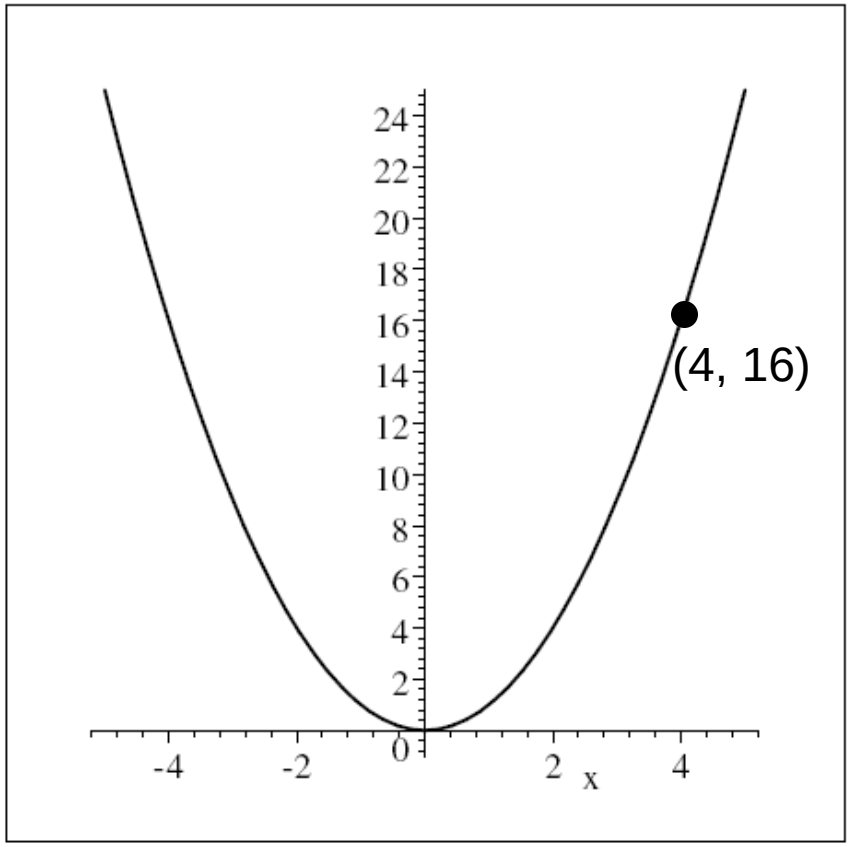
Shifting

$$f(x) = x^2$$

x	$f(x)$
0	0
1 or -1	1
2 or -2	4
3 or -3	9
4 or -4	16

$$f_1(x) = x^2 + 3$$

x	$f(x) + 3$
0	3
1 or -1	4
2 or -2	7
3 or -3	12
4 or -4	19


24
22
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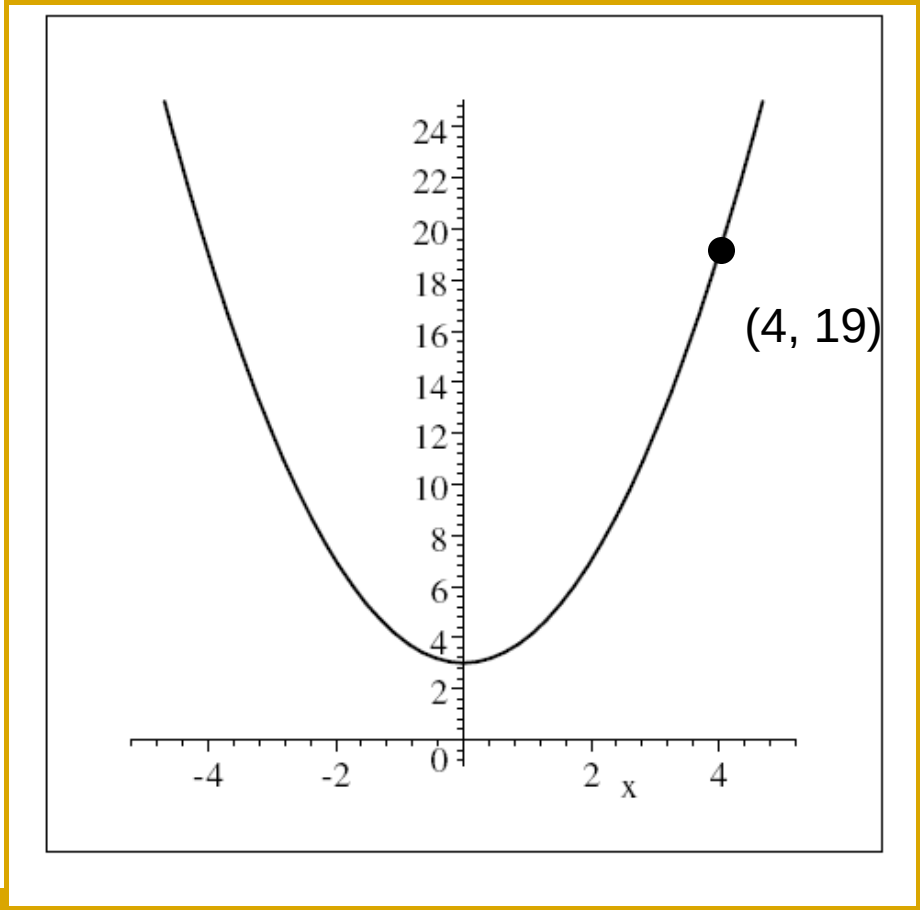
-4
-2
2
4
x
(4, 16)


