Addition for Lesson 25 Notes, Summer 2016

In the system of equations below, find the value of b such that the system will have…
(a) 1 solution
(b) 2 solutions
(c) no solution
Interpret each graphically.

Quadratic Formula for $Ax^2 + Bx + C = 0$ is $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$. The part of this formula under the square root sign is called the discriminant.

Value of Discriminant	**Number/Types of Solutions**
Zero	One Rational Solution
Positive Perfect Square	Two Rational Solutions
Positive, but not perfect square	Two Irrational Solutions
Negative	No Real Solutions

Find the value of b in the system of equations below, such that the system has….
(a) 1 solution
(b) 2 solutions
(c) no solution Interpret each graphically.

$$\begin{cases} x^2 + y^2 = 4 \\ y = x + b \end{cases}$$

(This is a system with a circle and a line.)

Using substitution of the value of y from second equation for y in the first equation…

$$x^2 + (x+b)^2 = 4$$
$$x^2 + x^2 + 2bx + b^2 = 4$$
$$2x^2 + 2bx + (b^2 - 4) = 0$$

Solve using the quadratic formula:
$A = 2, \quad B = 2b, \quad C = b^2 - 4$
I am using capital letters above, so there is no confusion with the variable b in the original problem.

(a) If there is only 1 solution, the discriminant must be equal to zero.

$$B^2 - 4AC = 0$$
$$(2b)^2 - 4(2)(b^2 - 4) = 0$$
$$4b^2 - 8(b^2 - 4) = 0$$
$$4b^2 - 8b^2 + 32 = 0$$
$$32 = 4b^2$$
$$8 = b^2$$
$$\pm\sqrt{8} = b \qquad \text{(or } b = \pm 2\sqrt{2} \text{ if the radical is simplified)}$$

There will be only 1 solution if the system has $b = \pm\sqrt{8}$. In other words, the system would be….

$\begin{cases} x^2 + y^2 = 4 \\ y = x + \sqrt{8} \end{cases}$ or $\begin{cases} x^2 + y^2 = 4 \\ y = x - \sqrt{8} \end{cases}$ **This would occur if the line was tangent to the circle.**

(b) If there are 2 solutions, the discriminant would be greater than zero (positive).

$$B^2 - 4AC > 0$$
$$(2b)^2 - 4(2)(b^2 - 4) > 0$$
$$4b^2 - 8(b^2 - 4) > 0$$
$$4b^2 - 8b^2 + 32 > 0$$
$$-4b^2 + 32 > 0$$
$-4(b^2 - 8) > 0$ Divide both sides by -4, switch ineqality symbol.
$$b^2 - 8 < 0$$
$b^2 < 8$ The zeros are $\pm\sqrt{8}$. Use the zeros to make a 'sign chart'.

	$(-\infty, -\sqrt{8})$	$(-\sqrt{8}, \sqrt{8})$	$(\sqrt{8}, \infty)$	
$b^2 - 8$	+	−	+	
result	positive	negative	positive	We want the negative result.

There will be 2 solutions if the system has a *b* value in the second equation between $-\sqrt{8}$ and $\sqrt{8}$.
This would occur if the line intersects the circle at two points.

c) If there is no solution, the discriminant would be less than zero (negative).

$$B^2 - 4AC < 0$$
$$(2b)^2 - 4(2)(b^2 - 4) < 0$$
$$4b^2 - 8(b^2 - 4) < 0$$
$$4b^2 - 8b^2 + 32 < 0$$
$$-4b^2 + 32 < 0$$
$-4(b^2 - 8) < 0$ Divide both sides by -4, switch ineqality symbol.
$$b^2 - 8 > 0$$
$b^2 > 8$ The zeros are $\pm\sqrt{8}$. Use the zeros to make a 'sign chart'.

	$(-\infty, -\sqrt{8})$	$(-\sqrt{8}, \sqrt{8})$	$(\sqrt{8}, \infty)$	
$b^2 - 8$	+	−	+	
result	positive	negative	positive	We want the positive results.

There will be no solution if the system has a *b* value in the second equation that is less than $-\sqrt{8}$ or greater than $\sqrt{8}$.
This would occur if the line and the circle do not intersect.