MA 16500
EXAM 1 INSTRUCTIONS
VERSION 01
September 20, 2016

Your name ____________________ Your TA's name ____________________

Student ID # ______________ Section # and recitation time __________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–12. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 12 questions, each worth 8 points. The maximum possible score is

$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the exam booklets.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

Questions

1. We have the information

$$\cos\theta = -\frac{12}{13} \quad \text{and} \quad \frac{\pi}{2} < \theta < \pi.$$

Determine the value of $\tan\theta$.

A. $\frac{5}{12}$

B. $-\frac{5}{12}$ (correct)

C. $\frac{12}{5}$

D. $-\frac{12}{5}$

E. $-\frac{5}{13}$

2. Find the exact value of each expression.

$$\text{(i)} \quad e^{\ln(\ln e^3)}$$
$$\text{(ii)} \quad \sin^{-1}\left(\sin \frac{7\pi}{5}\right)$$

HINT: The range of the function "$\sin^{-1}$" is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

A. (i) 3 (ii) $\frac{7\pi}{5}$

B. (i) 3 (ii) $\frac{2\pi}{5}$

C. (i) 3 (ii) $-\frac{2\pi}{5}$ (correct)

D. (i) e^3 (ii) $-\frac{2\pi}{5}$

E. (i) e^3 (ii) $\frac{7\pi}{5}$

3. Consider the following function defined over the domain $(-\infty, 4) \cup (4, \infty)$

$$y = f(x) = \frac{3x+2}{x-4}.$$

Choose the right answer for

(i) the formula for the inverse function $f^{-1}(x)$,
(ii) the range for the inverse function.

A. (i) $f^{-1}(x) = \dfrac{x-3}{3x+2}$ (ii) $(-\infty, 3) \cup (3, \infty)$
B. (i) $f^{-1}(x) = \dfrac{x-3}{3x+2}$ (ii) $(-\infty, 4) \cup (4, \infty)$
C. (i) $f^{-1}(x) = \dfrac{4x+2}{x-3}$ (ii) $(-\infty, 3) \cup (3, \infty)$
D. (i) $f^{-1}(x) = \dfrac{4x+2}{x-3}$ (ii) $(-\infty, 4) \cup (4, \infty)$ (correct)
E. The function $y = f(x)$ is not one-to-one over the specified domain, and hence its inverse function does not exist.

4. We want to compute the following limit

$$\lim_{x\to\infty} \frac{\sin x}{x}.$$

Choose the right answer with correct reasoning.

A. $\displaystyle\lim_{x\to\infty} \frac{\sin x}{x}$ is one of the basic limits involving the trigonometric functions, and it is well-known to be equal to 1.

B. $\displaystyle\lim_{x\to\infty} \frac{\sin x}{x} = \left[\lim_{x\to\infty} \sin x\right] \cdot \left[\lim_{x\to\infty} \frac{1}{x}\right]$. Since $\sin x$ oscillates as x approaches ∞, $\displaystyle\lim_{x\to\infty} \sin x$ does not exist. Therefore, the limit we want to compute does not exist, either.

C. $\displaystyle\lim_{x\to\infty} \frac{\sin x}{x} = \left[\lim_{x\to\infty} \sin x\right] \cdot \left[\lim_{x\to\infty} \frac{1}{x}\right]$. Since $\frac{1}{x}$ goes to 0 as x approaches ∞, we conclude that $\displaystyle\lim_{x\to\infty} \frac{1}{x} = 0$. Therefore, the limit we want to compute is also equal to 0.

D. Since $-1 \leq \sin x \leq 1$, we have $-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$ when x is a positive number. By observing both $-\frac{1}{x}$ and $\frac{1}{x}$ go to 0 as x approaches ∞ and by applying the Squeeze Theorem, we conclude that the limit we want to compute is equal to 0. (correct)

E. For a positive number a, we have $\frac{1}{a} \cdot \sin x = \sin\left(\frac{1}{a} \cdot x\right)$. Applying this rule, we have $\displaystyle\lim_{x\to\infty} \frac{\sin x}{x} = \lim_{x\to\infty} \frac{1}{x} \cdot \sin x = \lim_{x\to\infty} \sin\left(\frac{1}{x} \cdot x\right) = \sin 1$.

