Lesson 25 (part 2)

Operations with Matrices

A matrix is an array of numbers with rows and columns. Matrices are classified by the number of rows and number of columns. Below are some illustrations of matrices. (Notice the number of rows is listed first.)

2×3 matrix

$$\begin{bmatrix} -5 & 3 & 1 \\ 7 & 0 & -2 \end{bmatrix}$$

2×2 matrix

$$\begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$$

3×2 matrix

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 8 & 7 \end{bmatrix}$$

3×1 matrix

$$\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

Two matrices may be added/subtracted only if they have the same size. Add (or subtract) corresponding elements.

<u>Ex 11:</u>

$$\begin{bmatrix} 8 & 5 \\ -1 & 3 \\ 0 & -2 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ -4 & 0 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ -5 & 3 \\ 8 & -4 \end{bmatrix}$$

A product of a real number and a matrix is defined as follows. Let n = a real number and a, b, c, and d represent the numbers in the matrix. This product definition can be extended to any sized matrix.

$$n\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} na & nb \\ nc & nd \end{bmatrix}$$

<u>Ex 12:</u>

$$4\begin{bmatrix} 5 & 2 \\ -1 & 0 \\ 9 & 1 \end{bmatrix} - \begin{bmatrix} -2 & 3 \\ 1 & 8 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ -4 & 0 \\ 36 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ -1 & -8 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 5 \\ -5 & -8 \\ 34 & 7 \end{bmatrix}$$

Definition of the Product of Two Matrices:

Let A (a_{ij}) = an $m\times n$ matrix and B (b_{ij}) = an $n\times p$ matrix. (Notice: The number of columns of the first matrix equals the number of rows of the second matrix.) The product AB is the $m\times p$ matrix C where $c_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+a_{i3}b_{3j}+...+a_{in}b_{nj}$ for $i=1,2,3,...,m$ and $j=1,2,3,...,p$

Examine the following problem and how the numbers in C_{23} and C_{14} were found.

$$\begin{bmatrix}1&2&-3\\4&0&-2\end{bmatrix}\begin{bmatrix}5&-4&2&0\\-1&6&3&1\\7&0&5&8\end{bmatrix}=\begin{bmatrix}?&?&?&-22\\?&?&-2&?\end{bmatrix}$$

$$C_{23}=4\cdot2+0\cdot3+(-2)\cdot5=8+0-10=-2$$

$$C_{14}=1\cdot0+2\cdot1+(-3)\cdot8=0+2-24=-22$$

Find each product.

<u>Ex 13:</u> The first matrix is 3×2 and the second is 2×1. The product will be 3×1

$$\begin{bmatrix}-2&4\\0&-1\\5&3\end{bmatrix}\begin{bmatrix}-2\\1\end{bmatrix}=\begin{bmatrix}-2(-2)+4(1)\\0(-2)-1(1)\\5(-2)+3(1)\end{bmatrix}=\begin{bmatrix}0\\-1\\-7\end{bmatrix}$$

<u>Ex 14:</u> The first matric is 2×1 and the second is 1×2. The product will be 2×2

$$\begin{bmatrix}-2\\3\end{bmatrix}\begin{bmatrix}1&5\end{bmatrix}=\begin{bmatrix}-2(1)&-2(5)\\3(1)&3(5)\end{bmatrix}=\begin{bmatrix}-2&-10\\3&15\end{bmatrix}$$

Extra Example:

$$\begin{bmatrix}1&2&-3\\4&0&-2\end{bmatrix}\begin{bmatrix}5&-4&2&0\\-1&6&3&1\\7&0&5&8\end{bmatrix}$$

$$=\begin{bmatrix}1(5)+2(-1)-3(7)&1(-4)+2(6)-3(0)&1(2)+2(3)-3(5)&1(0)+2(1)-3(8)\\4(5)+0(-1)-2(7)&4(-4)+0(6)-2(0)&4(2)+0(3)-2(5)&4(0)+0(1)-2(8)\end{bmatrix}$$

$$=\begin{bmatrix}-18&8&-7&-22\\6&-16&-2&-16\end{bmatrix}$$

Ex 15: Given the matrices $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$, find AB.

Matrix A is 2 × 3. Matrix B is 3 × 3. The product AB will be 2 × 3.

$$AB = \begin{bmatrix} 2(1)+0(3)+1(0) & 2(-1)+0(1)+1(2) & 2(2)+0(0)+1(1) \\ -1(1)+2(3)+0(0) & -1(-1)+2(1)+0(2) & -1(2)+2(0)+0(1) \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 0 & 5 \\ 5 & 3 & -2 \end{bmatrix}$$

Ex 16: Using matrix B in example 15, find B^2.

Matrix B is 3 × 3. . The product B^2 be 3 × 3.

$$B^2 = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1(1)-1(3)+2(0) & 1(-1)-1(1)+2(2) & 1(2)-2(0)+2(1) \\ 3(1)+1(3)+0(0) & 3(-1)+1(1)+0(2) & 3(2)+1(0)+0(1) \\ 0(1)+2(3)+1(0) & 0(-1)+2(1)+1(2) & 0(2)+2(0)+1(1) \end{bmatrix}$$

$$B^2 = \begin{bmatrix} -2 & 2 & 4 \\ 6 & -2 & 6 \\ 6 & 4 & 1 \end{bmatrix}$$

Ex 17: Given matrix $M = \begin{bmatrix} 8 & -2 \\ -1 & 5 \end{bmatrix}$. Compute $3M - M^2$

$$3M - M^2 = 3\begin{bmatrix} 8 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 8 & -2 \\ -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 8 & -2 \\ -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & -6 \\ -3 & 15 \end{bmatrix} - \begin{bmatrix} 66 & -26 \\ -13 & 27 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & -6 \\ -3 & 15 \end{bmatrix} + \begin{bmatrix} -66 & 26 \\ 13 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} -42 & 20 \\ 10 & -12 \end{bmatrix}$$

Ex 18: Write an augmented matrix for this system of equations.

$$\begin{cases} x - 2y + 3z = 4 \\ 2x + y - 4z = 3 \\ -3x + 4y - z = -2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ -3 & 4 & -1 & -2 \end{array}\right]$$

Ex 18.5: Perform the row operation $2R_1 + R_2 \to R_2$ and write the resulting augmented matrix.

$$2R_1 + R_2 = 2(1) \quad 2(-2) \quad 2(3) \quad 2(4)$$
$$2 \quad 1 \quad -4 \quad 3$$

$2R_1 + R_2$ is $4 \quad -3 \quad 2 \quad 11$

Replace R_2 with the above result.

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 4 \\ 4 & -3 & 2 & 11 \\ -3 & 4 & -1 & -2 \end{array}\right]$$