Fall 2017

MA 16100
Study Guide - Exam # 3

1 **Related Rates Word Problems Method**:

1. Read problem carefully several times to understand what is asked.
2. Draw a picture (if possible) and label.
3. Write down the given rate; write down the desired rate.
4. Find an equation relating the variables.
5. Use Chain Rule to differentiate equation w.r.t to time and solve for desired rate.

2 The *Linear Approximation* (or tangent line approximation) to a function $f(x)$ at $x = a$ is the function $L(x) = f(a) + f'(a)(x - a)$; Approximation formula $\boxed{f(x) \approx f(a) + f'(a)(x - a)}$ for x near a; if $y = f(x)$, the differential of y is $dy = f'(x)dx$.

3 Definitions of absolute maximum, absolute minimum, local/relative maxmium, and local/relative minimum; c is a *critical number* of f if c is in the domain of f and either $f'(c) = 0$ or $f'(c)$ DNE.

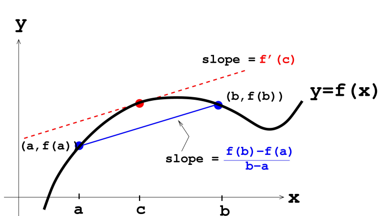
4 **Extreme Value Theorem**: If $f(x)$ is continuous on a closed interval $[a, b]$, then f always has an absolute maximum value and an absolute minimum value on $[a, b]$.

5 **Method to Find Absolute Max/Min of $f(x)$ over Closed Interval $[a, b]$**:

(i) Find all admissible critical numbers in (a, b);
(ii) Find endpoints of interval;
(iii) Make table of values of $f(x)$ at the points found in (i) and (ii).
The largest value = abs max value of f and the smallest value = abs min value of f.

6 **Rolle's Theorem**: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), and $f(a) = f(b)$, then $f'(c) = 0$ for some $c \in (a, b)$.

7 **Mean Value Theorem**: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is a number c, where $a < c < b$, such that $\dfrac{f(b) - f(a)}{b - a} = f'(c)$:



i.e., $\boxed{f(b) - f(a) = f'(c)\,(b - a)}$

If something about f' is known, then something about the sizes of $f(a)$ and $f(b)$ can be found.

8 Fact: (Useful for integration theory later)

(a) If $f'(x) = 0$ for all $x \in I$, then $f(x) = C$ for all $x \in I$.
(b) If $f'(x) = g'(x)$ for all $x \in I$, then $f(x) = g(x) + C$ for all $x \in I$.

9 Increasing functions: $f'(x) > 0 \Longrightarrow f\nearrow$; decreasing functions: $f'(x) < 0 \Longrightarrow f\searrow$.

10 **<u>First Derivative Test</u>**: Suppose c is a critical number of a continuous function f.

(a) If f' changes from $+$ to $-$ at $c \Longrightarrow f$ has local max at c
(b) If f' changes from $-$ to $+$ at $c \Longrightarrow f$ has local min at c
(c) If f' <u>does not</u> change sign at $c \Longrightarrow f$ has <u>neither</u> local max nor local min at c

(*Displaying this information on a number line is much more efficient, see above figure.*)

11 f concave up: $f''(x) > 0 \Longrightarrow f \bigcup$; and f concave down: $f''(x) < 0 \Longrightarrow f \bigcap$;
inflection point (i.e. point where concavity changes).

(*Displaying this information on a number line is much more efficient, see above figure.*)

12 **<u>Second Derivative Test</u>**: Suppose f'' is continuous near critical number c and $f'(c) = 0$.

(a) If $f''(c) > 0 \Longrightarrow f$ has a local min at c.
(b) If $f''(c) < 0 \Longrightarrow f$ has a local max at c.

Note: If $f''(c) = 0$, then 2^{nd} Derivative Test cannot be used, so then use 1^{st} Derivative Test.

13 <u>Indeterminate Forms</u>:

(a) **Indeterminate Form (Types)**: $\mathbf{\frac{0}{0}}$, $\mathbf{\frac{\infty}{\infty}}$, $\mathbf{0 \cdot \infty}$, $\mathbf{\infty - \infty}$, $\mathbf{0^0}$, $\mathbf{\infty^0}$, $\mathbf{1^\infty}$

(b) **<u>L'Hopital's Rule</u>**: Let f and g be differentiable and $g'(x) \neq 0$ on an open interval I containing a (except possibly at a). If $\lim_{x\to a} f(x) = 0$ and if $\lim_{x\to a} g(x) = 0$ [Type $\frac{0}{0}$]; or if $\lim_{x\to a} f(x) = \infty$ (or $-\infty$) and if $\lim_{x\to a} g(x) = \infty$ (or $-\infty$) [Type $\frac{\infty}{\infty}$], then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$

provided the limit on the right side exists or is infinite.

Use algebra to convert the different Indetermine Forms in (a) *into expressions where the above formula can be used.*

<u>Important Remark</u>: L'Hopital's Rule is also valid for one-sided limits, $x \to a^-$, $x \to a^+$, and also for limits when $x \to \infty$ or $x \to -\infty$.

14 **Curve Sketching Guidelines**:

(a) `Domain` of f

(b) `Intercepts` (if any)

(c) `Symmetry`:
$f(-x) = f(x)$ for *even* function;
$f(-x) = -f(x)$ for *odd* function;
$f(x+p) = f(x)$ for *periodic* function

(d) `Asymptotes`:
$x = a$ is a *Vertical Asymptote*: if either $\lim_{x \to a^-} f(x)$ or $\lim_{x \to a^+} f(x)$ is infinite

$y = L$ is a *Horizontal Asymptote*: if either $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$.

(e) `Intervals`: where f is increasing $\nearrow$ and decreasing $\searrow$; local max and local min

(f) `Intervals`: where f is concave up $\bigcup$ and concave down $\bigcap$; inflection points