MA 161, EXAM 3, FALL 2015

1. A spherical balloon is inflated at the rate of 8 cubic centimeters per second. Find the rate at which the radius is increasing when the radius is 5 centimeters.
Hint: Volume of a sphere, $V = \frac{4}{3}\pi r^3$

A. 100π cm/s

B. 800π cm/s

C. $\sqrt{\frac{2}{\pi}}$ cm/s

D. $\frac{3}{50\pi}$ cm/s

E. $\boxed{\frac{2}{25\pi}}$ cm/s

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\left.\frac{dr}{dt}\right|_{r=5} = \frac{8}{4\pi(5)^2}$$

2. A spotlight on the ground shines on a wall 12 m away. If a man 1.8 m tall walks from the wall to the spotlight at a speed of 1 m/s, how fast is the length of his shadow on the wall changing when he is 3.6 m from the spotlight?

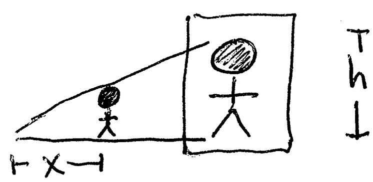
A. $-\frac{6}{10}$ m/s

B. $\frac{6}{10}$ m/s

C. $\boxed{\frac{10}{6} \text{ m/s}}$

D. $-\frac{10}{6}$ m/s

E. 6 m/s



$$\frac{h}{12} = \frac{1.8}{x}$$

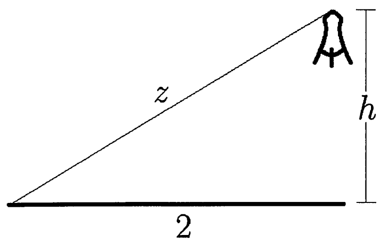
$$h = \frac{12(1.8)}{x}$$

$$\frac{dh}{dt} = \frac{12(1.8)}{-x^2}\left(\frac{dx}{dt}\right)$$

$$\left.\frac{dh}{dt}\right|_{x=3.6} = \frac{12(1.8)}{-(3.6)^2}(-1)$$

$$= \frac{6}{3.6} = \frac{60}{36} = \frac{10}{6}$$

3. An observer is stationed 2 miles from a rocket launch pad. The rocket rises vertically off the launch pad. h denotes the height of the rocket (in miles), and z denotes the distance from the observer to the rocket (in miles). Find a formula for $\dfrac{dz}{dt}$.



$$z^2 = 2^2 + h^2$$

$$2z\frac{dz}{dt} = 2h\frac{dh}{dt}$$

$$\frac{dz}{dt} = \frac{h\frac{dh}{dt}}{z} = \frac{h\frac{dh}{dt}}{\sqrt{4+h^2}}$$

A. $\sqrt{2h\dfrac{dh}{dt}}$

B. $\sqrt{4+\left(\dfrac{dh}{dt}\right)^2}$

C. $2h\dfrac{dh}{dt}$

D. $\boxed{\dfrac{h\frac{dh}{dt}}{\sqrt{4+h^2}}}$

E. $\dfrac{2+h\frac{dh}{dt}}{\sqrt{4+h^2}}$

4. Find the approximate value of

$$\sqrt{25.1}$$

by considering the linear approximation of the function $f(x) = x^{\frac{1}{2}}$ at $x = 25$.

A. 4.95

B. $\boxed{5.01}$

C. 5.05

D. 5.10

E. 5.15

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{10}$$

$$f(25.1) \approx f(25) + f'(25)(25.1-25)$$

$$= 5 + \frac{1}{10}(.1)$$

$$= 5.01$$

5. Let $f(x)$ be a polynomial with $f(2) = 1$. Assume that $f'(x) \geq 3$ for every x in $[2, 4]$. What is the smallest possible value of $f(4)$? *Hint: Apply the Mean Value Theorem.*

A. 1

B. 3

C. 4

D. 6

E. $\boxed{7}$

$$3 \leq f'(c) = \frac{f(4) - f(2)}{4-2}$$

$$3 \leq \frac{f(4) - 1}{2}$$

$$3 \cdot 2 + 1 \leq f(4)$$

6. Find the absolute minimum value of $f(x) = 3x^3 - x$ on the closed interval $\left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$.

A. 0

B. $-\frac{1}{\sqrt{3}}$

C. $-\frac{1}{3}$

D. $\boxed{-\frac{2}{9}}$

E. There is no absolute minimum value.

$$f'(x) = 9x^2 - 1$$

CRIT: $x = \pm \frac{1}{3}$

$$f\left(-\frac{1}{\sqrt{3}}\right) = 0$$

$$f\left(-\frac{1}{3}\right) = -\frac{1}{9} + \frac{1}{3} = 2/9$$

$$\rightarrow f\left(\frac{1}{3}\right) = -2/9$$

$$f\left(\frac{1}{\sqrt{3}}\right) = 0$$

7. Consider the function $f(x) = 1 + \dfrac{1}{x} - \dfrac{1}{x^2}$ for $x \neq 0$. This function has

A. one local minimum and one point of inflection

B. $\boxed{\text{one local maximum and one point of inflection}}$

C. one local minimum and two points of inflection

D. one local maximum and two points of inflection

E. one local minimum, one local maximum, and two points of inflection

$$f'(x) = -\frac{1}{x^2} + \frac{2}{x^3} = \frac{2-x}{x^3}.$$ $f''(2) < 0$, so LOCAL MAX at $x=2$.

$$f''(x) = \frac{2}{x^3} - \frac{6}{x^4} = \frac{2x-6}{x^4}.$$ INFL PT at $x=3$.

8. If $f(x) = xe^{bx^2}$, what is the value of the constant b such that $f(x)$ attains its maximum at $x = 2$?

A. $b = -\dfrac{1}{2}$

B. $b = \dfrac{1}{4}$

C. $\boxed{b = -\dfrac{1}{8}}$

D. $b = -1$

E. $b = 1$

Require $f'(2) = 0$.

$$f'(x) = e^{bx^2} + 2bx^2e^{bx^2}$$

$$f'(2) = e^{4b}(1+8b)$$

$$f'(2) = 0 \Leftrightarrow 1+8b = 0$$

$$\Leftrightarrow b = -1/8$$

9. Find the point(s) of inflection of the function $f(x) = \ln(1 - \ln(x))$ on the interval $0 < x < e$.

A. $x = \ln 2$

B. $x = \sqrt{e}$

C. $x = \frac{1}{2}$ and $x = \frac{3}{2}$

D. $x = 2$

E. $\boxed{x = 1}$

$$f'(x) = \frac{1}{1 - \ln x} \cdot \left(-\frac{1}{x}\right) = \frac{1}{x\ln x - x}$$

$$f''(x) = -\frac{1}{(x\ln x - x)^2}\left(\ln x + x\left(\frac{1}{x}\right) - 1\right)$$

$$= \frac{-\ln x}{(x\ln x - x)^2}$$

f'' changes sign at $x = 1$.

10. Find the limit.

$0 \cdot -\infty$?

$$\lim_{x\to 1^+} (\ln x)\left(\tan\frac{\pi x}{2}\right)$$

A. $\boxed{-\frac{2}{\pi}}$

B. $-\frac{\pi}{2}$

C. 0

D. $-\infty$

E. ∞

$$= \lim_{x\to 1^+} \frac{\ln x}{\cot\left(\frac{\pi x}{2}\right)} \overset{LH}{\underset{\frac{0}{0}}{=}} \lim_{x\to 1^+} \frac{1/x}{-\csc^2\left(\frac{\pi x}{2}\right)\left(\frac{\pi}{2}\right)}$$

$$= \frac{1/1}{-\csc^2\left(\frac{\pi}{2}\right)\left(\frac{\pi}{2}\right)} = \frac{1}{-\frac{\pi}{2}}$$

$\boxed{\text{OR}}$

$$= \lim_{x\to 1^+} \frac{(\ln x)\left(\sin\frac{\pi x}{2}\right)}{\cos\frac{\pi x}{2}} \overset{LH}{\underset{\frac{0}{0}}{=}} \lim_{x\to 1^+} \frac{\frac{1}{x}\sin\frac{\pi x}{2} + (\ln x)\frac{\pi}{2}\cos\frac{\pi x}{2}}{-\frac{\pi}{2}\sin\frac{\pi x}{2}}$$

$$= \frac{\frac{1}{1}(1) + 0\left(\frac{\pi}{2}\right)(0)}{-\frac{\pi}{2}(1)}$$

11. Suppose $f''(x) = e^{x^2}$ and $f'(1) = 0$. At $x = 1$, f has

A. A local maximum

B. $\boxed{\text{A local minimum}}$

C. An inflection point

D. None of these

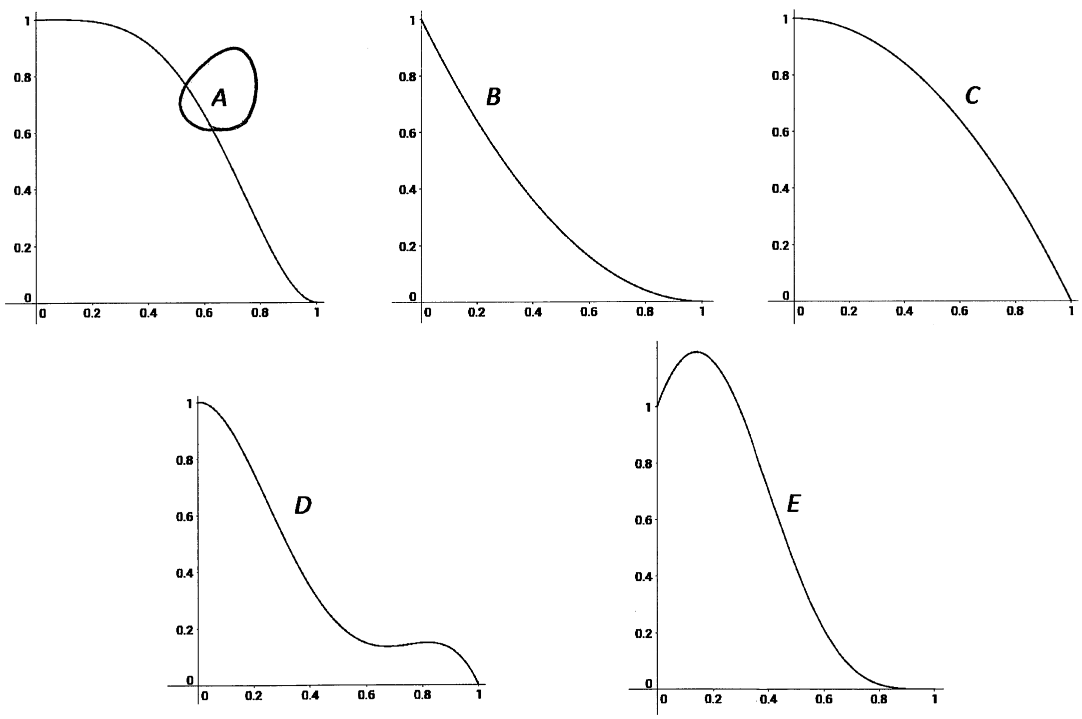
E. Impossible to determine

$f''(1) = e^{1^2} > 0$

By Second Deriv Test,

(f is concave up at critical number)

12. Which of these curves is the graph of $y = 1 + 4x^5 - 5x^4$ between $x = 0$ and $x = 1$?



$y' = 20x^4 - 20x^3 = 20x^3(x-1)$

CRIT: $x=0$, $x=1$

GRAPH HAS HORIZONTAL TANGENT