MA 15300 Exam 3A Summer 2014

Name: ____Purdue ID____________________

Instructions:

1. Fill in your name and Purdue ID above.
2. It is suggested you use a # 2 pencil, not ink.
3. There are 15 questions on the exam. Circle the capital letter in front of your choice for the answer. Do all of your work on the question sheets. Your work will not be graded, but I will examine your work to give you 'feedback'.
4. All questions are worth the same. **Please answer every question**. There is no penalty for guessing.
5. A TI-30XA scientific calculator is the ONLY calculator that may be used on the exam. **No other calculators are allowed. Cell phones, iPods, books, and scrap paper are also NOT allowed.**
6. The exam is self-explanatory. Do **NOT** ask any questions about any of the exam problems unless you believe there is a printing error or typing error.
7. When you are finished, take you exam to your instructor.

Circle your section number.

8:40 – 9:40 001 9:50 – 10:50 002

Use the functions, $f(x)=\dfrac{x-16}{x+5}$ and $g(x)=x^2-9$ to answer questions #1 and #2:

1. Find $(f-g)(-4)$.

$$=f(-4)-g(-4)$$
$$=\left(\frac{-4-16}{-4+5}\right)-((-4)^2-9)$$
$$=\left(\frac{-20}{1}\right)-(16-9)$$
$$=-20-7$$
$$=-27$$

A. -13

B. -27

C. -33

D. 13

E. None of the above.

2. Find x, such that $(f\circ g)(x)=0$.

$$f(g(x))=0$$
$$f(x^2-9)=0$$
$$\frac{(x^2-9)-16}{(x^2-9)+5}=0$$
$$\frac{x^2-25}{x^2-4}=0 \quad \text{Denominator cannot equal 0.}$$
$$x^2-25=0$$
$$(x+5)(x-5)=0$$
$$x+5=0 \qquad x-5=0$$
$$x=-5, \qquad x=5$$

A. $x=-2,2$

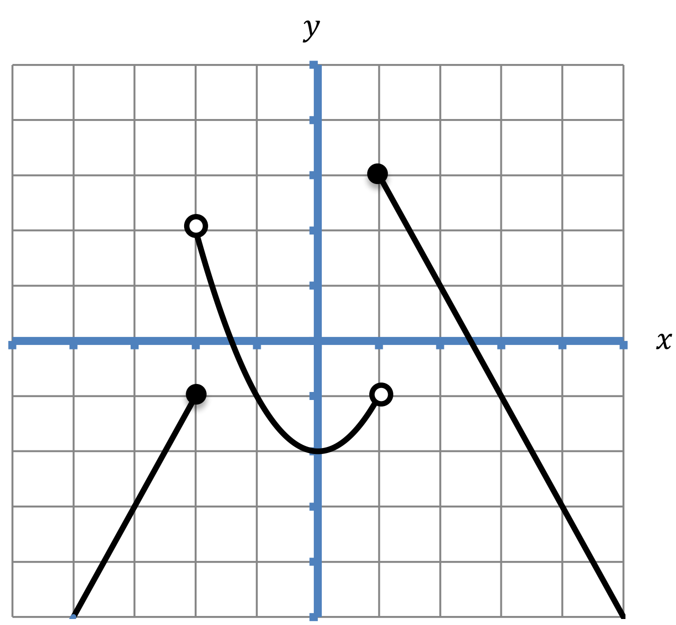
B. $x=-3,3$

C. $x=-5,5$

D. $x=16$

E. $(f\circ g)(x)\neq 0$

3. Which of the following statements about the graph of f is/are true?



I.	Decreasing intervals: $(-\infty,-2]\cup[0,1)$
II.	Domain: $(-\infty,3]$
III.	y-intecept: $(0,-2)$

A. I only

B. III only

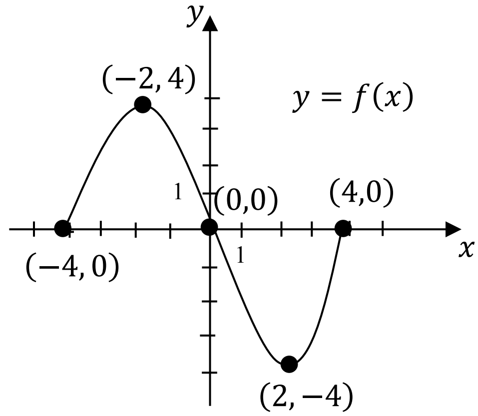
C. I and II only

D. I and III only

E. I, II, and III

I is not true. The function decreases on the intervals $(-2,0]$ and $[1,\infty)$.
II is not true. The domain is all real numbers.
III is true.

