

Study Guide for the Final Exam

1. You are supposed to know how to determine the domain and range of a function given its expression.

2. You are supposed to be able to determine where the solutions lie for a given equation, using the Intermediate Value Theorem and the Mean Value Theorem.

Example Problems:

- How many solutions are there for the equation

$$2 \cos(x) = \sin(x) + 1$$

over the interval $[0, 2\pi]$?

- Problems 19 and 20 on Page 292 on the textbook.

3. You are supposed to know how to use L'Hospital's Rule to compute the limits of the form $\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$, as well as those of the form $\pm\infty \times 0$, $\infty - \infty$.

4. You are supposed to be able to compute the derivative $\frac{dy}{dx}$ of a function of the form $y = [f(x)]^{g(x)}$.

5. You are supposed to know the meaning of a function being continuous.

Example Problems:

- Problem 18 in Webassign HW 5.
- Problem 1 in Webassign HW 6.

6. You are supposed to know the relation between the derivative of a function f and that of its inverse g , using the equation $g(f(x)) = x$ and the chain rule.

Example Problem:

- Suppose we have a function f such that $f(5) = 7$ and $f'(5) = -6$. What is the value of $g'(7)$ where g is the inverse of f .

7. You are supposed to be able to determine the equation of the tangent line at a given point on the curve, using the Implicit Differentiation.

8. You are supposed to know how to compute and derive the derivatives of the inverse trigonometric functions, where you should be able to express the trigonometric functions with angle θ (or 2θ) in terms of x when $\theta = \sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$. (Look at Pages 213-214 of the textbook.)

Example Problems:

- Problems 15-17 in Webassign HW 12.

9. You are supposed to be able to compute the limits of an intermediate form of type $\infty - \infty$ (Look at Page 309 of the textbook. You should also look at Section 2.6 of the textbook.)

10. You are supposed to know how to use the Linear Approximation to give a good estimate (approximation) of the value of a function.

11. You are supposed to know how to determine the absolute maximum and absolute minimum of a function over a closed interval, by comparing the values at the end points and at the critical points.

12. You are supposed to know, using the Intermediate Value Theorem, how to locate an interval where the solution of an equation lies. (Look at Pages 122 - 123 of the textbook.)

13. You are supposed to be able to evaluate the integral involving the absolute value sign, by splitting the given integration into the intervals where the expression of the absolute value sign changes.

14. You are supposed to be able to determine the local maximum and local minimum of a function, using the 1st derivative test and/or 2nd derivative test.

Example Problems:

- Problems 2 and 3 in Webassign HW 21.
- Problem 2 in Version 01 of Exam 3 of 2014

15. You are supposed to be able to compute the limits $\lim_{x \rightarrow a} [f(x)]^{g(x)}$ of the form $0^0, \infty^0, 1^\infty$.

16. You are supposed to be able to recognize the formula of the form $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \delta x$ as the Riemann sum, and accordingly compute its value using the integration.

Example Problems:

- Problems 5 and 6 in Webassign HW 28.
- Problems 24 and 25 on Page 377 of the textbook (evaluate the value of the integration).

17. You are supposed to be able to solve the related rates problems.

Example Problems:

- Problems in Webassign HW 15.
- In a right triangle, leg x is increasing at a rate of 3 m/s, while leg y is decreasing at a rate that keeps the area of the triangle constant at 30 m². What is the rate of the change of the hypotenuse when $x = 12$ m ?

18. You are supposed to be able to solve the optimization problems.
Example Problems:

- Problems 4-6 in Webassign HW 21.
- Problems 2,4,5 in Webassign HW 22.
- Example 3 on Page 333 of the textbook

19. You are supposed to compute the (indefinite) integral using the substitution rule, and also to evaluate the (definite) integral using the substitution rule.

20. You are supposed to know how to solve the differential equation $\frac{dP}{dt}/P = kP$ for the population growth and the formula for its solution $P(t) = P(0)e^{kt}$. In the case of radioactive decay, you should also know the formula using the half-life $m(t) = m(0)e^{kt} = m(0)2^{-\frac{t}{h}}$, including the relation between the value of k (the negative of the relative decay rate) and half-life h .

21. You are supposed to be able to sketch the graph of a function by using the information obtained from its first derivative and second derivative, determining the horizontal, vertical, slant asymptotes.

22. You are supposed to know how to use the 2nd part of the Fundamental Theorem to Calculus to compute the derivative of a function defined through the integration.

Example Problems:

- Problems 2-5 in Webassign HW 29.

23. You are supposed to be able to determine the velocity function, position function taking the anti-derivatives, given the acceleration function with some initial data. You should also know how to compute the total distance traveled.

Example Problems:

- Problem 9 in Webassign HW 30.
- Example 1 on Page 224 of the textbook.

24. You are supposed to know the following about the conic sections:

Parabola: Knowing the standard form of the equation, you should be able to derive the equation of the directrix and determine the focus. Conversely, knowing the directrix and the focus, you should be able to determine the standard form of the equation.

You should also know the geometric characterization of a parabola in terms of the directrix and focus.

Ellipse: Knowing the standard form of the equation, you should be able to determine the major axis, minor axis, and foci. Conversely,

knowing the major axis, minor axis, and foci (or only some sufficient part of them), you should be able to determine the standard form of the equation.

You should also know the geometric characterization of an ellipse in terms of the sum of the distances from the foci to the point on the ellipse.

Hyperbola: Knowing the standard form of the equation, you should be able to determine the vertices, asymptotes, and foci. Conversely, knowing the vertices, asymptotes, and foci (or only some sufficient part of them), you should be able to determine the standard form of the equation.

You should also know the geometric characterization of a hyperbola in terms of the difference of the distances from the foci to the point on the ellipse.

25. You are supposed to understand all the material about the conic section mentioned above when there are shifts in x - and y - coordinates. You should also be able to determine the shifts from the given equation of the conic section

Example Problems:

- Problems 33-35 in Final Exam Practice Questions.
- Problems 1-8 in Webassign HW 35.

26. You are supposed to be able to determine the concavity (concave up or down) of the graph of a function by looking at the behavior of its second derivative, and to determine where the inflection points are.

Example Problems:

- We have a function $y = f(x)$ whose second derivative is given by the formula

$$f''(x) = x(x-1)^2(x-2)^3(x-3)^4.$$

Find the x -coordinates of ALL the inflection points.

27. You are supposed to know how to use L'Hospital's Rule to compute the limits of the form $\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$, as well as those of the form $\pm\infty \times 0$, $\infty - \infty$.