MA 15910 Lesson 18 Notes
(Calculus part of text) Section 4.3 (part 2)

The Chain Rule

Chain Rule Forms:

If $y = f(u)$ and $u = g(x)$, such that $y = f(u) = f[g(x)]$, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad or \quad \frac{dy}{dx} = f'[g(x)] \cdot g'(x)$$

Derivative of a composite function = derivative of the 'outer' function (with respect to $g(x)$ or inner function) times the derivative of the 'inner' function (with respect to x).

EX 1) Use the table below to find: (a) $D_x(g[f(x)])$ at $x = 2$, (b) $D_x(g[f(x)])$ at $x = 1$.

x	1	2	3	4
$f(x)$	2	3	1	2
$f'(x)$	-8	-9	-10	-11
$g(x)$	2	1	4	3
$g'(x)$	0	5	6	9

a) $D_x(g[f(x)])$ (at $x = 2$) = $g'[f(2)] \cdot f'(2)$

b)

Problems 2 and 3:

Find the equation of the tangent lines to the graph of the given function at the given value of *x*.

2) $f(x) = (x^3 + 1)^{\frac{3}{2}}$ for $x = 2$

a) Find derivative of *f*. Evaluate when *x* = 2 to find slope.

b) Find the point of *f* when *x* = 2.

c) Use the point and slope to find the equation of the tangent line.

a)

$$f'(x) = \text{dee outer times dee inner}$$

$$f'(x) = dee\ (inner)^{3/2} \cdot dee(x^3 + 1)$$

$$f'(x) = \frac{3}{2}(inner)^{1/2} \cdot (3x^2 + 0)$$

$$=$$

b) $f(2) = (2^3 + 1)^{3/2} = \left(\sqrt{(8+1)}\right)^3$

c)

3) $g(x) = x^2\sqrt{x^4 - 12}$ for $x = 2$

a) Find the derivative of g. Evaluate when $x = 2$ to find slope.

b) Find the point of g when $x = 2$.

c) Use the point and slope to find the equation of the tangent line.

Find all values of x (and the points) for the given functions where the tangent line is horizontal for problems 4 and 5.

4) $f(x) = 2\left(\frac{3x-6}{5x}\right)^2$

The slope of a horizontal line is zero. The derivative of a function is the slope of the tangent line for every x. Therefore: Find the derivative and set it equal to zero. Solve.

5) $h(x) = 2\left(\sqrt{x}\right)^3 - \left(\sqrt{x}\right)^5$ Assume that $x \geq 0$. (Otherwise, the square root would be undefined.)

5 1/2) $j(x) = -x^2 - 4x - 5$

6) A sum of \$2500 is deposited in an account with an interest rate of *r*% per year, compounded daily. At the end of 5 years, the balance in the account is given by $A = 2500\left(1+\frac{r}{36500}\right)^{1825}$. Find the rate of change of *A* with respect to *r* if *r* = 4.

7) The value V of a machine t years after it is purchased is given by the equation $V(t)=\frac{10000}{\sqrt{t+1}}$. Find the rate of depreciation when (a) t = 1 and when (b) t = 3.

Hint: Let rewrite the function V as $V(t)=10000(t+1)^{-1/2}$