Use the following graph of $y = f(x)$ to answer questions #4 and #5:



4. For which of the following interval(s) is $f(x) > 0$?

$f(x) > 0$ means on what x interval(s) is the function values (y values) positive. Looking at the graph, the function is 'above the x-axis' on the interval $(-4,0)$.

A. $(-4,0)$

B. $(-2,2)$

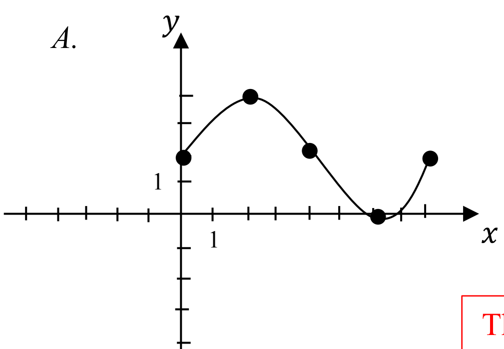
C. $(-4,-2) \cup (2,4)$

D. $(0,4)$

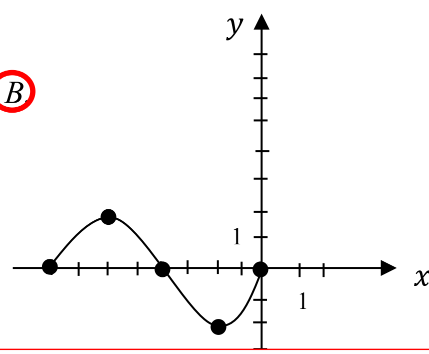
E. No such interval exists.

5. Choose the graph below that depicts $y = \frac{1}{2} f(x+4)$ if the graph $y = f(x)$ given above is the basic graph.

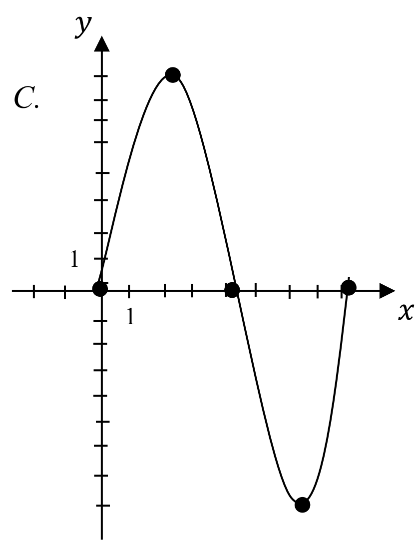
A.



B.

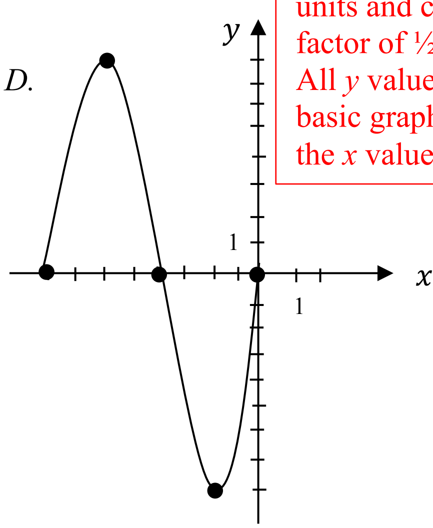


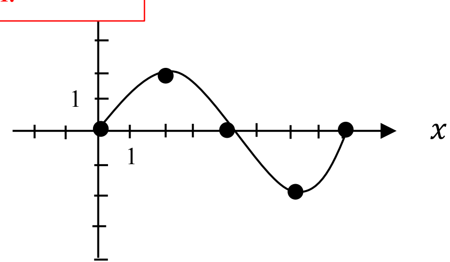
C.



The 'new' function is shifted left 4 units and compressed vertically by a factor of ½ from the basic graph f. All y values are half of those on the basic graph and all x values 4 less than the x values of the basic graph.

D.





