MA 15910 Lesson 23 Notes
2nd half of textbook, Section 5.1
Increasing and Decreasing Functions

A function is **increasing** if its graph goes **up (positive slope)** from left to right and **decreasing** if its graph goes **down (negative slope)** from left to right. When describing where a function is increasing, use open interval notation of *x* values (domain values, left to right). When describing where a function is decreasing, use open interval notation of *x* values (domain values, left to right).

The FIGURE 2 below (also found in your textbook on page 253), are pictures of 3 graphs (a, b, and c) where the function is increasing and pictures of 3 graphs (d, e, and f) where the function is decreasing. Increasing means 'rising' or 'climbing', always left to right (slopes of tangent lines to the curve would be positive). Decreasing means 'falling' or 'sliding down', always left to right (slopes of tangent lines to the curve would be negative). **In an increasing function; as the *x* values get larger, so do the *y* values or the function values. In a decreasing function; as the *x* values get larger, the *y* values (function values) get smaller.** Using 'algebra' language, the definitions of increasing or decreasing functions are found on page 253 of the textbook.

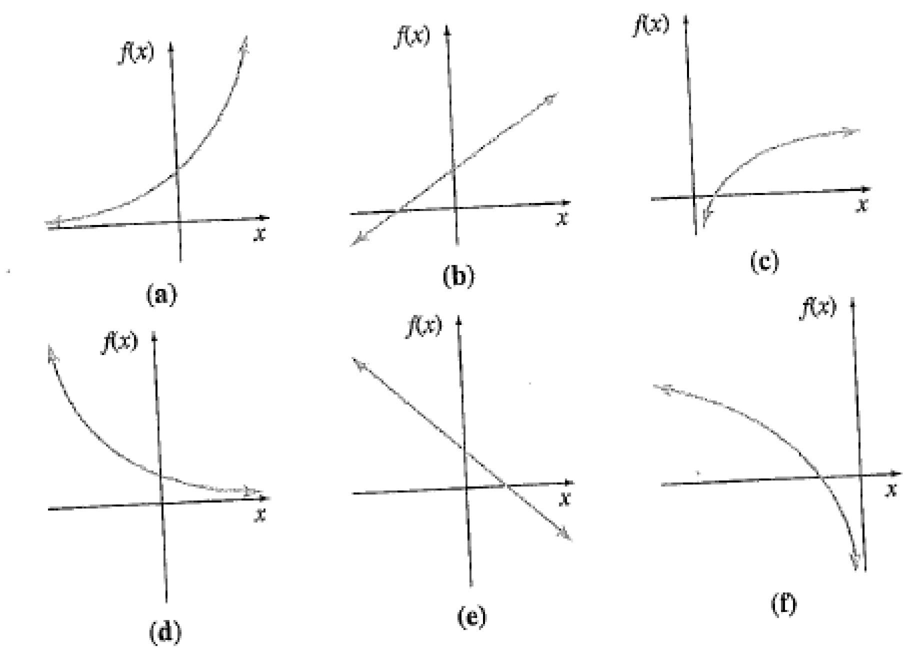
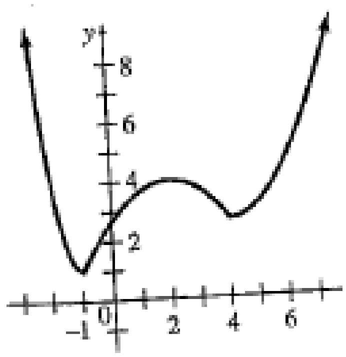


FIGURE 2

Graphs (a), (b), and (c) are increasing. Graphs (d), (e), and (f) are decreasing.

Example 1: Look at the graph below. Describe the **open interval(s)** where the function is increasing and the open interval(s) where the function is decreasing. (Always use 'open' intervals in MyMathLab, quizzes, exams.)



