MA 15800

Lesson 29 Limits from Graphs

One-Sided Limits:

A left-sided limit is approaching the *x* value from slightly less value than *x*.

Notation: $\lim\limits_{x \to c^-} f(x)$

Example: $\lim\limits_{x \to 5^-} \frac{1}{x-5} =$

The denominator would be negative. (For example, if $x = 4.999$, $x - 5 = -0.001$.

$$\lim_{x \to 5^-} \frac{1}{x-5} = \frac{1}{-0.001} \to -\infty$$

A right-sided limit is approaching the *x* value from slightly greater value than *x*.

Notation: $\lim\limits_{x \to c^+} f(x)$

Example: $\lim\limits_{x \to 5^+} \frac{1}{x-5} =$

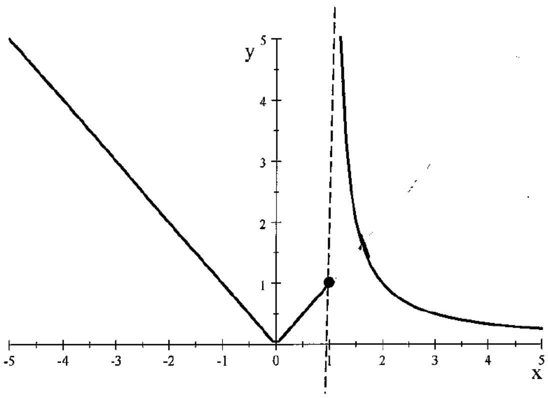
The denominator would be positive. (for example, if $x = 5.001, x - 5 = 0.001$.

$$\lim_{x \to 5^+} \frac{1}{x-5} = \frac{1}{0.001} \to \infty$$

The left-sided limit and right-sided limit above exist. However, $\lim\limits_{x \to 5} \frac{1}{x-5}$ in general does not exist because the left-sided and right-sided limits are not the same value.

Limits can also be determined from graphs; one-sided limits as well as general limits.

Ex 1: Find the denoted limits.

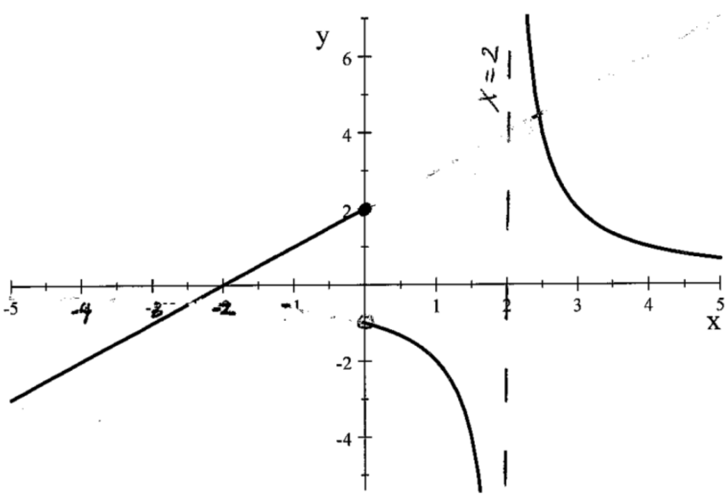


$$f(x)=\begin{cases} |x| & \text{if } x\le 1 \\ \dfrac{1}{x-1} & \text{if } x>1 \end{cases}$$

a) $\lim\limits_{x\to 1^-} f(x) =$

b) $\lim\limits_{x\to 1^+} f(x) =$

Ex 2: Find the denoted limits.



$$g(x)=\begin{cases} x+2 & \text{if } x\le 0 \\ \dfrac{2}{x-2} & \text{if } x>0 \end{cases}$$