6. Solve the following system of equations. Choose the answer that describes the solution(s).

$$\begin{cases} 2x - y = 4 \\ 6x - 3y = 7 \end{cases}$$

Multiply the top equation by -3, leave bottom as is.

$$\begin{cases} -3(2x - y = 4) \\ \quad 6x - 3y = 7 \end{cases} \rightarrow \begin{cases} -6x + 3y = -12 \\ \underline{\quad 6x - 3y = 7} \end{cases}$$

$$0 = -5$$

A false statement after elimination indicates there is no solution.

A. There is one solution. It is in Quadrant I.

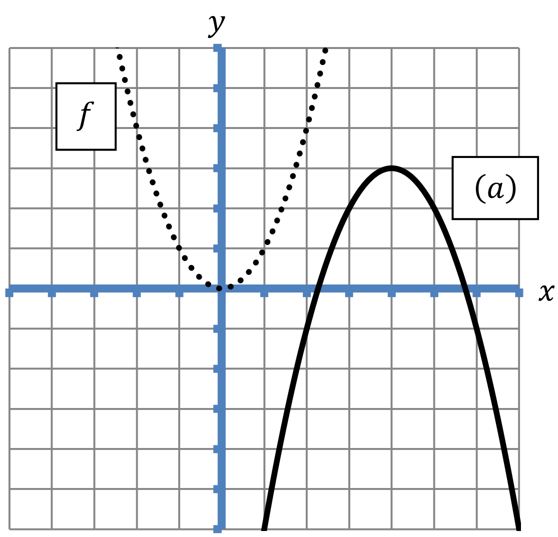
B. The solution is $(0,0)$.

C. The solutions are ordered pairs of the form $(x, 3x-4)$.

D. There is no solution.

E. None of the above.

7. The graph of a function f is shown below, together with the graph of another function (a). Use properties of symmetry, shifts, and reflecting to find the equation for the graph (a) in terms of f.



A. $y = f(-x+4) - 3$

B. $y = -f(x-4) + 3$

C. $y = f(-x-4) + 3$

D. $y = -f(x+4) + 3$

E. None of the above.

Function (a) has been shifted 4 units right (subtract 4 from *x*), then vertically reflected (negative sign before function *f*), and then shifted 3 units up from the original function *f* (add 3 to function value).

8. The height $s(t)$ (in feet) of an object projected vertically upward after t seconds is given by $s(t) = -16t^2 + 80t$. Which of the following intervals represents the times when the height of the object is <u>greater than</u> 64 feet?

height $s > 64$

$-16t^2 + 80t > 64$

$-16t^2 + 80t - 64 > 0$ Find the 'zeros' and make a sign chart.

$-16(t^2 - 5t + 4) > 0$

$-16(t-4)(t-1) > 0$ Zeros: 4 and 1 $t \neq$ a negative value

	$(0,1)$	$(1,4)$	$(4,?)$
-16	$-$	$-$	$-$
$t-4$	$-$	$-$	$+$
$t-1$	$-$	$+$	$+$
result	$-$	$+$	$-$

The positive result is for times of $(1,4)$.

A. $(1,3)$

B. $(2,4)$

C. $(2,3)$

D. $(1,4)$

E. $(0,5)$

9. Solve the following inequality. Express your answer in interval notation.

$\dfrac{x-4}{x^2-x-20} \geq 0$ Factor where possible.

$\dfrac{(x-4)}{(x-5)(x+4)} \geq 0$ Zeros are 4, 5, and -4. However, only 4 could be zero.

	$(-\infty,-4)$	$(-4,4]$	$[4,5)$	$(5,\infty)$
$x-4$	$-$	$-$	$+$	$+$
$x-5$	$-$	$-$	$-$	$+$
$x+4$	$-$	$+$	$+$	$+$
result	$-$	$+$	$-$	$+$

The inequality symbol is $\geq$. Select the positive and zero results.

