MA 15910, Lesson 28 Notes
(sketching graphs part 2)

Reminder of Steps for Sketching Curves:

GUIDELINES FOR SKETCHING CURVES

1. Consider the domain of the function, and note any restrictions.

2. Find the ***y*-intercept** (if it exists) and **any *x*-intercepts**, if it is not too difficult to do so.

3. If the function if a rational function, find **any vertical asymptotes** and **any horizontal asymptotes.** If the function is an exponential function, find any horizontal asymptotes; if it is a logarithmic function, find any vertical asymptotes.

4. Investigate possible symmetry. If $f(-x) = f(x)$, the function is even, so the graph is symmetric about the y-axis. If $f(-x) = -f(x)$, the function is odd, so the graph is symmetric about the origin.

5. Find the first derivative. Locate any critical values by finding where the first derivative is zero or undefined. **Find intervals where the function is increasing or decreasing and any relative extrema (maximums or minimums).**

6. Find the second derivative. Locate **possible inflection points** by finding where the second derivative is zero or does not exist. Determine intervals where the function is **concave upward or concave downward.**

7. Plot the intercepts, the critical points, the inflection points, the asymptotes, and other points as needed. Take advantage of any symmetry found in step 4.

8. Connect the points with a smooth curve using the correct concavity, being careful not to connect points where the function is not defined.

Hints to creating good sketches:

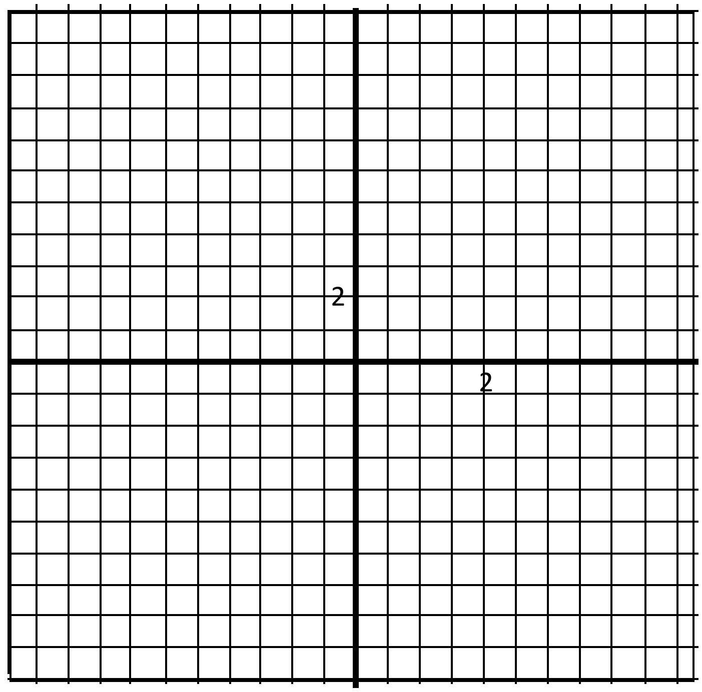
1) Use graph paper if possible. Draw straight perpendicular axes.
2) Use a uniform equally spaced reasonable scale for each axis. (You do not have to use the same scale for both axes.)
3) In addition to any intercepts, relative extrema, or point(s) of inflection, you may want to find some additional points to fill in the sketch.
4) Remember that a graph never touches or crosses a vertical asymptote, but may cross a horizontal asymptote.
5) Keep in mind where the graph is increasing/decreasing and concave upward/downward and carefully and neatly sketch your graph.

Example 1: $f(x)=\dfrac{x^2}{x-1}$

Domain: D =

Asymptote(s):

Intercept(s):



$$f'(x)=\frac{(x-1)(2x)-x^2(1)}{(x-1)^2}$$

$$=\frac{2x^2-2x-x^2}{(x-1)^2}$$

$$=$$

First Derivative 'Sign Chart':

conclusions:

$f''=$ (use back of paper)

Second derivative 'Sign Chart':

conclusions:

few other points

x	y
$1\frac{1}{2}$	4
-2	$-1\frac{1}{3}$
3	$4\frac{1}{2}$
4	$5\frac{1}{3}$
$\frac{1}{2}$	$-\frac{1}{2}$
-1	$-\frac{1}{2}$