5. Compute the following limits:

$$\text{(i)} \quad \lim_{x\to-\infty} \frac{\sqrt{3x^2+6}}{5-2x}$$

$$\text{(ii)} \quad \lim_{x\to\infty} \left(\sqrt{x^2+x}-x\right)$$

A. (i) $\frac{\infty}{\infty} = 1$ (ii) $\infty - \infty = 0$

B. (i) $\frac{\sqrt{3}}{2}$ (ii) 0

C. (i) $-\frac{\sqrt{3}}{2}$ (ii) 0

D. (i) $\frac{\sqrt{3}}{2}$ (ii) $\frac{1}{2}$ (correct)

E. (i) $-\frac{\sqrt{3}}{2}$ (ii) $\frac{1}{2}$

6. How many solutions are there on the interval $[0, 2\pi]$ for the equation $4\cos x = \sin(2x)$?

A. 0

B. 1

C. 2 (correct)

D. 3

E. 4

7. Mark asks the following question:

Is there a number c which is exactly 3 more than its cube c^3 ?

Nancy answers, saying "Yes. We can apply the Intermediate Value Theorem to the function $f(x) = (x^3 + 3) - x$ with $N = 0$."

Choose the interval where such a number c is guaranteed to exist as in Nancy's answer.

A. $(-3, -2)$

B. $(-2, -1)$ (correct)

C. $(-1, 0)$

D. $(0, 1)$

E. $(1, 2)$

8. Find the value of $f'(1)$ for the function $f(x) = \dfrac{4\sqrt{x}}{x^2 - 2}$.

A. 1

B. -1

C. 10

D. -10 (correct)

E. 0

9. Find the values of a and b so that the function

$$f(x) = \begin{cases} \dfrac{x^2 + x - a}{x + 3} & \text{if } \; x < -3 \\ \\ bx + 1 & \text{if } \; x \geq -3 \end{cases}$$

is continuous on $(-\infty, \infty)$.

A. $a = 6, b = 2$ (correct)

B. $a = 6, b = 1$

C. $a = 9, b = 2$

D. $a = 9, b = 1$

E. No matter how we choose the values for a and b, the function $f(x)$ can never be continuous on $(-\infty, \infty)$.

10. Let $f(x)$ be the function defined as follows:

$$f(x) = \begin{cases} 1 & \text{if} \quad x < 0 \\ e^x & \text{if} \quad x \geq 0. \end{cases} .$$

Compute the value of $f'(0)$ if it exists.

If $f'(0)$ does not exist, then write DNE.

A. 0

B. 1

C. 2

D. DNE (correct)

E. We can not determine the exact value of $f'(0)$ from the given information.

11. Find the equation(s) of

(i) the horizontal asymptote(s)
(ii) the vertical asymptote(s)

for the graph of the function

$$y = f(x) = \frac{5x^2 + 5x - 30}{x^2 + 2x - 3}.$$

A. (i) $y = 5$ (ii) $x = -3$ and $x = 1$

B. (i) $y = 5$ and $y = -5$ (ii) $x = -3$ and $x = 1$

C. (i) $y = -5$ (ii) $x = 1$

D. (i) $y = 5$ (ii) $x = 1$ (correct)

E. The graph has the slant asymptote $y = 5x - 10$, but neither horizontal nor vertical asymptote.

12. Choose the right description of the interval(s) where the function

$$f(x) = |(x+2)(x-1)^2|$$

is

(i) continuous,
(ii) differentiable.

A. (i) $(-\infty, \infty)$, (ii) $(-\infty, -2) \cup (-2, \infty)$ (correct)

B. (i) $(-\infty, \infty)$, (ii) $(-\infty, \infty)$

C. (i) $(-\infty, \infty)$, (ii) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

D. (i) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$ (ii) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

E. (i) $(-\infty, -2) \cup (1, \infty)$, (ii) $(-2, 1)$