solution: $(-4,4] \cup (5,\infty)$

A. $[-4,4] \cup [5,\infty)$

B. $(-\infty,-4) \cup [4,5)$

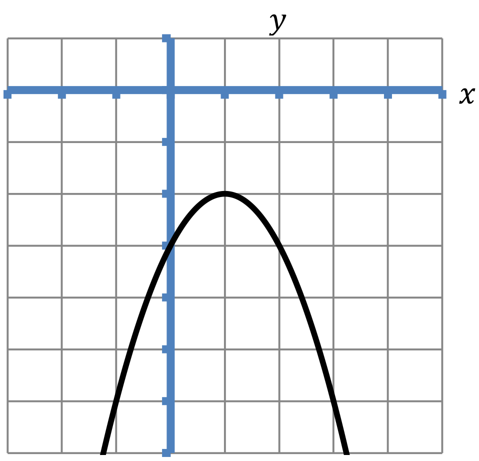
C. $(-4,4] \cup (5,\infty)$

D. $[4,5)$

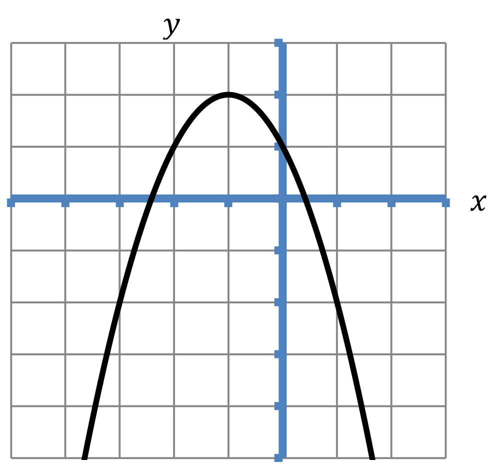
E. $(-\infty,-4] \cup [4,5]$

10. Which of the following is the graph of $f(x) = -x^2 - 2x - 3$?

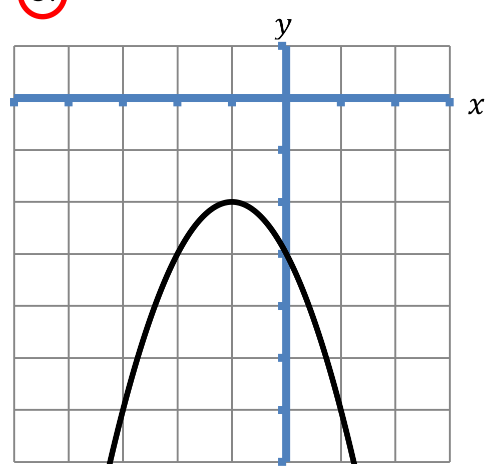
A.



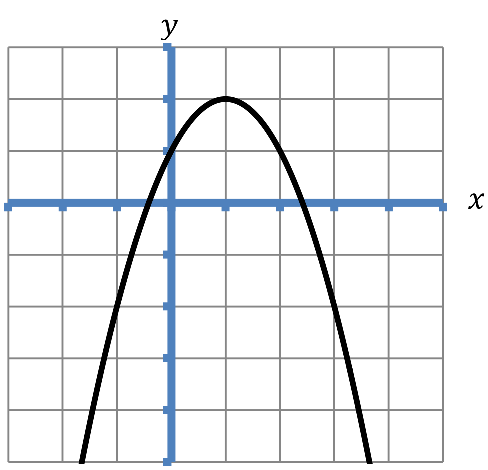
B.



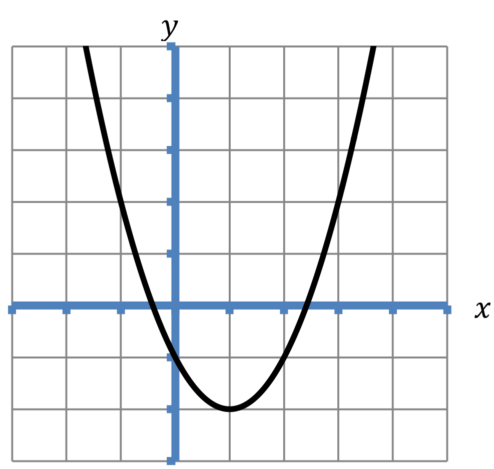
C.



D.



E.



Find the vertex.

$h = \dfrac{-b}{2a}$

$= \dfrac{-(-2)}{2(-1)} = -1$

$k = f(-1)$

$= -(-1)^2 - 2(-1) - 3$

$= -1 + 2 - 3 = -2$

$V(-1,-2)$

Parabola should open down, since $a = -1$.