24
22
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12
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8
6
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2
0
-4
-2
2
4
x
(4, 19)

$$f(x) = x^2$$

$$f_1(x) = x^2 + 3$$

Vertical Shifting

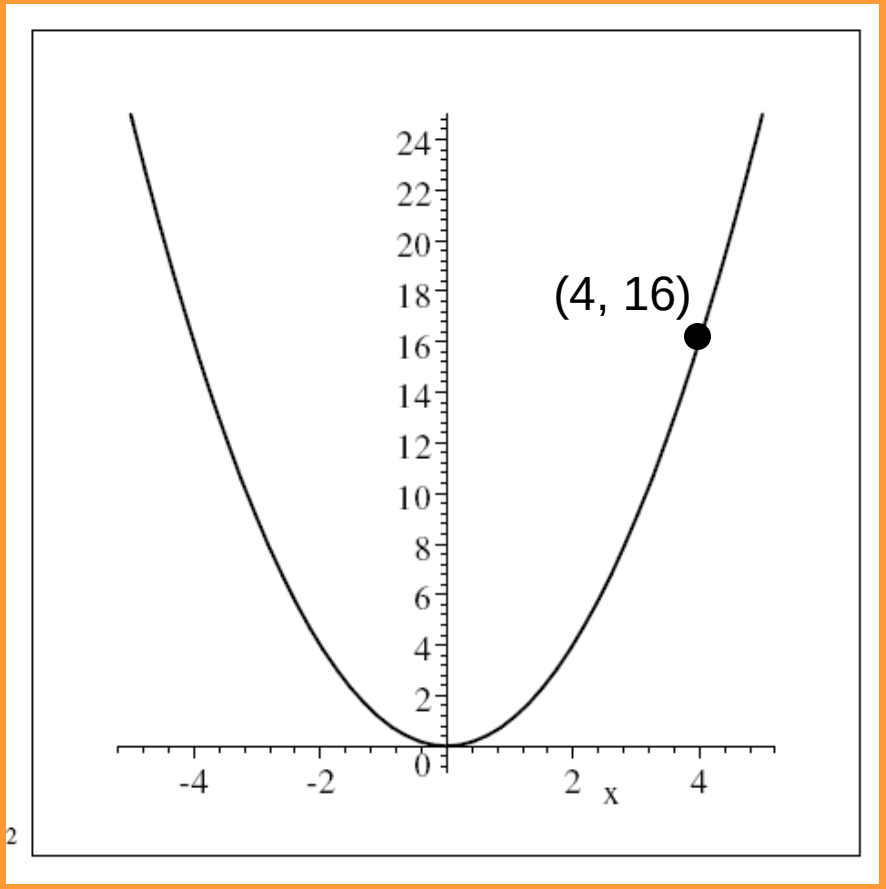
- If c is positive, then a graph of $y = f(x) + c$ will be shifted c units vertically upward.
- If c is positive, then a graph of $y = f(x) – c$ will be shifted c units vertically downward.

$f(x) = x^2$

x	*f(x)*
0	0
1 or -1	1
2 or -2	4
3 or -3	9
4 or -4	16

$f_2(x) = (x+3)^2$

x	*f(x)*
-3	0
-4 or -2	1
-5 or -1	4
-6 or 0	9
- 7 or 1	16


(4, 16)
24
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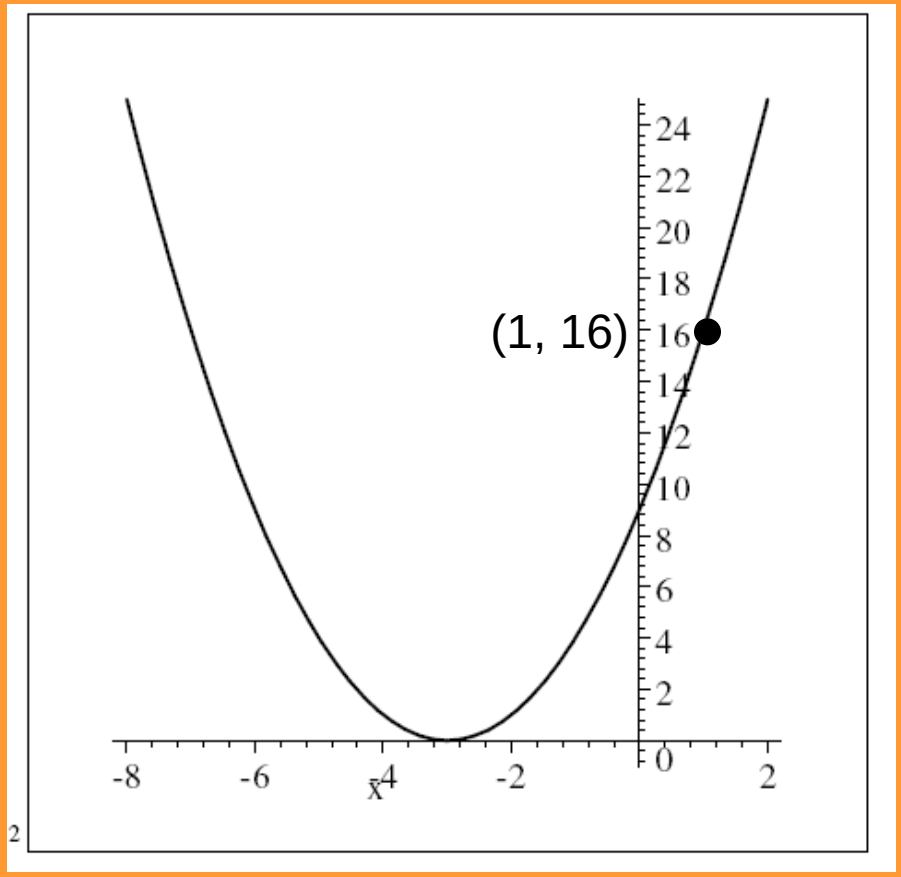
8
6
4
2
0
-4
-2
2
4
x


(1, 16)
24
22
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12
10
8
6
4
2
0
-8
-6
-4
-2
2
x

$$f(x) = x^2$$

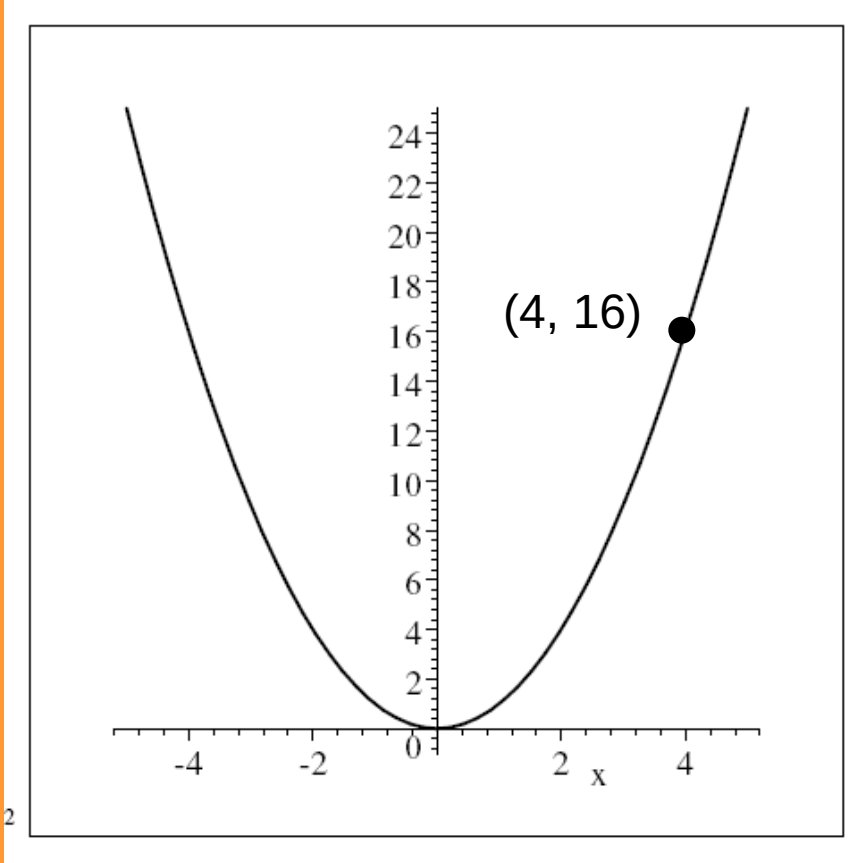
$$f_2(x) = (x+3)^2$$

$$f(x) = x^2$$

x	*f(x)*
0	0
1 or -1	1
2 or -2	4
3 or -3	9
4 or -4	16

$$f_3(x) = (x-3)^2$$

x	*f(x)*
3	0
2 or 4	1
1 or 5	4
0 or 6	9
-1 or 7	16


24
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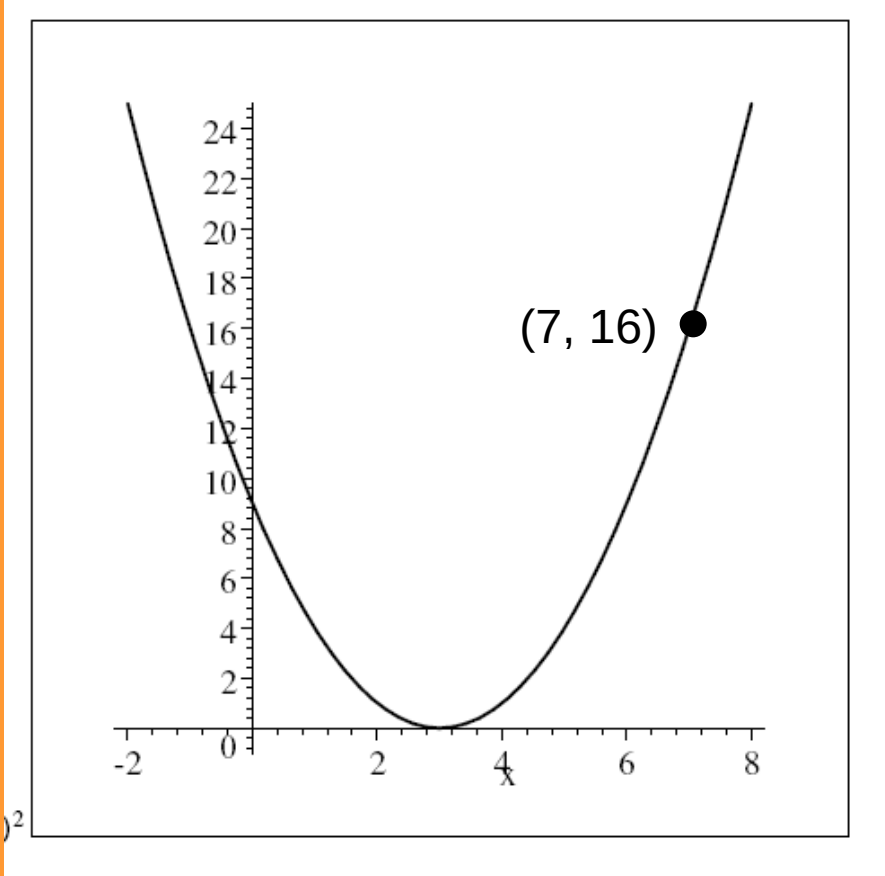
-4
-2
2
4
x
(4, 16)


24
22
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16
14
12
10
8
6
4
2
0
-2
2
4
6
8
x
(7, 16)

$$f(x) = x^2$$

$$f_3(x) = (x-3)^2$$

Horizontal Shifting

- If c > 0, the graph of $y = f(x - c)$ will be shifted horizontally *c* units **right** from the graph of $y = f(x)$.
- If $c > 0$, the graph of $y = f(x + c)$ will be shifted horizontally *c* units **left** from the graph of $y = f(x)$.

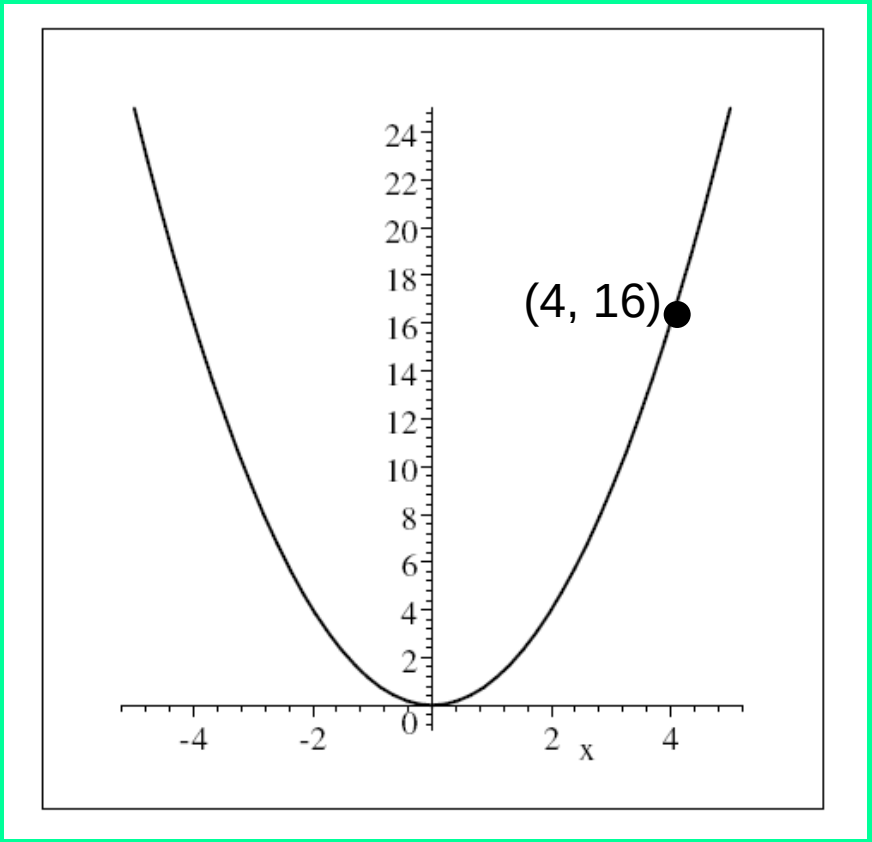
REFLECTIONS

$$f(x) = x^2$$

x	*f(x)*
0	0
1 or -1	1
2 or -2	4
3 or -3	9
4 or -4	16

$$f_4(x) = -x^2$$

x	*f(x)*
0	0
1 or -1	-1
2 or -2	-4
3 or -3	-9
4 or -4	-16


24
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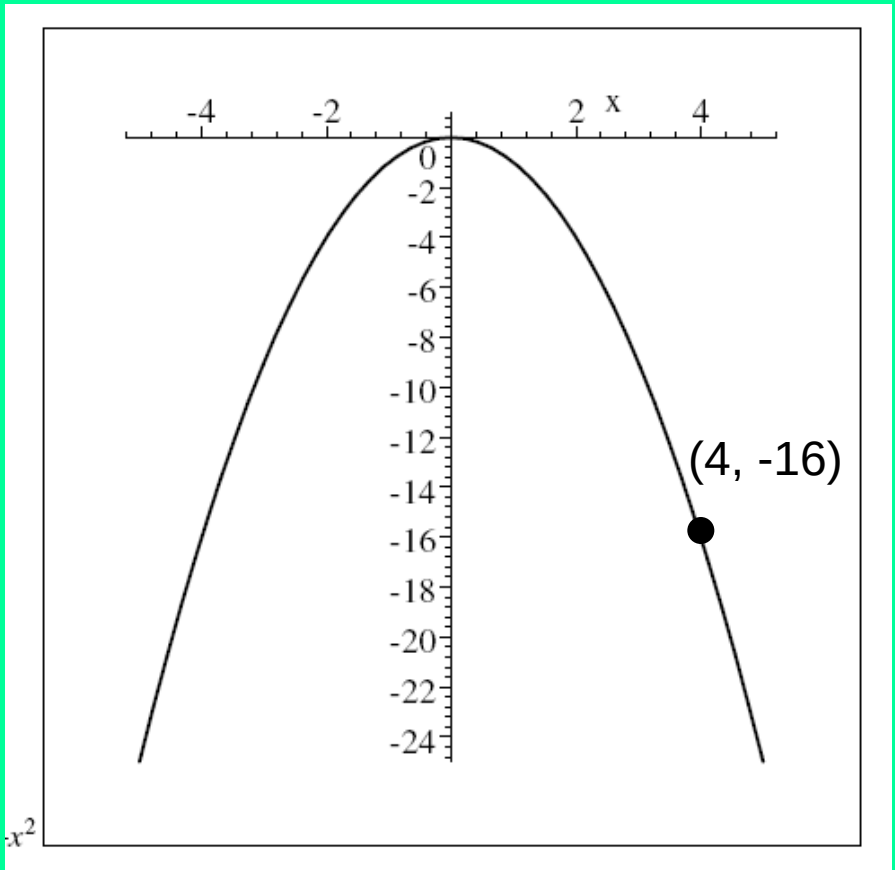
14
12
10
8
6
4
2
0
-4
-2
2
x
4
(4, 16)

$$f_4(x) = -x^2$$

$$f(x) = x^2$$


-4
-2
2
x
4
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
(4, -16)
$-x^2$

Vertical Reflection

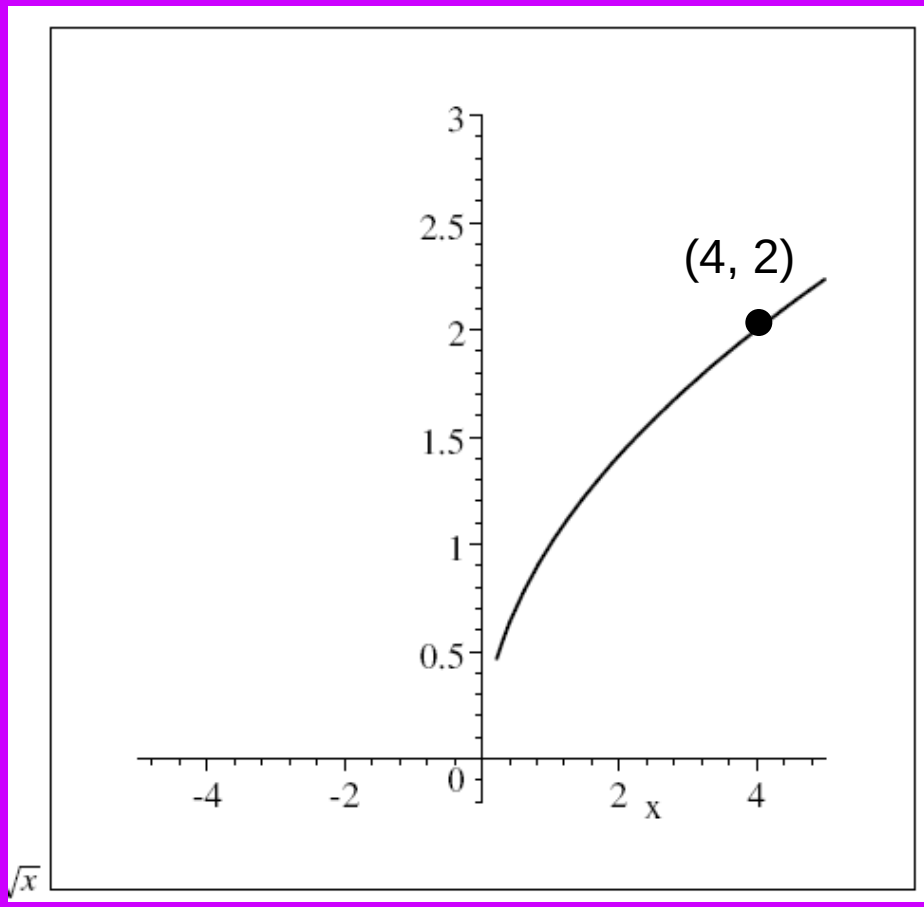
- The graph of $y = -f(x)$ will be reflected vertically over the x-axis from $y = f(x)$.

