Notation for the Derivative:

The derivative of a function $y = f(x)$ may be written in any of these ways (these notations).

1) 'prime' notation: $f'(x)$ or $y'$ (read "$f$ prime of $x"$ or "$y$ prime")

2) \( \frac{dy}{dx} \) (read "dee $y$ to dee $x"$, derivative of $y$ with respect to $x$)

3) \( \frac{d}{dx} [f(x)] \) (derivative of function $f(x)$ with respect to $x$)

4) $D_x[f(x)]$ (derivative of function $f(x)$ with respect to $x$)

Note: In the notations above, the independent variable is $x$. Other letters could be used for the independent variable and other names could be given to the function (other than $f$).

A) **Constant Rule:**

If $f(x) = k$, where $k$ represents any real number (a constant), then $f'(x) = 0$. The derivative of a constant is zero.

This rule is reasonable. Derivative represents a 'rate of change'. **If something is constant, it has no 'change'.** Also, the graph of $f(x) = k$ is a horizontal line. At any point on this line, the tangent to $f(x)$ at that point would be the line itself and the slope of a horizontal line is 0. (Remember, the derivative is the slope of a tangent line to a graph at a specified point.)

Examples:

1a) If $g(x) = 9$, find $g'(x)$. $g'(x) = ?$

b) If $y = \frac{\pi}{2}$, find $y'$. $y' = ?$

c) Find $D_t[2^{300}]$. $D_t[2^{300}] = ?$
B) Power Rule:

If \( f(x) = x^n \), where \( n \) is a real number, then \( f'(x) = nx^{n-1} \).
(The derivative of a power is found by multiplying the exponent by \( x \) to one less power.)

The proof of this rule is found in the textbook on page 199. It is tedious, so I will not prove this rule during class time.

Examples:

2a) \( g(x) = x^{10} \), \( g'(x) = \)

b) \( y = \frac{1}{x^5} \), \( \frac{dy}{dx} = \)

c) \( D_x[x^{3/2}] = \)

C) Derivative of a Constant time a Function:

If \( k \) is any real number and if the derivative of \( g \) exists, then the derivative of \( f(x) = k \cdot g(x) \) is \( f'(x) = k \cdot g'(x) \). (The derivative of a constant times a function is the constant times the derivative of the function.)

Examples 3:

a) \( y = 12x^4 \), \( \frac{dy}{dx} = \)

b) \( g(x) = -\frac{3}{4}x^4 \), \( g'(x) = \)

c) \( D_x[-5t] = \)
D) **Sum or Difference Rule:**

If \( f(x) = u(x) \pm v(x) \), then \( f'(x) = u'(x) \pm v'(x) \) (as long as the derivatives of \( u \) and \( v \) exist. *(The derivative of a sum or difference of functions if the sum or difference of the derivatives.)*

Examples 4:

a) \( y = 5x^3 + 2x^2 - 5x + 9 \), \( \frac{dy}{dx} = \)

b) \( p(n) = 6n^2 - 3\sqrt{n} + \frac{2}{n} \), \( p'(n) = \)

c) \( y = \frac{x^4 - 3x^2 + 2\sqrt{x}}{x} \) (Hint: Rewrite equation without a denominator.)

\( y' = \)

d) \( f(x) = \left(2x^2 - 3x\right)^2 \) (Hint: Rewrite by finding the product.)

\( f'(x) = \)

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**Marginal cost, marginal revenue, or marginal profit:**

In business and economics the rates of change of variables such as cost, revenue, and profit are called **marginal cost, marginal revenue, or marginal profit**. Since the derivative of a function gives the instantaneous rate of change of the function; a marginal cost (or revenue or profit) function is found by taking the derivative. **Roughly, the marginal cost at \( x \) represents the cost of the next \( (x + 1) \) item and approximates the value** \( C(x + 1) - C(x) \). Similar statements can be made for marginal profit or marginal revenue.
Example 5:
If the total cost (in hundreds of dollars) to produce \( x \) thousand barrels of a beverage is given by the cost function \( C(x) = 3x^2 + 900x + 450 \), find and interpret \( C'(4) \). Compare with the value of \( C(5) - C(4) \).

Marginal cost evaluated at \( x \) is a good approximation of the actual cost to produce the \((x + 1)\)st unit.
Marginal revenue evaluated at \( x \) is a good approximation of the actual revenue from the sale of the \((x + 1)\)st unit.
Marginal profit evaluated at \( x \) is a good approximation of the actual profit from the sale of the \((x + 1)\)st unit.

The demand function relates the number of units \( x \) of an item that consumers are willing to purchase at the price \( p \). The revenue function can be found if the demand function is known and is \( R(x) = xp \) (number of items times the price/item).
Example 6:
The demand function for a certain product is given by $p = \frac{5000 - 2x}{2500}$ dollars (where $x$ is number of products made and sold). Write a revenue function of the number of items sold. Find the marginal revenue when 1000 units are sold and interpret.

Example 7:
Suppose the revenue function from the sale of $x$ items is given by $R(x) = 3x - 0.01x^2$ and the cost of $x$ items is given by $C(x) = 210 + 0.2x$ for $0 \leq x \leq 10,000$. Write a profit function for this situation. Find the marginal profit (or loss) for 1500 items and interpret.
Example 8 (example 9 of textbook):
The number of Americans (in thousands) who are expected to be over 100 years old can be approximated by the function \( f(t) = 0.00943t^3 - 0.470t^2 + 11.085t + 23.441 \) where \( t \) is the year, with \( t = 0 \) corresponding to 2000 and \( 0 \leq t \leq 50 \). Find the derivative of \( f \). Evaluate \( f'(25) \) and interpret.