1. A spherical balloon is inflated at the rate of 8 cubic centimeters per second. Find the rate at which the radius is increasing when the radius is 5 centimeters.

*Hint: Volume of a sphere, $V = \frac{4}{3} \pi r^3$*

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\left. \frac{dr}{dt} \right|_{r=5} = \frac{8}{4\pi (5)^2}$$

A. $100\pi$ cm/s 
B. $800\pi$ cm/s 
C. $\sqrt{\frac{2}{\pi}}$ cm/s 
D. $\frac{3}{50\pi}$ cm/s 
E. $\frac{2}{25\pi}$ cm/s

2. A spotlight on the ground shines on a wall 12 m away. If a man 1.8 m tall walks from the wall to the spotlight at a speed of 1 m/s, how fast is the length of his shadow on the wall changing when he is 3.6 m from the spotlight?

$$\frac{h}{12} = \frac{1.8}{x}$$

$$h = \frac{12(1.8)}{x}$$

$$\frac{dh}{dt} = \frac{12(1.8)}{-x^2} \left( \frac{dx}{dt} \right)$$

$$\left. \frac{dh}{dt} \right|_{x=3.6} = \frac{12(1.8)}{-(3.6)^2} (-1)$$

$$= \frac{6}{3.6} = \frac{60}{36} = \frac{10}{6}$$

A. $-\frac{6}{10}$ m/s 
B. $\frac{6}{10}$ m/s 
C. $\frac{10}{6}$ m/s 
D. $-\frac{10}{6}$ m/s 
E. 6 m/s
3. An observer is stationed 2 miles from a rocket launch pad. The rocket rises vertically off the launch pad. \( h \) denotes the height of the rocket (in miles), and \( z \) denotes the distance from the observer to the rocket (in miles). Find a formula for \( \frac{dz}{dt} \).

\[
2z \frac{dz}{dt} = 2h \frac{dh}{dt}
\]

\[
\frac{dz}{dt} = \frac{h \frac{dh}{dt}}{z} = \frac{h}{\sqrt{4+h^2}}
\]

A. \( \sqrt{2h \frac{dh}{dt}} \)
B. \( \sqrt{4 + \left( \frac{dh}{dt} \right)^2} \)
C. \( 2h \frac{dh}{dt} \)
D. \( \frac{h \frac{dh}{dt}}{\sqrt{4+h^2}} \)
E. \( \frac{2 + h \frac{dh}{dt}}{\sqrt{4+h^2}} \)

4. Find the approximate value of \( \sqrt{25.1} \) by considering the linear approximation of the function \( f(x) = x^{\frac{1}{2}} \) at \( x = 25 \).

\[
f'(x) = \frac{1}{2\sqrt{x}}
\]

\[
f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{10}
\]

\[
f(25.1) \approx f(25) + f'(25)(25.1-25)
\]

\[
= 5 + \frac{1}{10}(0.1)
\]

\[
= 5.01
\]
5. Let \( f(x) \) be a polynomial with \( f(2) = 1 \). Assume that \( f'(x) \geq 3 \) for every \( x \) in \([2, 4]\). What is the smallest possible value of \( f(4) \)? *Hint: Apply the Mean Value Theorem.*

A. 1  
B. 3  
C. 4  
D. 6  
E. \( 7 \)

\[
3 \leq f'(c) = \frac{f(4) - f(2)}{4 - 2} \\
3 \leq \frac{f(4) - 1}{2} \\
3 \cdot 2 + 1 \leq f(4)
\]

6. Find the absolute minimum value of \( f(x) = 3x^3 - x \) on the closed interval \( \left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right] \).