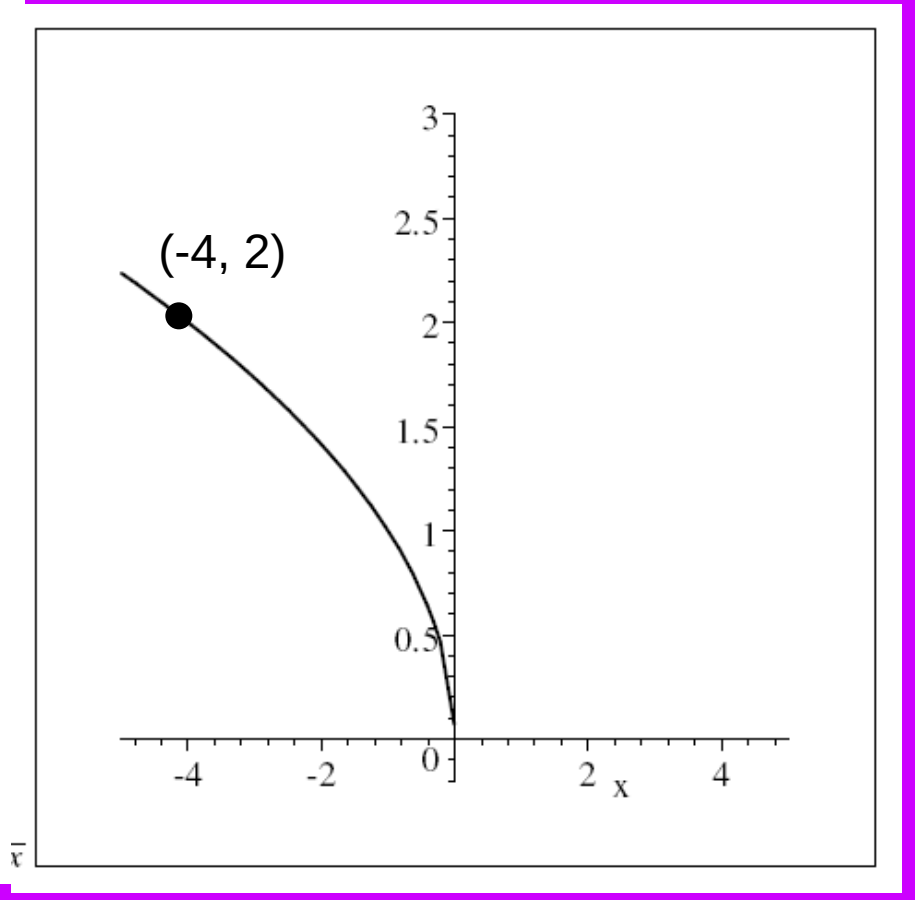
$$g(x) = \sqrt{x}$$

x	$f(x)$
0	0
1	1
4	2
9	3
16	4

$$g_1(x) = \sqrt{-x}$$

x	*f(x)*
0	0
-1	1
-4	2
-9	3
-16	4





$$g(x) = \sqrt{x}$$

$$g_1(x) = \sqrt{-x}$$

Horizontal Reflection

- The graph of $y = f(-x)$ would be reflected horizontally about the y-axis from $y = f(x)$.

