Study Guide for Exam 1

1. Having the information on the range of a given rotation angle θ and knowing the value of a trigonometric function, you are supposed to be able to determine the values of the other trigonometric functions.

2. You are supposed to be able to determine the exact values of the formulas involving the exponential functions, logarithmic functions (which are inverse to each other) and the ones involving the trigonometric functions and their inverses. Special attention has to be paid about the range of the inverse trigonometric functions.

Example: $\sin^{-1}\left(\sin\frac{9\pi}{4}\right) \neq \frac{9\pi}{4}$

3. Given a one-to-one function, you are supposed to be able to find the formula, domain, and range of its inverse function.

4. You are supposed to be able to compute the limits, using the Squeeze Theorem and some basic limits such as $\lim_{\theta\to 0}\dfrac{\sin\theta}{\theta}$.

5. You are supposed to be able to compute the limits at infinity, especially when they are of the indetrminate forms $\dfrac{\infty}{\infty}$ and $\infty - \infty$.

6. You are supposed to be able to solve the equations involving the trigonometric functions, using the double angle formula (for sines and cosines).

7. You are supposed to know how to use the Intermediate Value Theorem and how to determine an interval where a solution of the given equation is guranteed to exist.

8. You are supposed to be able to compute the derivatives, using the definition of the derivative and using the product and quotient rules.

9. When a function is defined piecewise and depending on some variables, you are supposed to know how to determine those variables so that the function becomes continuous entirely over its domain.

10. You are supposed to be able to determine the equations of the horizontal asymptote(s) and vertical asymptote(s).

Example: $y = f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 1}$

11. Given a function, you should be able to tell where the function is continuous and/or differentiable.

Example: Determine the interval(s) where the following function is continuous and/or differentiable:

$$\begin{array}{ll} \text{(i)} & f(x) = |2x+1| \\ \text{(ii)} & g(x) = |(x+5)(x-3)| \\ \text{(ii)} & h(x) = |(x+5)^2(x-3)| \\ \text{(ii)} & \phi(x) = |(x+5)^2(x-3)(x-7)^2| \end{array}$$

Example: Determine whether the following functions are continuous and/or differentiable at $x = 0$.

$$\text{(i)} \quad f(x) = \begin{cases} \sin\left(\frac{1}{x}\right) & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$$

$$\text{(ii)} \quad g(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$$

$$\text{(ii)} \quad h(x) = \begin{cases} x^2\sin\left(\frac{1}{x}\right) & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$$