Slope:
Definition: The slope of a line is the ratio of the ‘change’ in y to the ‘change’ in x (ratio of vertical change to horizontal change) often referred to as rise compared to run.

\[
slope = m = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

(where \(x_1 \neq x_2\))

Ex 1: Find the slope of a line containing each pair of points. If the slope is undefined, write ‘undefined’.

a) \((-3, -4), (2, -2)\)  
b) \((4, 2)\) and \((-7, 2)\)

c) \((-5, 8)\) and \((2, 3)\)  
d) \((3, 9)\) and \((3, 6)\)

As illustrated above and below, there are 4 types of slopes.

1. A line with a **positive slope** rises left to right.
2. A line with a **negative slope** falls left to right.
3. A line with a **zero slope** is horizontal.
4. A line with an **undefined slope** (zero denominator) is vertical.
Linear Equations:
Begin with the slope formula, letting \((x_i, y_i)\) be one specific point of a line and any other point be \((x, y)\). Cross multiply and the result is called the Point-Slope form of a line.

\[
m = \frac{y - y_i}{x - x_i} \quad \text{Cross multiply...}
\]

\[y - y_i = m(x - x_i) \quad \text{Point - Slope Form of a Linear Equation}
\]

Represent the point \((x_i, y_i)\) as the y-intercept \((0, b)\). Solve the equation for \(y\). The result is called the Slope-Intercept form of a line.

\[
y - b = m(x - 0)
\]
\[
y - b = mx
\]
\[
y = mx + b \quad \text{Slope - Intercept Form}
\]
slope is \(m\) and the y-intercept is \((0, b)\)

Every line can be written in General Form, \(Ax + By + C = 0\). Some textbooks also define Standard Form as \(Ax + By = C\).

The equation for a horizontal line (slope 0) would be \(y = b\). A vertical line has no slope and cannot be written in slope-intercept form. However, its equation would be \(x = a\), where \((a, 0)\) is the x-intercept.

Find equations in (a) slope-intercept form and (b) standard form for the line through the pair of points. Use the slope-intercept form to (c) graph the line using the y-intercept and the rise/run (slope).

Ex 2: \((4, -3), \left(\frac{2}{3}, 2\right)\)
Ex 3:  (a) Find the general form for the equation of a line with a slope of $-\frac{8}{5}$ containing the point (5, -3).  (b) Use the general form to find the $x$-intercept and $y$-intercept and use the intercepts to graph the line.

Ex 4:  Find the equation of a horizontal line through the point (-3, 2) and the equation of a vertical line through the point (7, 10).

Ex 5:  The average weekly salary for a certain job in 2008 was $248.  For the same job, in 2011, the average weekly salary was $285.  Assume the salary can be modeled by a linear equation.  Write a linear function for the average weekly salary for this job in terms of the number of years since 2008.  ($t = 0$ for 2008, $t = 1$ for 2009, etc.).  Use this function to predict the average weekly salary for this job in 2015, if this trend continues.
**Ex 6:** A certain car at a rental agency rents for $75 a week plus $0.18 per mile driven during the week. (a) Write a linear function that will give the weekly cost \(C\) in terms of the number of miles \(n\) driven during the week.

b) Find the cost, if 75 miles are driven during a week.

c) How many miles were driven in a week, if the cost of the rental was $97.50?

**Ex 7:** A company purchased a carpet shampoo machine for $375. The machine cost an average of $15 per day for shampoo supplies and maintenance and the employee who operates the machine is paid $75 per day.

a) Write a linear function giving the total cost \(C\) of operating this carpet shampoo machine for \(d\) days.

b) If revenue from customers who have carpets shampooed averages $125 per day, write a revenue function \(R\) giving the total revenue for \(d\) days of use.

c) Write a profit function \(P\) that will give the profit from the machine after \(d\) days of rental use.

d) Find the approximate number of days the machine must be operated before the company will ‘break even’. The company will ‘break even’ when costs = revenue or profit = 0. (Round to the nearest day.)