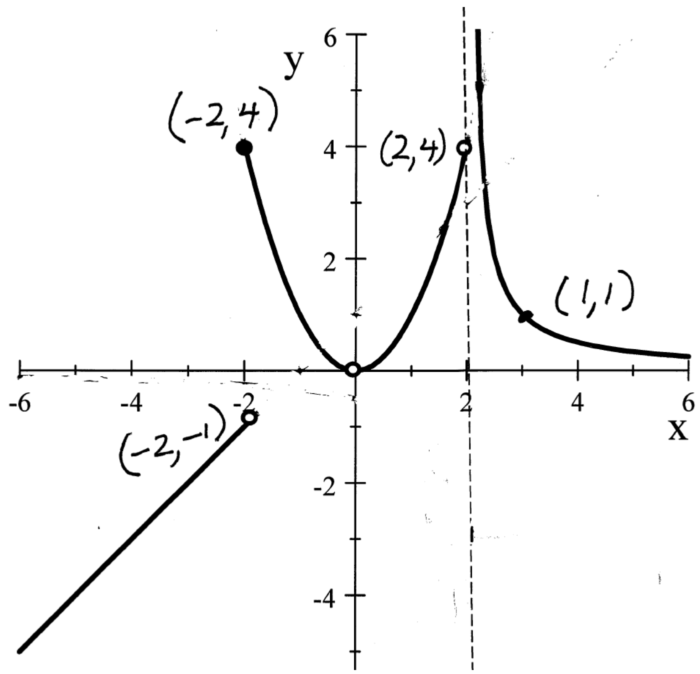
a) $\lim\limits_{x\to 0^-} g(x) =$

b) $\lim\limits_{x\to 0^+} g(x) =$

c) $\lim\limits_{x\to 2^-} g(x) =$

d) $\lim\limits_{x\to 2^+} g(x) =$

Ex 3: Find the denoted limits.



$h(x)$

a) $\lim_{x\to -2^-} h(x) =$

b) $\lim_{x\to -2^+} h(x) =$

c) $\lim_{x\to 0^-} h(x) =$

d) $\lim_{x\to 0^+} h(x) =$

e) $\lim_{x\to 2^-} h(x) =$

f) $\lim_{x\to 2^+} h(x) =$

g) $\lim_{x\to \infty} h(x) =$

h) $\lim_{x\to -\infty} h(x) =$

Find the denoted limits analytically, if they exist

Ex 4: $\lim_{x\to 2}(2x^2+3) =$

Ex 5: $\lim_{x\to \frac{6}{5}}\left(\frac{1}{5x-6}\right) =$

Ex 6: $\lim_{x\to -3}\left(\frac{1}{(x+3)^2}\right) =$

Ex 7: $\lim_{x\to 0}\left(\frac{x^2-5x}{x^2+8x}\right) =$

Ex 8: $\lim_{x\to -8}\left(\dfrac{x^2-5x}{x^2+8x}\right)=$

Ex 9: $\lim_{x\to -2}\left(\dfrac{x^2-3x-10}{x^2+3x+2}\right)=$

Ex 10: $\lim_{x\to 5}\left(\dfrac{x^2-3x-10}{x^2+3x+2}\right)=$

Ex 11: $\lim_{x\to -2}\left(\dfrac{x^2-3x-10}{x^2+3x+2}\right)=$

Use the function $f(x)=\begin{cases}3x+1 & if\ \ x\neq 2\\ 5 & if\ \ x=2\end{cases}$ for examples 12. If the limit does not exist, write DNE.

Ex 12: $\lim_{x\to 2^-} f(x)=$ $\lim_{x\to 2^+} f(x)=$ $\lim_{x\to 2} f(x)=$

Use the function $g(x)=\begin{cases}2x^2-3 & if\ \ x\leq 0\\ 5x+205 & if\ \ x>0\end{cases}$ for example 13. If the limit does not exist, write DNE.

Ex 13: $\lim_{x\to 0^-} g(x)=$ $\lim_{x\to 0^+} g(x)=$ $\lim_{x\to 0} g(x)=$

Use the function $j(x)=\begin{cases}x+11 & if\ \ x<10\\ \sqrt{x} & if\ \ x\geq 10\end{cases}$ for example 14. If the limit does not exist, write DNE.

Ex 14: $\lim_{x\to 10^-} j(x)=$ $\lim_{x\to 10^+} j(x)=$ $\lim_{x\to 10} j(x)=$

Use the function $q(x)=\begin{cases} 2x^2+3 & if\ \ x\le 0 \\ 5x+3 & if\ \ 0<x<2 \\ -x+10 & if\ \ x\ge 2 \end{cases}$ for example 15. If the limit does not exist, write DNE.

Ex 15:

$\lim_{x\to 0^-} q(x)=$ $\lim_{x\to 0^+} q(x)=$ $\lim_{x\to 2^-} q(x)=$

$\lim_{x\to 2^+} q(x)=$ $\lim_{x\to 0} q(x)=$ $\lim_{x\to 2} q(x)=$

Use the function $r(x)=\begin{cases} -2x-\frac{\pi}{2} & if\ \ x\le -\frac{\pi}{2} \\ \sin x & if\ \ -\frac{\pi}{2}<x<\frac{\pi}{2} \\ \cos x-1 & if\ \ x\ge \frac{\pi}{2} \end{cases}$ for example 16. If the limit does not exist, write DNE.

Ex 16:

$\lim_{x\to -\frac{\pi}{2}^-} r(x)=$ $\lim_{x\to -\frac{\pi}{2}^+} r(x)=$ $\lim_{x\to -\frac{\pi}{2}} r(x)=$

$\lim_{x\to \frac{\pi}{2}^-} r(x)=$ $\lim_{x\to \frac{\pi}{2}^+} r(x)=$ $\lim_{x\to \frac{\pi}{2}} r(x)=$

Use the function $v(x)=\frac{x^2+2x}{x^2-5x}$ for example 17. If the limit does not exist, write DNE.

Ex 17:

$\lim_{x\to -2} v(x)=$ $\lim_{x\to 5} v(x)=$ $\lim_{x\to 0} v(x)=$

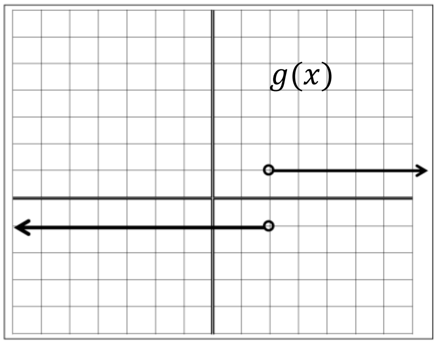
Use the function $g(x) = \dfrac{\frac{3}{x+4} - \frac{3}{4}}{x}$ for example 18. If the limit does not exist, write DNE.

Ex 18: $\lim_{x \to 3} g(x) =$ $\lim_{x \to 0} g(x) =$

Use the function $f(x) = \dfrac{\sqrt{3x+2} - \sqrt{2}}{x}$ for example 19.

Ex 19: $\lim_{x \to 9} f(x) =$ $\lim_{x \to 0} f(x) =$

Ex 20: The scale on each axis below is 1 unit per hash mark.

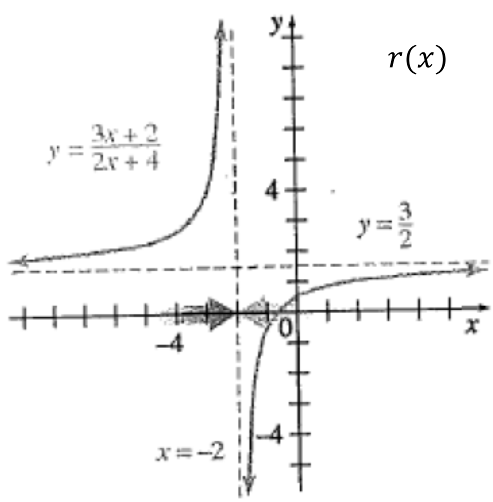


Find the following limits.

$\lim_{x \to 2^-} g(x) =$ $\lim_{x \to 2^+} g(x) =$ $\lim_{x \to 2} g(x) =$

Ex 21: Use the graph of $r(x)$ below to find these limits, if they exist. If the limit does not exist, write DNE. Note: The equation of this graph is written next to the graph.

$\lim_{x \to -8} r(x) =$ $\lim_{x \to -2^-} r(x) =$ $\lim_{x \to -2^+} r(x) =$ $\lim_{x \to -2} r(x) =$



Ex 22: Use the graph of $h(x)$ below to find the following limits. If the limit does not exist, write DNE.

$\lim_{x \to -1} h(x) =$ $\lim_{x \to 2} h(x) =$

What is the value of $h(2)$?