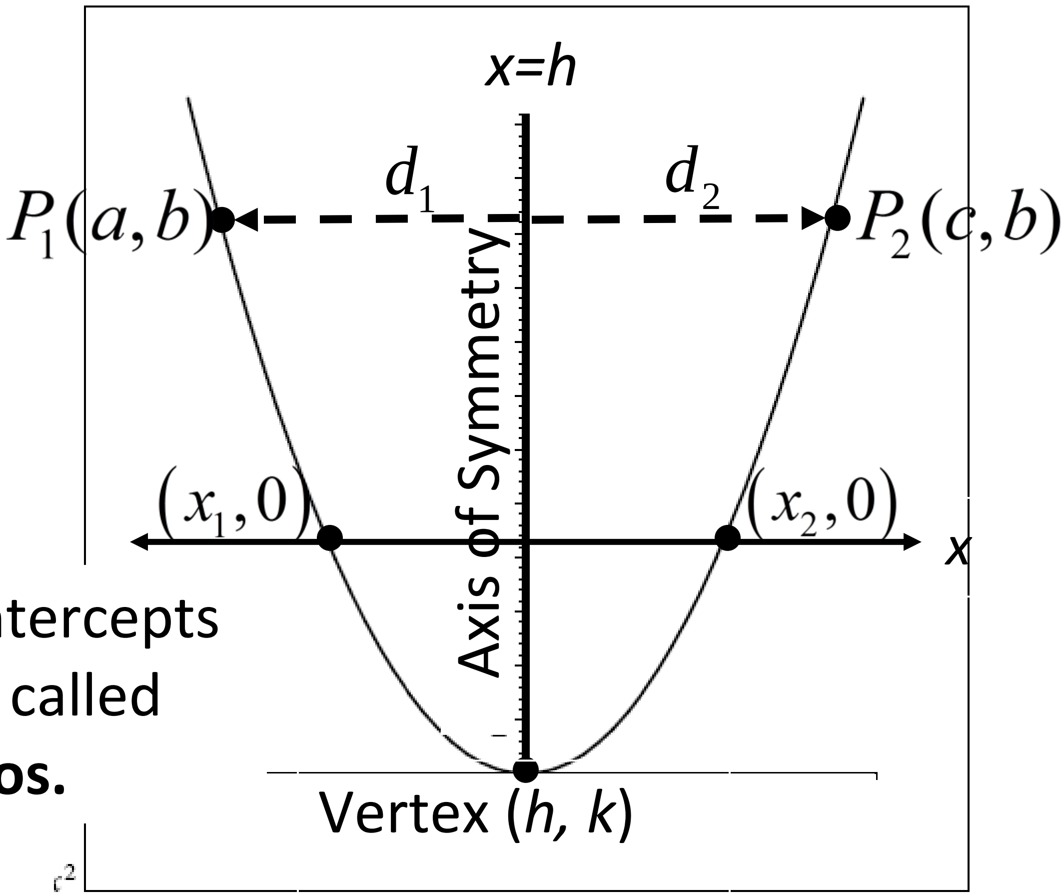
Definition:
A **quadratic function** is of the form $\boldsymbol{f(x) = y = ax^2 + bx + c}$**;** where a, b, and c are real numbers and $a \neq 0$. This form of the quadratic function is called general form.

The graph of a quadratic function will be a **parabola** with a vertical axis. See the picture below.

Quadratic Function Parabola Shape

There is symmetry about the axis of symmetry. Distances from P_1 to axis of symmetry and P_2 to axis of symmetry are equal. $d_1 = d_2$



x-intercepts are called **zeros.**

The average of the **zeros** will equal h, the x-coordinate of the vertex. $h = \dfrac{x_1 + x_2}{2}$

Quadratic Functions

Reminder: If general form of a quadratic function is $y = f(x) = ax^2 + bx + c$. The **value of *a*** will give the direction that the parabola 'opens'. If $a > 0$, the parabola opens upward. If $a < 0$, the parabola opens downward.

If the parabola **opens upward**, the quadratic function has a **minimum value of *k*** (the *y*-coordinate of the vertex) and it occurs when $x = h$ or at the point $V(h, k)$. If the parabola **opens downward**, the quadratic function has a **maximum value of *k*** (again, the *y*-coordinate of the vertex) and it occurs when $x = h$ or at the point $V(h, k)$. Notice: The maximum or minimum value occurs at the vertex.