11. Which of the following statements is/are true of the parabola given by $f(x) = 2x^2 + 4x - 30$?

Zeros:
$0 = 2x^2 + 4x - 30$
$0 = 2(x^2 + 2x - 15)$
$0 = 2(x+5)(x-3)$
$x + 5 = 0$ or $x - 3 = 0$
$x = -5$ $\quad$ $x = 3$
I is false.

I. The zeros of f are given by $x = 5, x = -3$.
II. The minimum value of $f(x)$ is -32.
III. The standard form is $f(x) = 2(x+1)^2 - 24$.

A. I only.
B. II only.
C. I and II only.
D. I and III only.
E. I, II, and III.

$h = \frac{-b}{2a} = \frac{-4}{2(2)} = -1$ $\quad$ $k = f(-1) = 2(-1)^2 + 4(-1) - 30$

$k = 2 - 4 - 30 = -32$

II is true.

$a = 2$ $\quad h = -1$ $\quad k = -32$
$y = a(x-h)^2 + k$
$y = 2(x+1)^2 - 32$
III is false.

12. Solve the following system of equations for x:

$$\begin{cases} x + y = 10 \\ xy = 16 \end{cases}$$

Using substitution:
Solve top equation for y: $\quad y = 10 - x$
Substitute this value for y in the bottom equation.
$x(10 - x) = 16$
$10x - x^2 = 16$
$0 = x^2 - 10x + 16$
$0 = (x-2)(x-8)$
$x - 2 = 0$ or $x - 8 = 0$
$x = 2$ $\quad$ $x = 8$

A. $x = -2, \quad x = -8$
B. $x = 0, \quad x = 10$
C. $x = 0, \quad x = -10$
D. $x = 2, \quad x = 8$
E. None of the above.

13. A golf ball is hit off the ground and projected upward. Its height above the ground $h(t)$ (in feet) t seconds after it is hit is given by the function $h(t) = -16t^2 + 144t$. Find the **time (in seconds)** when the ball reaches its maximum height above the ground. Which choice describes this time?

The maximum height of the ball will occur at the vertex of the quadratic function (parabola). Find the vertex.

$$h = \frac{-b}{2a} = \frac{-144}{2(-16)} = \frac{144}{32} = \frac{9}{2}$$

h represents the time.
The ball reached the maximum height in 4.5 seconds.

A. t is between 1 and 2 seconds
B. t is between 2 and 3 seconds
C. t is between 3 and 4 seconds
D. t is between 4 and 5 seconds
E. t is between 5 and 6 seconds

14. The kinetic energy E of a moving object is directly proportional to the product of the object's mass m and the square of its speed v. A rock with mass 10 kg that is moving at a rate of 6 $\frac{m}{s}$ has a kinetic energy of 120 J (joules). Determine the value of k, the constant of variation.

Variation format: $E = kmv^2$
Substitute and solve for k: $120 = k(10)(6^2)$
$120 = 360k$
$\frac{120}{360} = k \quad \frac{1}{3} = k$

A. $k = 3$

B. $k = \dfrac{18}{\sqrt{6}}$

C. $k = \dfrac{1}{3}$

D. $k = \dfrac{1}{2}$

E. None of the above.

15. A certain county taxes the first \$40,000 of an individual's income at 4%, and all income over \$40,000 is taxed at 9%. Find a piecewise-defined function that expresses the total tax, T, as a function of income, x. Simplify the function.

1st piece: Tax is 4% of x for $0 < x \le 40000$.
$T = 0.04x$
2nd piece: Tax is 4% of the first \$40000 plus 9% of all income over \$40000.
$T = 0.04(40000) + 0.09(x - 40000)$
$= 1600 + 0.09x - 3600$
$= 0.09x - 2000$

A. $T(x) = \begin{cases} .04x & \text{if } 0 < x \le 40,000 \\ .09x & \text{if } x > 40,000 \end{cases}$

B. $T(x) = \begin{cases} .04x & \text{if } 0 < x \le 40,000 \\ .09x + 1,600 & \text{if } x > 40,000 \end{cases}$

C. $T(x) = \begin{cases} .04x & \text{if } 0 < x \le 40,000 \\ .09x - 2,000 & \text{if } x > 40,000 \end{cases}$

D. $T(x) = \begin{cases} .04x & \text{if } 0 < x \le 40,000 \\ .09x - 3,600 & \text{if } x > 40,000 \end{cases}$

E. Cannot be determined.