NAME________________________INSTRUCTOR________________________

INSTRUCTIONS

1. Fill in your name and your instructor's name above.
2. You must use a #2 pencil on the scantron answer sheet.
3. Fill in your name, your four digit section number, "01" for the Test/Quiz Number, and your student identification number. Make sure to blacken in the appropriate spaces. If you do not know your section number, ask your instructor. Sign your name.
4. There are 12 questions. Blacken in your choice of the correct answer in the spaces provided on the scantron answer sheet. **Only the scantron answer sheet will be graded. When you have completed the exam, turn in the scantron answer sheet only. You may take the exam booklet with you.**
5. **The exam is self-explanatory. Do not ask your instructor any questions about the exam problems.**
6. **Only one-line calculators (any brand) are allowed. Cell phones and PDA's may not be used as a calculator and must be put away during the exam. NO BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.**

THE SECOND DERIVATIVE TEST

Suppose f is a function of two variables x and y, and that all the second-order partial derivatives are continuous. Let

$$d = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose (a, b) is a critical point of f.

1. **If $d(a,b) > 0$ and $f_{xx}(a,b) > 0$, then f has a relative minimum at (a,b).**
2. **If $d(a,b) > 0$ and $f_{xx}(a,b) < 0$, then f has a relative maximum at (a,b).**
3. **If $d(a,b) < 0$, then f has a saddle point at (a,b).**
4. **If $d(a,b) = 0$, the test is inconclusive.**

LAGRANGE EQUATIONS

For the function $f(x,y)$ subject to the constraint $g(x,y) = c$, the Lagrange equations are

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x,y) = c$$

VOLUME & SURFACE AREA

Right Circular Cylinder

$V = \pi r^2 h$

$SA = \begin{cases} 2\pi r^2 + 2\pi r h \\ \pi r^2 + 2\pi r h \end{cases}$

Sphere

$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

Right Circular Cone

$V = \frac{1}{3}\pi r^2 h$

$SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$

1. Starting with the matrix $\begin{bmatrix} 5 & 7 & 2 \\ 1 & -2 & 3 \end{bmatrix}$, find the row-equivalent matrix obtained by performing the following sequence of row operations:

   (i) Multiply Row 2 by -3.

   (ii) 2 times Row 2 is added to Row 1.

   A. $\begin{bmatrix} -1 & 19 & -16 \\ -3 & 6 & -9 \end{bmatrix}$

   B. $\begin{bmatrix} 7 & 3 & 8 \\ -3 & 6 & -9 \end{bmatrix}$

   C. $\begin{bmatrix} 5 & 7 & 2 \\ -1 & 19 & -16 \end{bmatrix}$

   D. $\begin{bmatrix} 7 & 3 & 8 \\ 1 & -2 & 3 \end{bmatrix}$

   E. $\begin{bmatrix} 5 & 7 & 2 \\ -3 & 6 & -9 \end{bmatrix}$

2. A rectangular box with a square base is changing in size with time. At a particular moment in time, the length of a side on the square base is 3 ft and is increasing at a rate of 1 ft/s. The height of the box is 5 ft and is decreasing at a rate of 2 ft/s. At what rate is the surface area of the box changing at that moment in time?

   A. -4 ft^2/s

   B. 2 ft^2/s

   C. 8 ft^2/s

   D. 50 ft^2/s

   E. 56 ft^2/s

3. The number of math problems, N(x,y), a student can solve in an evening is dependent upon x hours of studying notes and y hours doing problems. If $N(x, y) = -x^2 + 2x - y^2 + 4y + 15$, find the maximum number of problems that can be solved in an evening.

   A. 13

   B. 15

   C. 18

   D. 20

   E. 23

4. Compute $\int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin y}{y} \, dy dx$.

   A. $-\pi/2$

   B. π

   C. $\pi/2$

   D. $-\pi$

   E. 1

5. If

$$c = 2, \quad A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 4 & 2 \\ 0 & 3 \end{bmatrix},$$

find $(cA - B)^2$.

A. $\begin{bmatrix} 0 & 4 \\ 4 & 94 \end{bmatrix}$

B. $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

C. $\begin{bmatrix} 0 & -2 \\ 2 & 7 \end{bmatrix}$

D. $\begin{bmatrix} 68 & 42 \\ 42 & 137 \end{bmatrix}$

E. $\begin{bmatrix} -4 & -14 \\ 14 & 45 \end{bmatrix}$

6. Mitch Daniels wants to build a new building that will be constructed to look like the surface $f(x, y) = 50 - \frac{1}{2}x^2 - \frac{1}{8}y^2$ above the $xy-$plane with $y \geq 0$. Which double integral gives the volume of the constructed building?