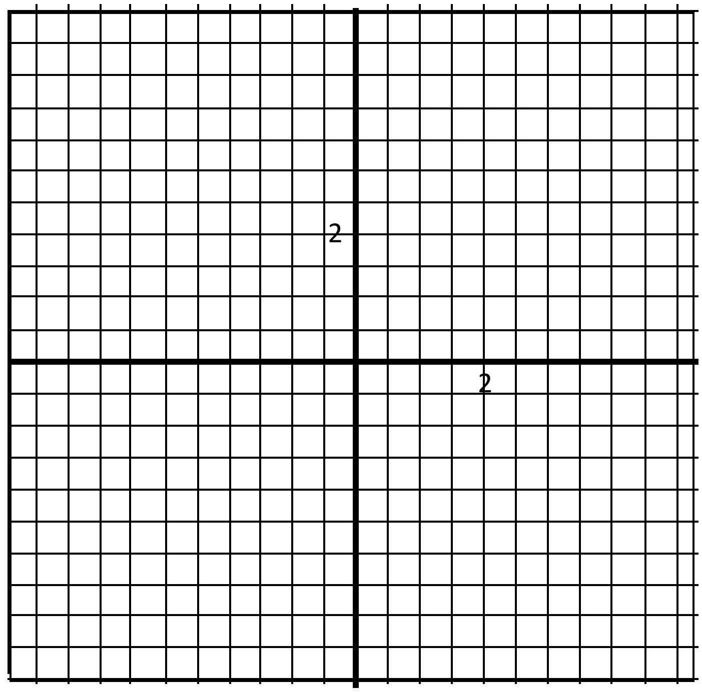
Example 2: $f(x)=\dfrac{2}{x^2+1}$

Domain: $D=$
Intercepts:

There are no x-intercept.

Horizontal Asymptote:

$f(-x)=f(x)$ Symmetry about y-axis



1^{st} derivative 'Sign Chart'

Conclusions…

A few other points…

x	y
± 2	$\frac{2}{5}$
± 3	$\frac{1}{5}$
$\pm\frac{3}{2}$	~ 0.6
± 1	1
$\pm\frac{1}{2}$	1.6

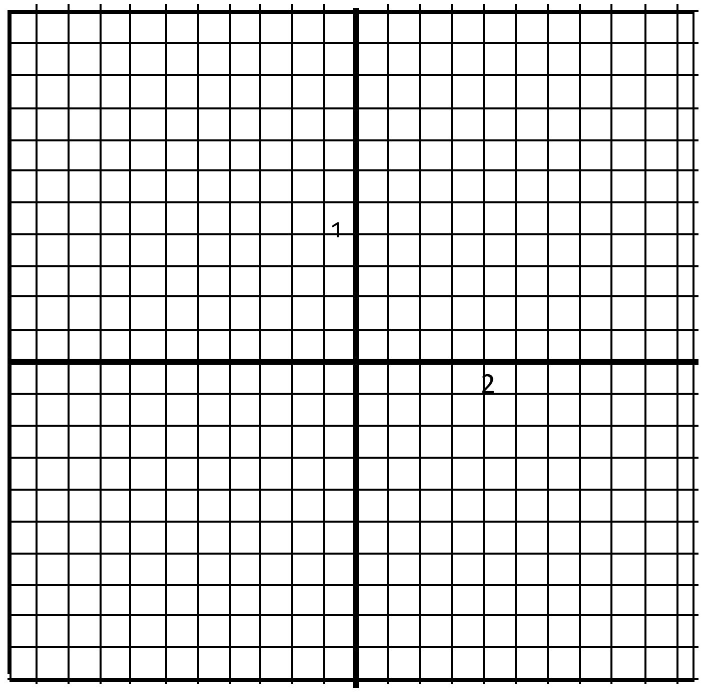
Use back of paper for finding second derivative and 2^{nd} derivative 'sign chart'.

Example 3: $g(x) = \dfrac{x}{(x+1)^2}$

Domain:

Intercept(s):

Vertical Asymptote:
Horizontal Asymptote:



First Derivative 'Sign Chart':

Conclusions…

A few other points…

x	y
2	
-2	
-3	
-4	
$-\frac{1}{2}$	

2nd Derivative 'Sign Chart':