Increasing:______________________

Decreasing:______________________

Using the first derivative to help determine intervals of increasing/decreasing:

Suppose a function *f* has a derivative at each point in an open interval; then

1) if $f'(x) > 0$ for each *x* in the interval, *f* is **increasing** on that interval.
2) if $f'(x) < 0$ for each *x* in the interval, *f* is **decreasing** on that interval.
3) if $f'(x) = 0$ for each *x* in the interval, *f* is **constant** on that interval (not increasing nor Decreasing, graph would be a constant horizontal line).

In MML, a critical value must be in the domain.

*****Critical Values or Critical Numbers and Critical Points*****

To find possible intervals of increasing or decreasing, we use **critical values.** The critical values of a function are those numbers **in the domain** of the function for which the derivative is zero or the derivative does not exist. Critical values are the *x*-coordinates of the critical points. A **critical point** is the ordered pair whose *x*-coordinate is the critical value *c* and whose *y*-coordinate is $f(c)$; $(c, f(c))$. The critical values and a **sign chart** can be used to determine intervals of increasing, decreasing, or constant.

A first derivative 'sign chart' (where the intervals are found using critical values and/or values where the function is undefined) is used to find where a function is increasing and/or where a function is decreasing. These types of problems will now be demonstrated.

Example 2: Find the intervals, if any, where the following function f is increasing and the intervals, if any, where the function f is decreasing.

$$f(x) = \frac{2}{3}x^3 - x^2 - 4x + 2$$

step 1) Find the critical value(s); values of x (in domain) where the derivative is zero (or undefined).

$$f'(x) = 2x^2 - 2x - 4$$

$$2x^2 - 2x - 4 = 0$$

$$2(x^2 - x - 2) = 0$$

$$(x-2)(x+1) = 0$$

$$x - 2 = 0 \qquad x + 1 = 0$$

$x = 2 \qquad x = -1$ ⟶ Note: Critical points would be $(2, -\frac{14}{3})$ and $(-1, \frac{13}{3})$.

step 2) Use the critical values to write intervals to be tested and make a sign chart using those intervals.

The intervals to be checked would be $(-\infty, -1)$, $(-1, 2)$, and $(2, \infty)$.

Make a sign chart, such as the one below.

SIGN CHART

	$(-\infty,-1)$	-1	$(-1,2)$	2	$(2,\infty)$
$x-2$	-	o	-	o	+
$x+1$	-		+		+
Result	+ INC.		- DEC.		+ INC.

I used open circles on the number line because these are 'open' intervals.

Rather than the union sign below, MyMathLab uses commas between intervals.

3) Give the answer.

The function is increasing on $(-\infty, -1) \cup (2, \infty)$ **and decreasing on** $(-1, 2)$.

Example 3: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation (open intervals).

$$g(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + 2$$

Example 4: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation.

$$y = x^4 + 8x^3 + 18x^2 - 8$$

Example 5: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation.

$$y = -\frac{3}{2}x + 2$$

Example 6: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation.

$$f(x) = \frac{x+3}{x-4}$$

Example 7: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation.

$$y = x^{2/3}$$

Example 8: Find all open intervals where the function below is increasing, decreasing, or constant. Write answers using interval notation.

$$g(x) = x \cdot e^{-x^2}$$

Example 9:
A manufacturer of CD players has determined that the profit $P(x)$ (in thousands of dollars) is related to the quantity x of CD players produced (in hundreds) per month by the model,
$P(x) = -(x-4)e^x - 4,\ \ 0 < x \le 3.9$.
At what production levels is the profit increasing? Decreasing?

$P(x) = (-x+4)e^x - 4$

	(0,3)	(3, 3.9)
e^x		
$4-x$		
Result		

Example 10:
The percent of concentration of a drug in the bloodstream x hours after the drug is administered is given by $K(x) = \dfrac{4x}{3x^2 + 27}$. On what time intervals is the concentration of the drug increasing? Decreasing?

	(0,3)	(3, ∞)
4		
$3 + x$		
$3 - x$		
3		
$(x^2 + 9)^2$		
Result		