MA 15910 Lesson 22
(2^{nd} half of textbook, Section 4.5)
Derivatives of Natural Logarithmic Functions

Derivative of $\ln x$**:**

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \text{ for } x > 0$$

The derivative of ln *x* is the reciprocal of *x*.

*****There is a justification for this rule on page 237 of the textbook. We will accept it as true without proof.

There is also a rule on page 237 of the text for finding derivatives of logarithmic expressions to a base other than base *e*. You may study some of these types of problems, but none are on homework or quizzes/tests. This rule is as follows: Let *a* be any logarithmic base other than *e*, then $\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$

CHAIN RULE FOR DERIVATIVE OF A NATURAL LOGARITHMIC FUNCTION

$$\frac{d}{dx}(\ln g(x)) = \frac{1}{g(x)}(g'(x)) \quad (\text{for } g(x) > 0)$$

The derivative of $\ln g(x)$ ($g(x)$ is a function of x) is the reciprocal of $g(x)$ times the derivative of $g(x)$.

For the next few problems, ***assume the value of the variable is positive***.

*****Note: There is one problem in MyMathLab where the variable may not be a positive value. That problem has an absolute value symbol around the variable. You must give the answer with the absolute value sign included. (I believe it is problem 5 in MML.)*****

<u>Before finding derivatives of the examples below, use rules of logarithms (properties of logarithms) where possible</u>.

Ex 1: Find the derivative of $y = \ln(12x)$

NOTE:
If *c* is a positive constant,

$$\frac{d}{dx}\ln(cx) = \frac{d}{dx}(\ln c + \ln x)$$

$$= \frac{d}{dx}(\text{constant}) + \frac{d}{dx}(\ln x)$$

$$= 0 + \frac{1}{x} = \frac{1}{x}$$

Ex 2: Find $D_x[\ln(x^2 - 4x)]$ Note: No rule of logarithms exist for a difference. The chain rule will have to be used.

Ex 3: Find the derivative of *f*, where $f(x) = \ln(3x^3 - x)^{3/2}$.

Ex 4: If $y = \frac{3x+2}{2\ln x}$, find y'. Note: The rule for finding the derivative of a natural logarithmic function is used within the quotient rule.

Ex 5: Find the derivative of $g(x) = (\ln 2)[\ln(12x)]$
(Note: natural log of 2 is a constant.)

Ex 6: $y = e^x(\ln x)$

Ex 7: $f(x) = \sqrt{e^x - \ln(3x)}$

(Use chain rule. Let the 'outer function' be (inner)$^{1/2}$ and the 'inner function' be $e^x - \ln(3x)$.)

Ex 8:

The total revenue (in dollars) received from the sale of x items is given by $R(x) = 20\ln(x+1)$ and the total cost (in dollars) to produce x items is given by $C(x) = \frac{2x}{3}$. (Use the back of the paper for extra space.)

a) Find the marginal revenue.
b) Find the profit function and the marginal profit function.
c) Find the marginal revenue when $x = 40$. Interpret.
d) Find the marginal profit when $x = 40$. Interpret.

Ex 9: Suppose the demand function for x units of a certain product is

$$p = D(x) = 70 + \frac{10}{\ln x}, \quad x > 1 \text{ and } p \text{ is in dollars.}$$

a) Find the revenue function and marginal revenue function.

b) Approximate the revenue from one more unit when 10 units are sold.