The value of *h*, the *x*-coordinate of the vertex, can also be found by averaging any two corresponding point that are equal distance from the axis of symmetry, such as the *x*-intercepts. $\boxed{h = \frac{x_1 + x_2}{2}}$ (Note: A parabola does not always have *x*-intercept(s), but will always have a *y*-intercept.) In general, if P_1 *and* P_2 are two corresponding point that are symmetric about the axis of symmetry, $P_1(a, b)$ and $P_2(c, b)$, then $\boxed{h = \frac{a+c}{2}}$.

Probably many of you are familiar with general form of a quadratic function, $y = ax^2 + bx + c$. There is another more valuable form of an equation for a quadratic function called **standard form.**

Definition:
A **quadratic function** is in **standard form** when written as $\boldsymbol{f(x) = y = a(x - h)^2 + k}$**.** This equation form represents a parabola (quadratic function) that has the same shape as $y = ax^2$, but has been shifted *h* units right and *k* units down from $y = ax^2$.
The value of *a* in this form describes the direction of opening, such as in general form. The vertex is $V(h, k)$. (Notice the minus sign in front of the *h*; *h* is always the opposite of the number inside that parentheses.) The vertex is the location of a **maximum** (if the parabola opens downward) or **minimum** (if it opens upward). The value of *k* is the maximum or minimum value, the *h* or the ordered pair for *V* is the location of where the maximum or minimum value occurs.

Ex 1: Describe the direction of opening for each quadratic function. Does the function have a maximum or minimum value?

a) $y = -\frac{1}{2}x^2 + 6x - 9$

b) $g(x) = 4(x+2)^2 - 5$

A 'completing the square' process may be used to convert from general form to standard form. See the example below.

Ex 2: Convert $y = \frac{2}{3}x^2 - 6x + 8$ from general form to standard form.

$y = \frac{2}{3}x^2 - 6x + 8$ First, factor out the $\frac{2}{3}$ from the first 2 terms.

$y = \frac{2}{3}(x^2 - 9x) + 8$ Second, complete the square inside the parentheses.

To complete the square, add the square of half of the -9. However, because the $\frac{2}{3}$ is multiplied by the parentheses, so will any number added to complete the square. You also must 'balance' the equation by adding the opposite on the outside of the parentheses.

$$y = \frac{2}{3}(x^2 - 9x + \frac{81}{4}) + 8 - \frac{2}{3} \cdot \frac{81}{4}$$

Rewrite the parentheses with the square power that equals the parentheses. Also, add the values at the end by getting an LCD.

$$y = \frac{2}{3}(x - \frac{9}{2})^2 + \frac{16}{2} - \frac{27}{2}$$

$$y = \frac{2}{3}(x - \frac{9}{2})^2 - \frac{11}{2}$$

This is a quadratic function that represents a parabola that open upward, with a vertex of $\left(\frac{9}{2}, -\frac{11}{2}\right)$, a minimum value of $-\frac{11}{2}$ when $x = \frac{9}{2}$, and an axis of symmetry at $x = \frac{9}{2}$.

Let's try to derive some formulas to determine the values of h and k (coordinates of the vertex) given general form.

$y = ax^2 + bx + c$ Factor a from the first two terms.

$y = a(x^2 + \frac{b}{a}x) + c$ Complete the square inside the parentheses and balance the equation by adding the opposite outside the parentheses.

$$y = a(x^2 + \frac{b}{a}x + (\frac{1}{2}\cdot\frac{b}{a})^2) + c - (\frac{1}{2}\cdot\frac{b}{a})^2$$

$$y = a\left(x^2 + \frac{b}{a}a + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a^2}$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a^2}\right)$$

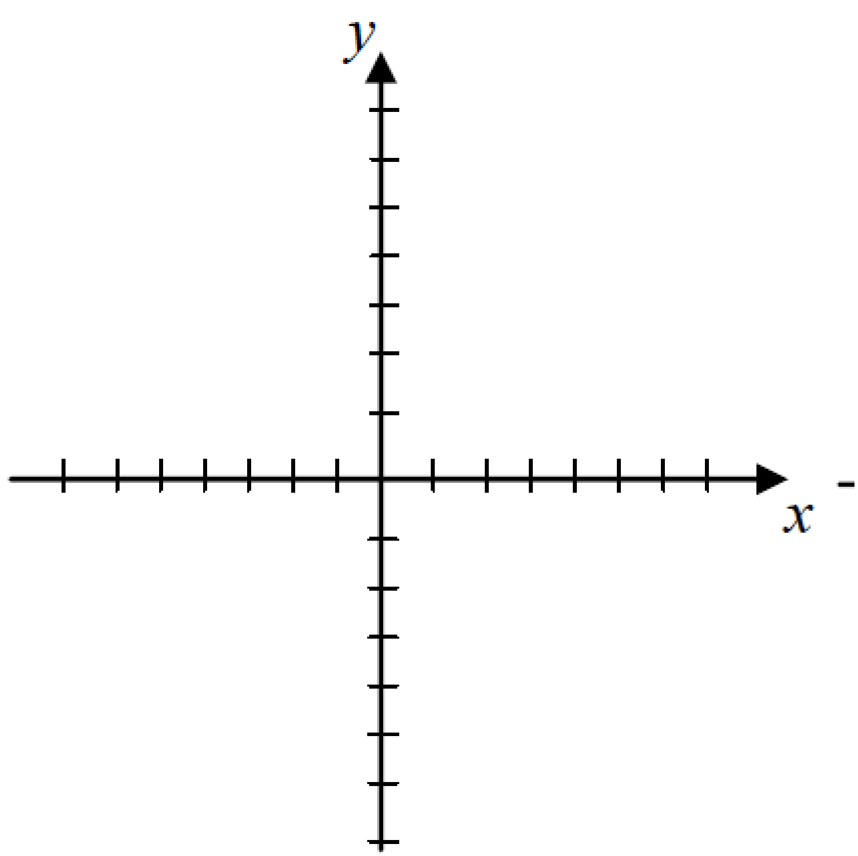
The equation above is in standard form, $y = a(x-h)^2 + k$, where $\boxed{h = -\frac{b}{2a}}$ and $k = c - \frac{b^2}{4a^2}$. I expect you to memorize the formula for h ($h = -\frac{b}{2a}$), but you do not have to memorize the formula to find k. Instead just memorize that $k = f(h) or\ f\left(-\frac{b}{2a}\right)$ (replace h back in the equation of the function and solve for y, the function value).

For the following examples 1 and 2, find the following and make a sketch of the function.

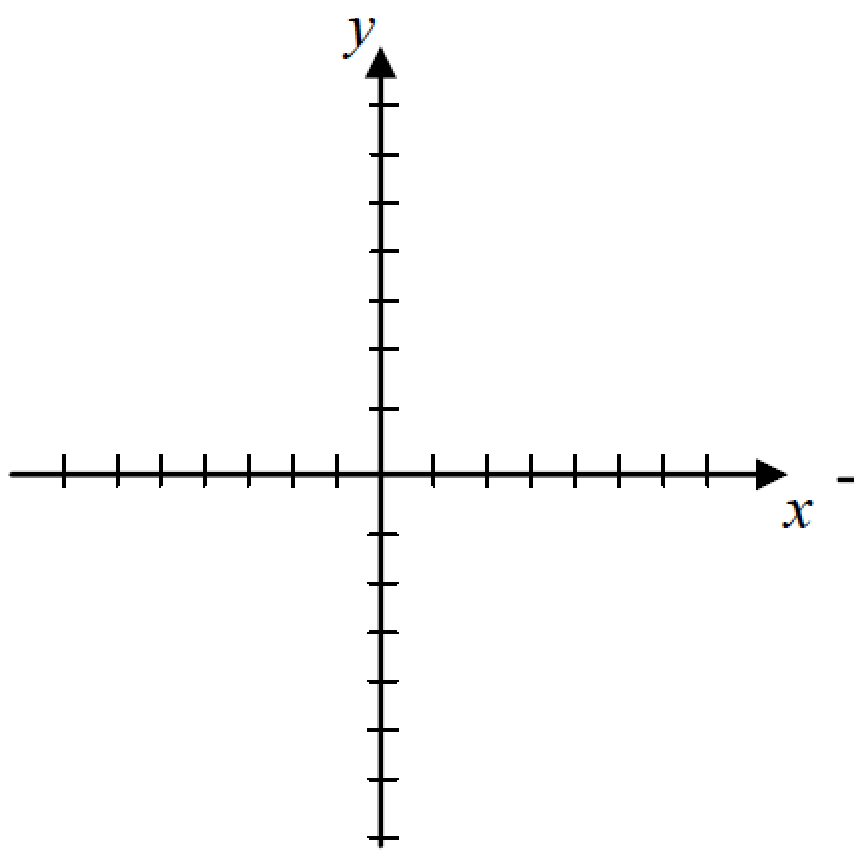
a) Convert the general form to standard form.
b) Give the vertex.
c) Give the direction of opening.
d) Give the maximum or minimum value.
e) Find the zeros (the x-intercepts).
f) Find the y-intercept.

Ex 1: $f(x) = 2x^2 - 4x - 11$

There is more space of the next page for example 1 as well as a coordinate system for the graph.


y
x

Ex 2: $y = -3x^2 - 7x + 6$ (Use back of the sheet for more space.)



If the trinomial of a quadratic function written in general form is factorable, we can write the equation in what is called **intercept form.**

$$y = ax^2 + bx + c$$
$$y = a(x - x_1)(x - x_2)$$

Definition of Intercept Form: A quadratic function written in the form $y = a(x - x_1)(x - x_2)$ is written in intercept form.

Below is a summary of the 3 different forms for a quadratic function.

FORM	**Vertex (*h, k*)**	***x*-intercepts (if possible)**
$y = f(x) = a(x-h)^2 + k$ **Standard Form**	*h* and *k* as in the form	Let $y = 0$ and solve for *x*. **if they exist**
$y = f(x) = a(x-x_1)(x-x_2)$ **Intercept Form**	$h = \frac{x_1 + x_2}{2}$, $k = f(h)$	$(x_1, 0)$ and $(x_2, 0)$
$y = f(x) = ax^2 + bx + c$ **General Form**	$h = -\frac{b}{2a}$, $k = f(h)$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **if they exist**

<u>**Ex 3:**</u> Find the vertex and intercepts of each quadratic function.

a) $y = -2x^2 + 4x - 8$

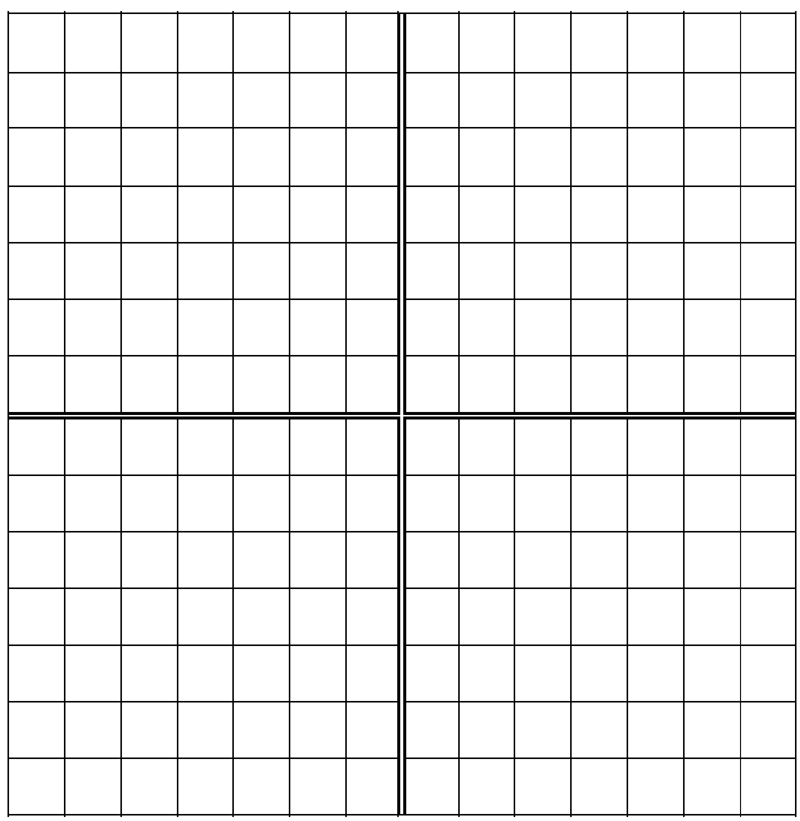
b) $f(x) = -\frac{1}{3}(x+3)(x-1)$

c) $g(x) = \frac{2}{9}(x+1)^2 + 2$

<u>Steps to sketch a graph of a quadratic function:</u>

1. Find the y-intercept.
2. Find the zeros (x-intercepts) if possible by using factoring or the quadratic formula.
3. Find the vertex.
4. Find a few other points using symmetry where possible.

<u>Ex 4:</u> Sketch the graph of the quadratic function $y = \frac{3}{2}x^2 - 3x - 12$.



Quadratic functions' equations may be found if given any point and the vertex, or other information that helps find a point and the vertex. Substitute the given values for *x, y, h, and k* in standard form and solve for *a*.

Ex 5: Find the equation in standard form for a quadratic function with a vertex $V(-4,3)$ and a point $P(2,1)$.

Ex 6: Find the equation in general form for a quadratic function with vertex $V(3,5)$ and an *x*-intercept $(5,0)$.

Ex 7: Find the equation in intercept form for a quadratic function with the two *x*-intercepts of $(-2,0)$ and $(6,0)$ and another point $P(5,3)$.