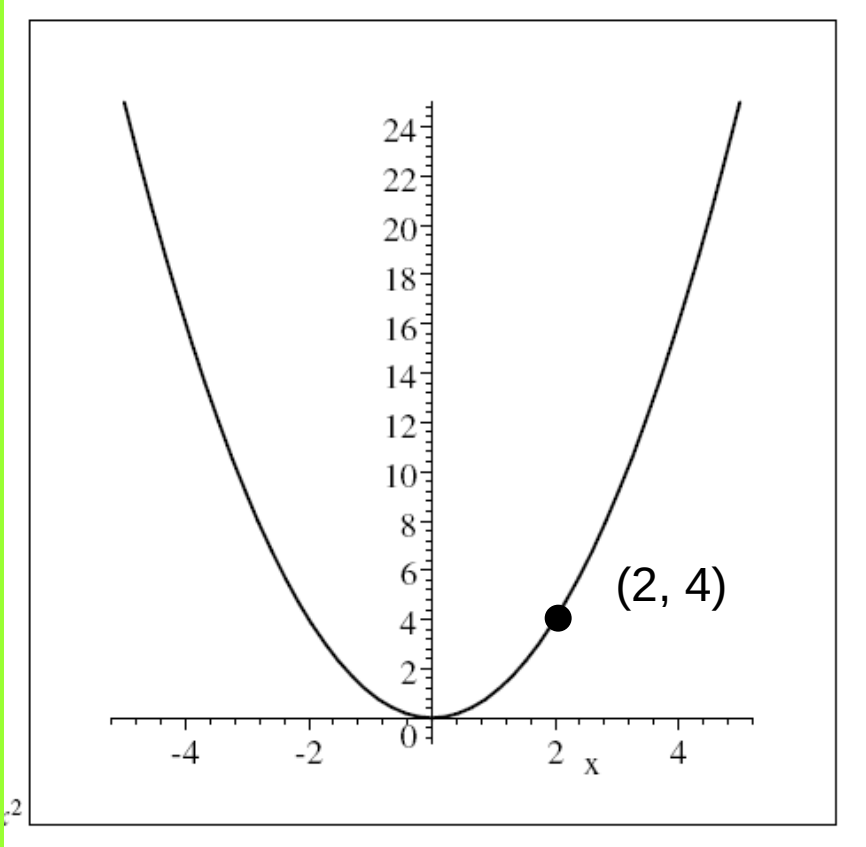
STRETCHING/COMPRESSING

$$f(x) = x^2$$

x	*f(x)*
0	0
1 or -1	1
2 or -2	4
3 or -3	9
4 or -4	16

$$f_5(x) = 2x^2$$

x	*f(x)*
0	0
1 or -1	2
2 or -2	8
3 or -3	18
4 or -4	32


24
22
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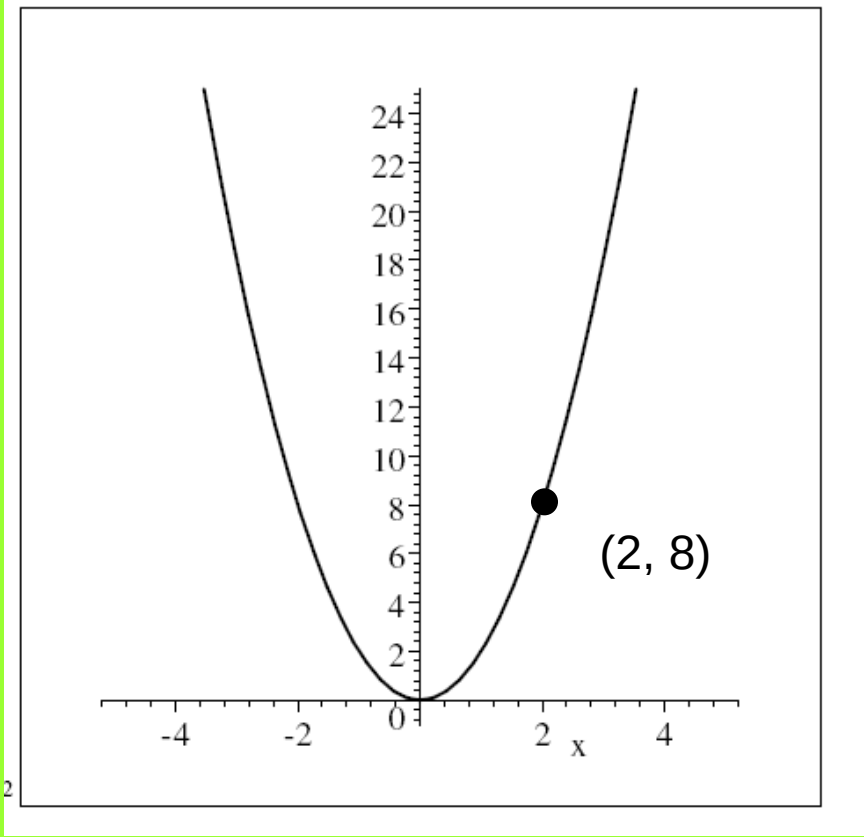
14
12
10
8
6
4
2
0
-4
-2
2
x
4
(2, 4)

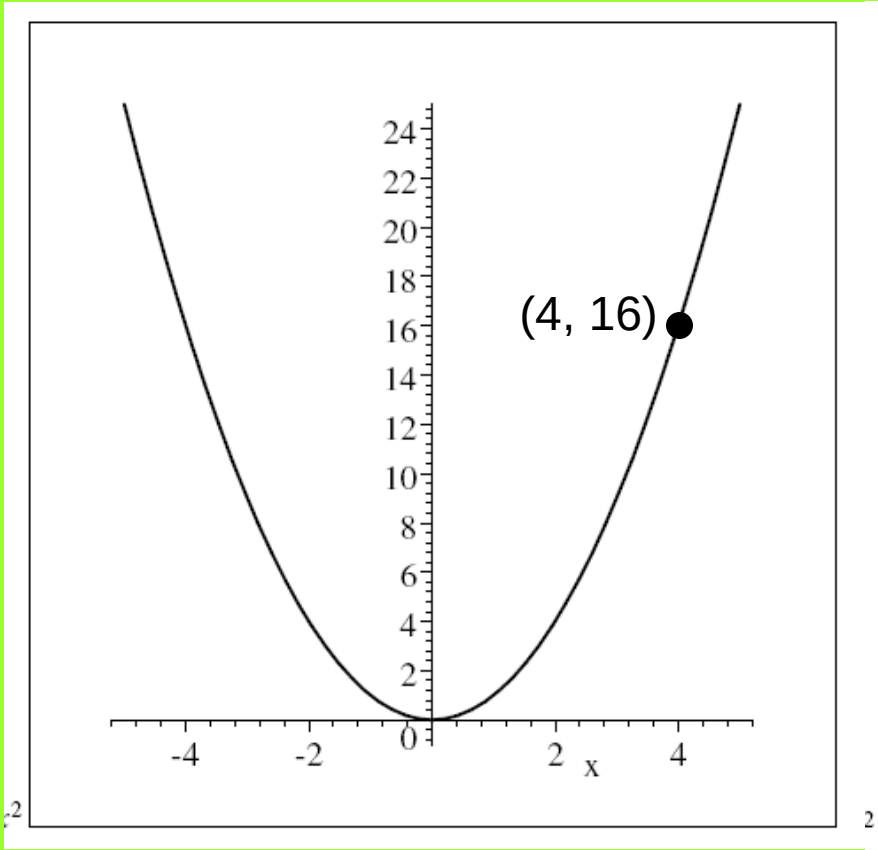

24
22
20
18
16
14
12
10
8
6
4
2
0
-4
-2
2
x
4
(2, 8)