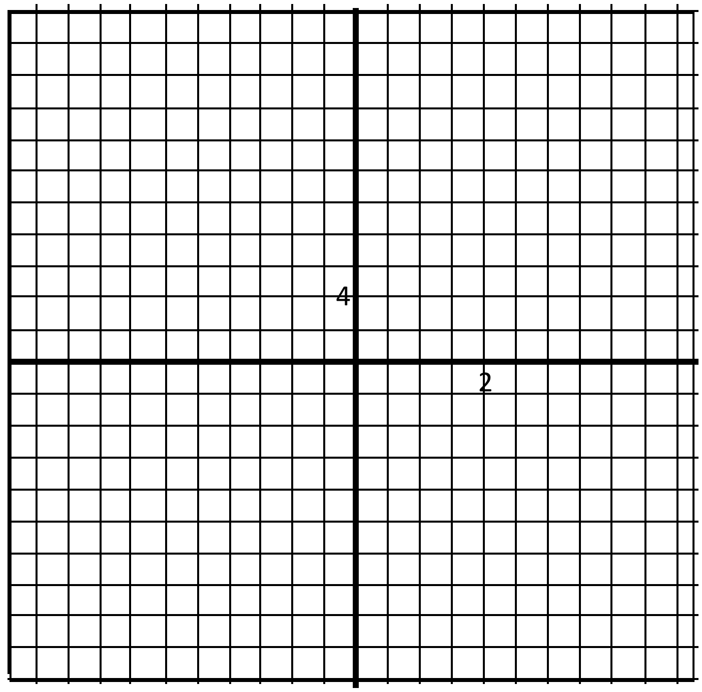
conclusions…

Example 4: $y = \dfrac{x^2 - 3x}{x+1}$

Asymptotes:

Domain:

Intercepts:



Symmetry?

1st derivative 'sign chart'

conclusions…

Few other points…

x	y
-2	-10
-6	-10.8
-4	$-9\frac{1}{3}$
$-\frac{1}{2}$	3.5
2	$-\frac{2}{3}$
-1.5	-13.5

Use the back of the sheet to find 2nd derivative and make a 2nd derivative 'sign chart'.

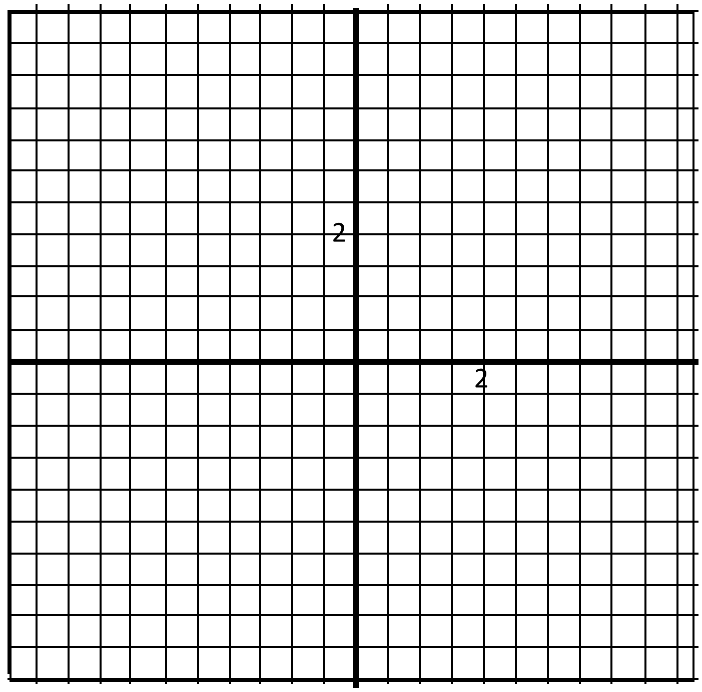
<u>Ex 5:</u> $f(x) = xe^{x}$

Domain:

Intercepts:

Asymptotes:

Symmetry?



1^{st} derivative 'sign chart'

conclusions…

Use the back of the sheet to find 2^{nd} derivative and make a 2^{nd} derivative 'sign chart'.

Ex 6: $y = \dfrac{\ln x}{2x}$

Domain:

Vertical Asymptote:

Intercept(s);

1st derivative 'sign chart':

$$y' = \frac{2x\left(\frac{1}{x}\right) - (\ln x)(2)}{(2x)^2} = \frac{2 - 2\ln x}{4x^2}$$

$$= \frac{2(1 - \ln x)}{4x^2} = \frac{1 - \ln x}{2x^2}$$

Conclusions from sign chart:

2nd derivative sign chart:

$$y'' = \frac{2x^2(0 - \frac{1}{x}) - (1 - \ln x)(4x)}{(2x^2)^2} = \frac{-2x - 4x + 4x(\ln x)}{4x^4}$$

$$= \frac{-6x + 4x(\ln x)}{4x^4} = \frac{2x(-3 + 2(\ln x))}{4x^4} = \frac{2\ln x - 3}{2x^3}$$

Conclusions from sign chart: