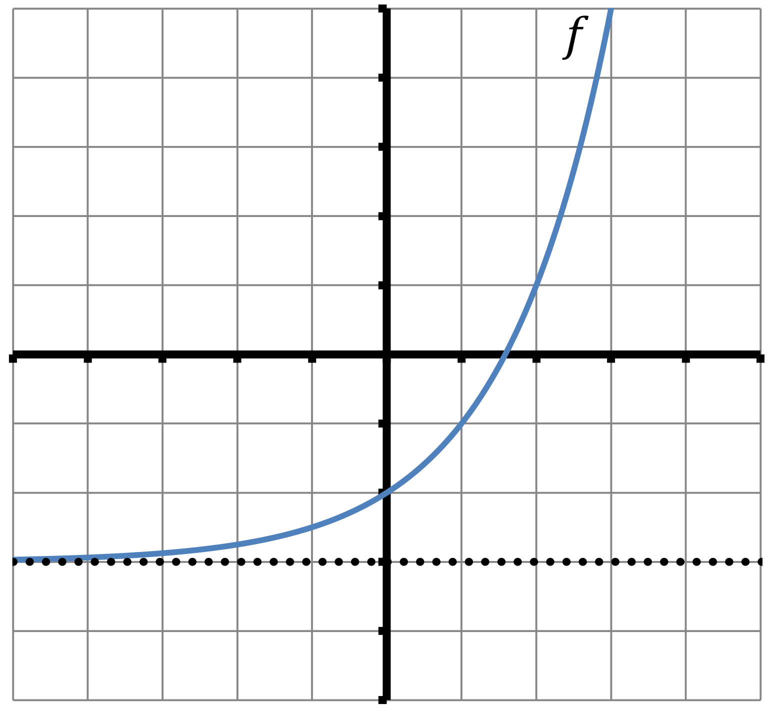
Example 4: Find the domain, range, zeros, and y-intercepts of the following functions. Approximate the zeros to four decimal places.

a. $f(x) = 5^x - 10$

b. $g(x) = 3^x + 5$

Example 5: Using the table/graph of $f(x) = 2^x - 3$, find the following:

Inputs	**Outputs**
x	$f(x) = 2^x - 3$
$x \to -\infty$	$f(x) \to -3$
-4	$-2\frac{15}{16}$
-3	$-2\frac{7}{8}$
-2	$-2\frac{3}{4}$
-1	$-2\frac{1}{2}$
0	-2
1	-1
2	1
3	5
4	13
$x \to \infty$	$f(x) \to \infty$



Domain:

Range:

$f(x) = 0$ when $x =$

x-intercepts:

$f(x) > 0$:

$f(0) =$

$f(x) < 0$:

y-intercepts:

Increasing:

Decreasing:

$f(x)$ is even/odd/neither: $f(-1) =$ $f(1) =$

Asymptotes:

Doubling Time:

- the amount of time needed for some initial quantity to grow to twice its original amount
- doubling time does **NOT** dependent on the initial quantity

Example 6: Find the doubling time for an investment, compounded continuously ($A = Pe^{rt}$) at 5%.

Example 7: Find the time needed for an investment compounded semiannually $\left(A = P\left(1 + \frac{r}{n}\right)^{nt}\right)$ at 5% to triple.

Half-Life:

- the amount of time needed for some initial quantity to decay to half its original amount
- if the half-life of a substance is 21 days and the initial amount is 10 units, the substance decays in the following way:

Day	0	21	42	63	84	…	
Amount	$10\ u$	$5\ u$	$2.5\ u$	$1.25\ u$	0.625	…	

Example 8: The trade-in value of a particular automobile t years after it is purchased is $V(t) = 0.75C(0.85)^{t-1}$, where C is the original purchase price. When will the trade-in value be half of the purchase price? Round to the nearest hundredth of a year.

Example 9: If we start with c milligrams of the polonium isotope ^{120}Po, the amount remaining after t days is given by $A = ce^{-0.00495t}$. Find the half-life of this isotope. Round to the nearest tenth of a day.