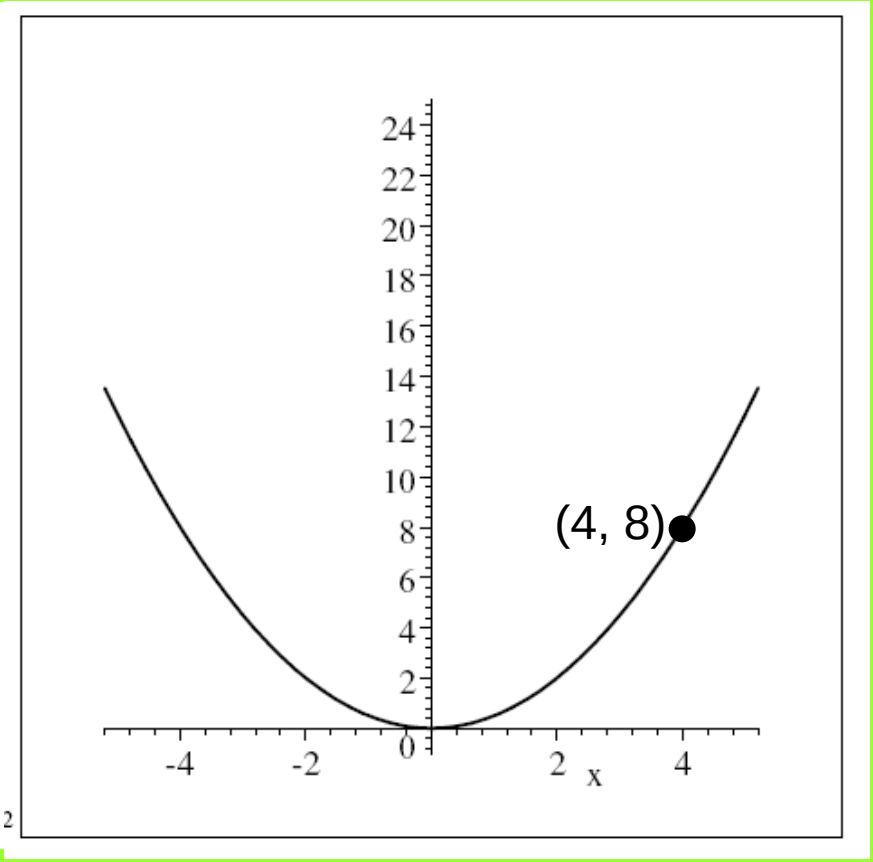
$$f(x) = x^2$$

$$f_5(x) = 2x^2$$

$$f_6(x) = \frac{1}{2}x^2$$

x	*f(x)*
0	0
1 or -1	½
2 or -2	2
3 or -3	9/2
4 or -4	8





$$f(x) = x^2$$

$$f_6(x) = \frac{1}{2}x^2$$

Vertical Stretch or Compression

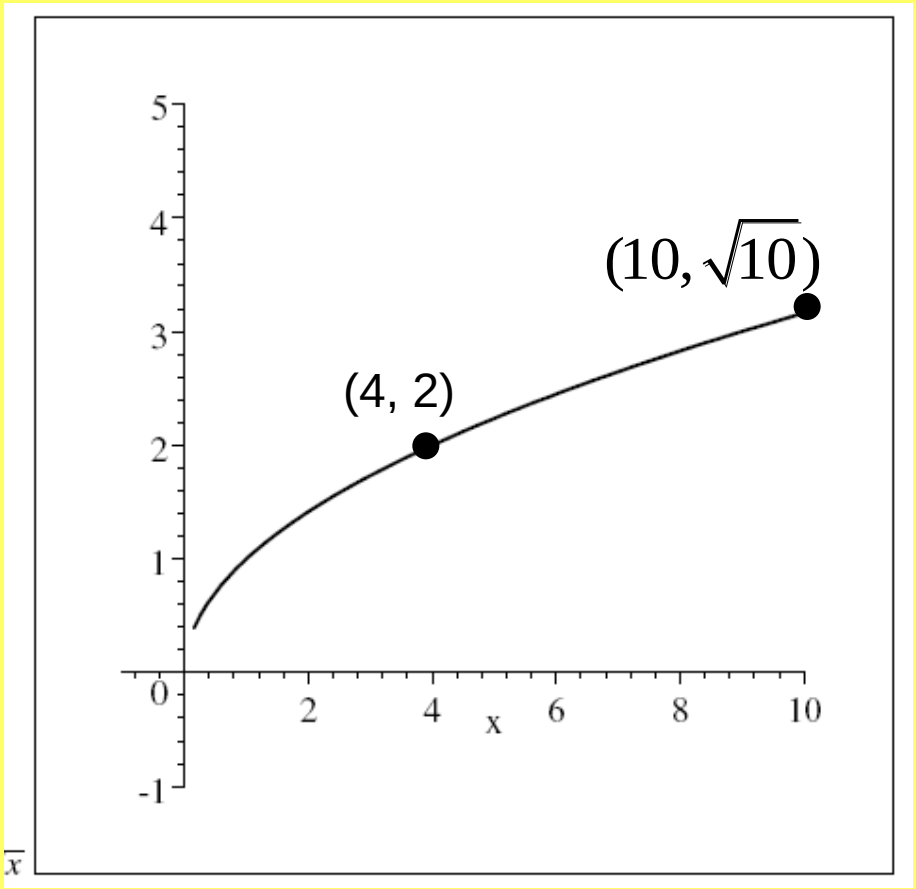
- If c > 1, the graph of $y = c\ f(x)$ will be stretched vertically by a factor of c from the graph of $y = f(x)$.
- If $0 < c < 1$, the graph of $y = c\ f(x)$ will be compressed vertically by a factor of c from the graph of $y = f(x)$.

