Slant or Oblique Asymptotes

Given a rational function $f(x)=\frac{g(x)}{h(x)}$: A slant or oblique asymptote occurs if the degree of $g(x)$ is exactly 1 greater than the degree of $h(x)$. To find the equation of the slant asymptote, use long division dividing $g(x)$ by $h(x)$ to get a quotient $ax+b$ with a remainder, $r(x)$. The slant or oblique asymptote has the equation $y=ax+b$.

Ex 1: Find the asymptotes (vertical, horizontal, and/or slant) for the following function.

$$f(x)=\frac{x^2-9}{2x-4}$$

A vertical asymptote is found by letting the denominator equal zero.

$$2x-4=0$$

$$2x=4$$

$$\boxed{x=2 \quad \text{equation for the vertical asymptote}}$$

A horizontal asymptote is found by comparing the leading term in the numerator to the leading term in the denominator. Since the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.

The oblique or slant asymptote is found by dividing the numerator by the denominator. A slant asymptote exists since the degree of the numerator is 1 greater than the degree of the denominator.

$$\begin{array}{r|l} & \frac{1}{2}x \quad +1 \\ \hline 2x-4 & x^2+0x-9 \\ & \underline{x^2-2x} \\ & \quad\quad 2x-9 \\ & \quad\quad \underline{2x-4} \\ & \quad\quad\quad -5 \end{array}$$

The quotient is $\frac{1}{2}x+1$ with a remainder of -5.

The equation $y=\frac{1}{2}x+1$ is a slant asymptote.

Ex 2: Find the asymptotes (vertical, horizontal, and/or slant) for the following function.

$$g(x)=\frac{2x^2-x-3}{x-2}$$

A vertical asymptote is found by letting the denominator equal zero.

$$x-2=0$$

$$\boxed{x=2, \quad \text{the vertical asymptote}}$$

A horizontal asymptote is found by comparing the leading term in the numerator to the leading term in the denominator. The degree of the numerator is greater than the degree of the denominator, so there is no horizontal asymptote.

The slant asymptote is found by dividing the numerator by the denominator.

$$
\begin{array}{r|l}
 & 2x \quad +3 \\
x-2 & 2x^2 - x - 3 \\
 & \underline{2x^2 - 4x} \\
 & \qquad 3x - 3 \\
 & \qquad \underline{3x - 6} \\
 & \qquad\quad 3
\end{array}
$$

The quotient is $2x + 3$ with a remainder of 3.

The equation $y = 2x + 3$ is a slant asymptote.

Ex 3: Find the asymptotes (vertical, horizontal, and/or slant) for the following function.

$$h(x) = \frac{x^3 + 1}{x^2 - 9}$$

The vertical asymptotes are found by letting the denominator equal zero.

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x + 3 = 0 \quad x - 3 = 0$

$x = -3, \quad x = 3$ equations of the vertical asymptotes

The horizontal asymptote, if it exists, if found by comparing the leading term in the numerator to the leading term in the denominator. Since the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.

The oblique or slant asymptote is found by dividing the numerator by the denominator. A slant asymptote exists, since the degree of the numerator is 1 greater than the degree of the denominator.

$$
\begin{array}{r|l}
 & x \\
x^2 + 0x - 9 & x^3 + 0x^2 + 0x + 1 \\
 & \underline{x^3 \;\; + 0x^2 - 9x} \\
 & \qquad\qquad 9x + 1
\end{array}
$$

The quotient is x with a remainder of $9x + 1$

The equation $y = x$ is a slant asymptote.