A. $\int_0^{10} \int_0^{\sqrt{400-4x^2}} \left(50 - \frac{1}{2}x^2 - \frac{1}{8}y^2\right) dy\, dx$

B. $\pi \int_{-10}^{10} \int_0^{\sqrt{400-4x^2}} \left(50 - \frac{1}{2}x^2 - \frac{1}{8}y^2\right)^2 dy\, dx$

C. $\int_{-10}^{10} \int_0^{\sqrt{400-4x^2}} \left(50 - \frac{1}{2}x^2 - \frac{1}{8}y^2\right) dy\, dx$

D. $\int_{-10}^{10} \int_{-\sqrt{400-4x^2}}^{\sqrt{400-4x^2}} \left(50 - \frac{1}{2}x^2 - \frac{1}{8}y^2\right) dy\, dx$

E. $\int_0^{20} \int_0^{\sqrt{400-y^2/4}} \left(50 - \frac{1}{2}x^2 - \frac{1}{8}y^2\right) dx\, dy$

7. What is the minimum of $f(x, y) = 2x - y$ subject to the constraint $x^2 + y^2 = 1$?

   A. $2\sqrt{2}$
   B. $-2\sqrt{2}$
   C. $\sqrt{5}$
   D. $-\sqrt{5}$
   E. -1

8. For shipping watermelons a company has three crate sizes that hold different amounts of watermelons. One small and one large crate used together will hold 20 watermelons. Two small crates and one medium crate used together will hold 15 watermelons. And two medium crates and a large crate used together will hold 35 watermelons. How many watermelons can be sent in a shipment that uses one small crate, one medium crate, and one large crate?

   A. 27
   B. 29
   C. 31
   D. 33
   E. 35

9. Set up integrals of the function $f(x, y) = \cos x \sin y$ for the region bounded by the curves $y = 2, x = 4$ and $y = \sqrt{x} + 2$ for both orders of integration.

A. $\int_2^4 \int_{\sqrt{x}+2}^{2} \cos x \sin y \, dy \, dx, \quad \int_2^4 \int_0^{y^2-4y+4} \cos x \sin y \, dx \, dy$

B. $\int_2^4 \int_2^{\sqrt{x}+2} \cos x \sin y \, dy \, dx, \quad \int_2^4 \int_{y^2-4y+4}^{4} \cos x \sin y \, dx \, dy$

C. $\int_0^4 \int_2^{\sqrt{x}+2} \cos x \sin y \, dy \, dx, \quad \int_2^4 \int_{y^2-4y+4}^{4} \cos x \sin y \, dx \, dy$

D. $\int_0^4 \int_2^{\sqrt{x}+2} \cos x \sin y \, dy \, dx, \quad \int_0^4 \int_{y^2-4y+4}^{4} \cos x \sin y \, dx \, dy$

E. $\int_2^4 \int_2^{\sqrt{x}+2} \cos x \sin y \, dy \, dx, \quad \int_2^4 \int_{\sqrt{y}+2}^{4} \cos x \sin y \, dx \, dy$

10. The centripetal acceleration of a particle moving in a circle is $a = \frac{v^2}{r}$, where v is the velocity and r is the radius of the circle. Approximate the maximum percent error in calculating the acceleration given the respective measuring errors are 2% for v and 15% for r.

A. 12%

B. 19%

C. 11%

D. 29%

E. 20%

11. Find and classify the critical points of $f(x, y) = -x^2 + 6xy - y^3$.

A. $(0, 0)$ saddle point; $(18, 6)$ relative maximum

B. $(0, 0)$ relative maximum; $(18, 6)$ relative maximum

C. $(0, 0)$ saddle point; $(18, 6)$ relative minimum

D. $(0, 0)$ saddle point; $(18, 6)$ saddle point

E. $(0, 0)$ relative maximum; $(18, 6)$ saddle point

12. A company's output is given by the production function $P = 840L^{2/3}K^{1/3}$, where L and K are the numbers of units of labor and capital that are to be used. To stay within its budget of \$2520, L are K are constrained by $35L + 140K = 2520$. Find the company's maximum output subject to its budget.

A. $7,331$

B. $10,080$

C. $3,665$

D. $20,160$

E. $16,141$

MA 16020 Exam 3 – Answer Key

Question Number	Green Version Form 01
1	A
2	C
3	D
4	E
5	E
6	C
7	D
8	B
9	C
10	B
11	A
12	D

The exam is worth 120 points

Your score = #correct * 10 points