MA 16010 - Exam 2 Practice Exam 2

1. Given $f(x) = \dfrac{x^2}{\sin x}$. Find $f'(x)$.

   A. $f'(x) = \dfrac{x^2 \cos x - 2x \sin x}{\sin^2 x}$

   B. $f'(x) = \dfrac{2x \sin x + x^2 \cos x}{\sin^2 x}$

   C. $f'(x) = \dfrac{2x \sin x - x^2 \cos x}{\sin^2 x}$

   D. $f'(x) = \dfrac{2x \sin x - x^2 \cos x}{x^4}$

   E. $f'(x) = \dfrac{2x}{\cos x}$

2. Find the slope of the tangent line to the graph of $y = x \cot x$ at $x = \dfrac{\pi}{4}$.

   A. -1

   B. $\dfrac{1}{2}$

   C. 3

   D. $1 - \dfrac{\pi}{2}$

   E. $1 + \dfrac{\pi}{8}$

3. Given $f(x) = \dfrac{x^3}{3} + x + \sqrt{x^3}$. Find $f''(4)$.

   A. $\dfrac{67}{8}$
   B. $\dfrac{49}{6}$
   C. $\dfrac{19}{2}$
   D. $\dfrac{26}{3}$
   E. $\dfrac{35}{4}$

4. A bowling ball is launched off the top of a 240-foot tall building. The height of the bowling ball above the ground t seconds after being launched is $s(t) = -16t^2 + 32t + 240$ feet above the ground. What is the velocity of the ball as it hits the ground?

   A. 0 ft/s
   B. 5 ft/s
   C. 32 ft/s
   D. -76 ft/s
   E. -128 ft/s

5. If $y = \left(\dfrac{2x-1}{2x+1}\right)^3$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x} =$

   A. $\dfrac{48}{(2x+1)^4}$

   B. $3\left(\dfrac{2x-1}{2x+1}\right)^2$

   C. $\dfrac{6(2x-1)^2}{(2x+1)^3}$

   D. $\dfrac{12(2x-1)^2}{(2x+1)^4}$

   E. $\dfrac{24x-12}{(2x+1)^3}$

6. Given $f(x) = e^{5x}\ln(7x+e)$. Find $f'(0)$.

   A. $1+\dfrac{1}{e}$

   B. $5+\dfrac{7}{e}$

   C. $\dfrac{1}{e}$

   D. $\dfrac{5}{e}$

   E. $\dfrac{35}{e}$

7. The price of one kilowatt-hour of electricity is given by

$$p(t) = (t^2 + 2t)^2,$$

where $p(t)$ is the price in dollars and t is years after 2014 (so 2015 corresponds to $t = 1$.) At what rate is the price changing in the year of 2024?

A. \$4800/year
B. \$2400/year
C. \$5280/year
D. \$1680/year
E. \$900/year

8. Find $\phi'(x)$ if $\phi(x) = \tan^2(3x^2 + 2)$.

A. $2\tan(6x)$
B. $2\sec^2(6x)$
C. $12x\sec^2(3x^2 + 2)$
D. $12x\tan(3x^2 + 2)$
E. $12x\tan(3x^2 + 2)\sec^2(3x^2 + 2)$

9. Use implicit differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $x^2 + y^2 = 2xy + 5$.

   A. 0
   B. 1
   C. $\frac{x}{1-y}$
   D. $\frac{x}{x-y}$
   E. $\frac{2y-2x+5}{2y-2x}$

10. All edges of a cube are expanding at a rate of 2 centimeters per second. How fast is the surface area changing when each edge is 3 centimeters?

    A. 12 cm^2/sec
    B. 46 cm^2/sec
    C. 72 cm^2/sec
    D. 36 cm^2/sec
    E. 54 cm^2/sec

11. Water flows into a right cylindrical can with a circular base at a rate 4 m^3/min. The radius of the base is 3 m. How fast is the water level rising inside the can? The volume of a right cylinder with a circular base is $V = \pi r^2 h$, where r is the radius of the base and h is the height of the cylinder.

    A. $\frac{4}{9\pi}$ m/min
    B. $\frac{3}{16\pi}$ m/min
    C. $\frac{2}{3\pi}$ m/min
    D. $\frac{3}{8\pi}$ m/min
    E. $\frac{4}{3\pi}$ m/min

12. A 10-ft ladder, whose base is sitting on level ground, is leaning at an angle against a vertical wall when its base starts to slide away from the vertical wall. When the base of the ladder is 6 ft away from the bottom of the vertical wall, the base is sliding away at a rate of 4 ft/sec. At what rate is the vertical distance from the top of the ladder to the ground changing at this moment?

    A. -3 ft/sec
    B. $-\frac{3}{4}$ ft/sec
    C. $\frac{1}{4}$ ft/sec
    D. 4 ft/sec
    E. 8 ft/sec