A. 0  
B. \( -\frac{1}{\sqrt{3}} \)  
C. \( -\frac{1}{3} \)  
D. \( -\frac{2}{9} \)  
E. There is no absolute minimum value.

\[
f'(x) = 9x^2 - 1 \\
\text{CRIT: } x = \pm \frac{1}{3} \\
f\left( -\frac{1}{\sqrt{3}} \right) = 0 \\
f\left( -\frac{1}{3} \right) = -\frac{1}{9} + \frac{1}{3} = \frac{2}{9} \\
f\left( \frac{1}{3} \right) = -\frac{2}{9} \\
f\left( \frac{1}{\sqrt{3}} \right) = 0
\]
7. Consider the function \( f(x) = 1 + \frac{1}{x} - \frac{1}{x^2} \) for \( x \neq 0 \). This function has

A. one local minimum and one point of inflection
B. **one local maximum and one point of inflection**
C. one local minimum and two points of inflection
D. one local maximum and two points of inflection
E. one local minimum, one local maximum, and two points of inflection

\[
\begin{align*}
 f'(x) &= -\frac{1}{x^2} + \frac{2}{x^3} = \frac{2-x}{x^3} , \quad f''(2) < 0 , \text{ so local max at } x=2. \\
 f''(x) &= \frac{2}{x^3} - \frac{6}{x^4} = \frac{2x-6}{x^4} , \quad \text{INF PT at } x=3.
\end{align*}
\]

8. If \( f(x) = xe^{bx^2} \), what is the value of the constant \( b \) such that \( f(x) \) attains its maximum at \( x = 2 \)?

A. \( b = \frac{-1}{2} \)
B. \( b = \frac{1}{4} \)
C. \( b = \frac{-1}{8} \)
D. \( b = -1 \)
E. \( b = 1 \)

Require \( f'(2) = 0 \).

\[
\begin{align*}
 f'(x) &= e^{bx^2} + 2bx^2e^{bx^2} \\
 f'(2) &= e^{4b}(1+8b) \\
 f'(2) &= 0 \iff 1+8b = 0 \iff b = -\frac{1}{8}
\end{align*}
\]
9. Find the point(s) of inflection of the function \( f(x) = \ln(1 - \ln(x)) \) on the interval \( 0 < x < e \).

A. \( x = \ln 2 \)
B. \( x = \sqrt{e} \)
C. \( x = \frac{1}{2} \) and \( x = \frac{3}{2} \)
D. \( x = 2 \)
E. \( x = 1 \)

\[
f'(x) = \frac{1}{1-\ln x} \left( -\frac{1}{x} \right) = \frac{1}{x\ln x - x}
\]

\[
f''(x) = -\frac{1}{(x\ln x - x)^2} \left( \ln x + x(\frac{1}{x}) - 1 \right)
\]

\[
= -\frac{\ln x}{(x\ln x - x)^2}
\]

\( f'' \) changes sign at \( x = 1 \).

10. Find the limit.

\[
\lim_{x \to 1^+} \frac{\ln x}{\cot \left( \frac{\pi x}{2} \right)}
\]

A. \( -\infty \)
B. \( \frac{2}{\pi} \)
C. 0
D. \( \frac{\pi}{2} \)
E. \( -\infty \)

\[
= \lim_{x \to 1^+} \frac{\ln x}{\cos \left( \frac{\pi x}{2} \right)} \quad \text{LH}
\]

\[
= \lim_{x \to 1^+} \frac{\frac{1}{x} \sin \left( \frac{\pi x}{2} \right) + \left( \frac{\pi x}{2} \right) \cos \left( \frac{\pi x}{2} \right)}{-\frac{\pi}{2} \sin \left( \frac{\pi x}{2} \right)}
\]

\[
= \frac{1}{-\csc^2 \left( \frac{\pi}{2} \right) \left( \frac{\pi}{2} \right)}
\]

\[
= \frac{1}{-\pi^2}
\]
11. Suppose \( f''(x) = e^{x^2} \) and \( f'(1) = 0 \). At \( x = 1 \), \( f \) has

A. A local maximum
B. A local minimum
C. An inflection point
D. None of these
E. Impossible to determine

\[ f''(1) = e^1 > 0 \]

By Second Deriv Test,

\( f \) is concave up at critical number.

12. Which of these curves is the graph of \( y = 1 + 4x^5 - 5x^4 \) between \( x = 0 \) and \( x = 1 \)?

\[ y' = 20x^4 - 20x^3 = 20x^3(x-1) \]

\( \text{CRIT: } x=0, x=1 \)

\( \text{GRAPH HAS HORIZONTAL TANGENT} \)