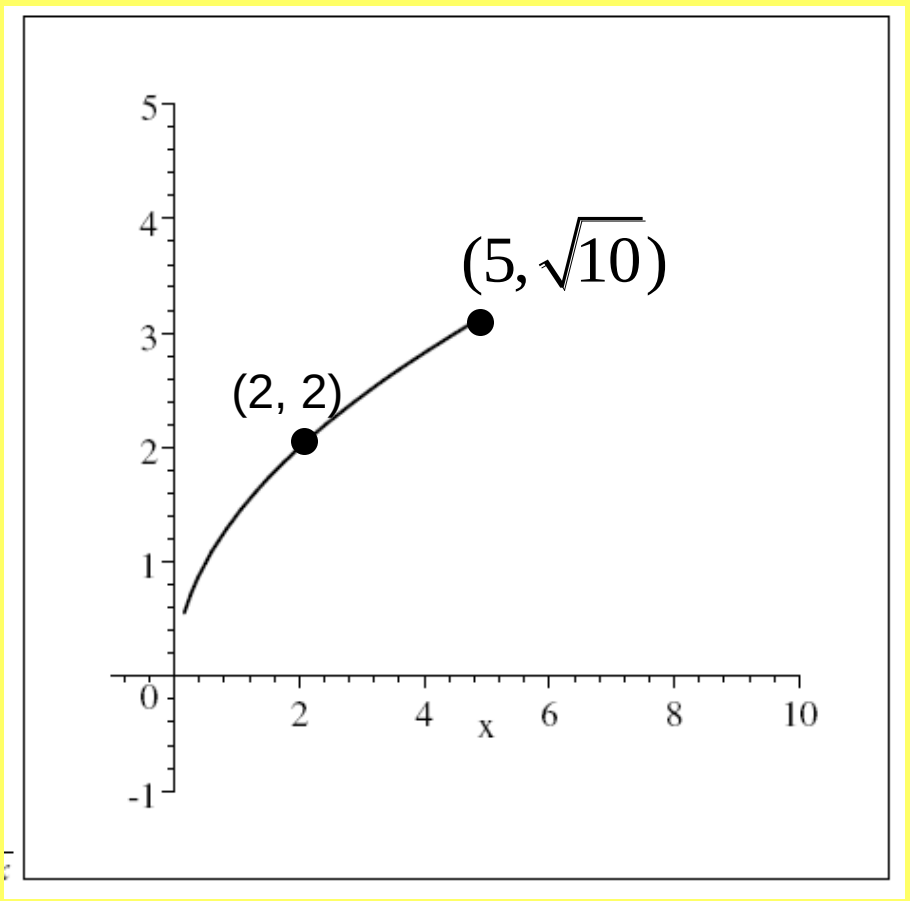
x	*f(x)*
0	0
1	1
4	2
9	3
16	4

x	*f(x)*
0	0
½	1
2	2
9/2	3
8	4

$$g(x) = \sqrt{x}$$

$$g_2(x) = \sqrt{2x}$$





$$g(x) = \sqrt{x}$$

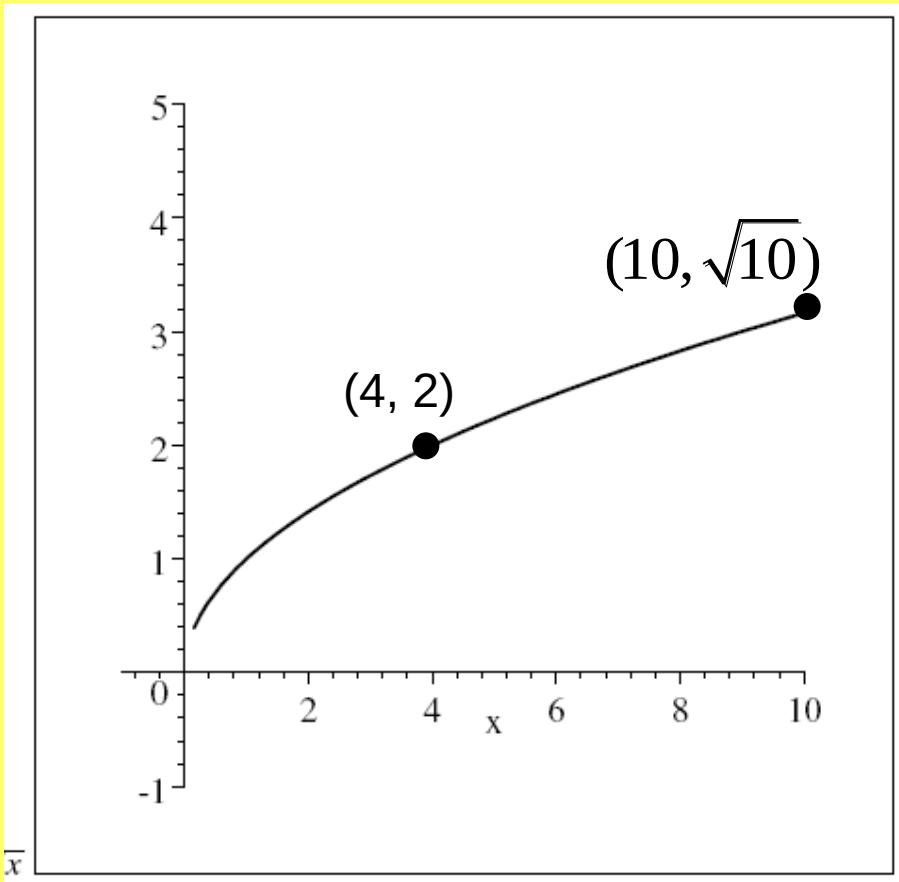
$$g_2(x) = \sqrt{2x}$$

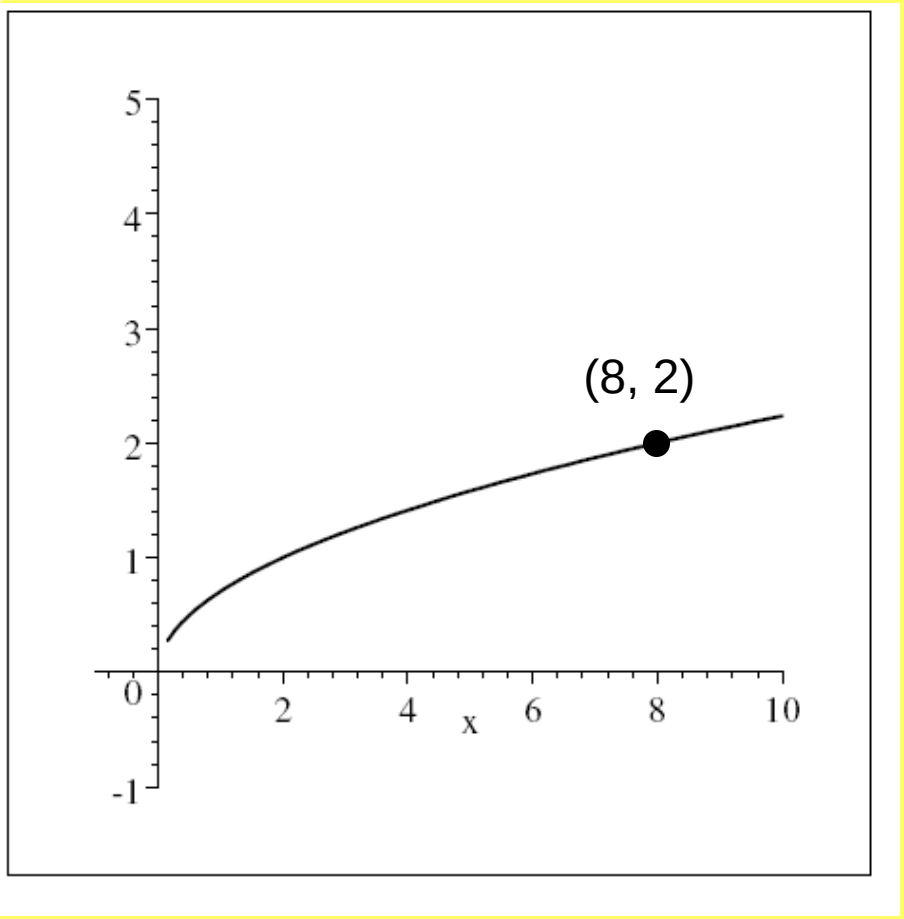
x	$f(x)$
0	0
1	1
4	2
9	3
16	4

x	f(x)
0	0
2	1
8	2
18	3
32	4

$$g(x) = \sqrt{x}$$

$$g_3(x) = \sqrt{\frac{1}{2}x}$$





$$g(x) = \sqrt{x}$$

$$g_3(x) = \sqrt{\frac{1}{2}x}$$

Horizontal Stretch or Compression

- If $c > 1$, the graph of $y = f(cx)$ will be horizontally **compressed** by a factor of $1/c$ from the graph of $f(x)$.
- If $0 < 1/c < 1$, the graph of $y = f((1/c)x)$ will be horizontally **stretched** by a factor of c from the graph of $f(x)$.