Study Guide for Exam 1

1. You are supposed to be able to determine the domain of a function, looking at the conditions for its expression to be well-defined. Some examples of the conditions are:

- What is inside of the square root must be non-negative (where there is no restriction on what is inside of the cubic root).
- The denominator must not be zero.
- What is inside of the logarithm must be strictly positive.

2. Having the information on the range of a given rotation angle θ and knowing the value of a trigonometric function, you are supposed to be able to determine the values of the other trigonometric functions.

Example: We have the information

$$\sin\theta = \frac{3}{5} \text{ and } \frac{\pi}{2} < \theta < \pi.$$

Determine the values of $\cos\theta, \tan\theta, \csc\theta, \sec\theta, \cot\theta$.

3. You are supposed to be able to solve the equations involving the trigonometric functions and find solutions on the giveen interval, using the basic formulas of the trigonometric functions (e.g., double angle formula for sine andd cosine, $\sin^2 x + \cos^2 x = 1$, etc.).

Example: Find the values of x on the interval $[0, 2\pi]$ which satisfy the equation

$$\cos x = \cos(2x).$$

4. Starting from the graph of a given function, you are supposed to know how to shift vertically/horizontally and/or reflect with respect to a certain line, to draw the graph of a new function, depending on the formula for the new function. Conversely, you are supposed to know how to derive the formula for the new function, once you are given the information on its graph through shifts, reflection compared to the original graph.

Example: Find the formua for a function

(i) whose graph is symmetric with respect to a line $y = -3$ to the graph of $y = e^x$.

(ii) whose graph is symmetric with respect to a line $x = 1$ to the graph of $y = |x - 5|$.

5. Given a function which is one-to-one, you are supposed to be able to find the formula and draw the graph of its inverse function.

Example: Find the formula, and state the domain and range of the inverse of the function

$$y = \frac{1 - e^{-x}}{1 + e^{-x}}.$$

6. You are supposed to be able to solve the equations involving the exponential and logarithmic functions.

Example: Look at the problems 51, 52, 53, 54 on Page 67 of the textbook.

7. You are supposed to be able to compute the right/left hand side limit, understanding its proper meaning. You are also supposed to determine the exact value of the limit who has an indeterminate form on the surface.

Example: Compute the following limits:

(i) $\lim_{x\to 5^-} \dfrac{x^2 - 3x - 5}{|x - 5|}$

(ii) $\lim_{x\to(\pi/2)^-} \tan x$

(iii) $\lim_{x\to 0} \left(\dfrac{5}{x^2 - x} + \dfrac{5}{x} \right)$

(iv) $\lim_{x\to\infty} \left(\sqrt{x^2 + 3x + 1} - x \right)$

(v) $\lim_{x\to 0} \dfrac{|3x - 4| - |5x + 4|}{x}$

8. You are supposed to understand the meaning of the defining formula of the derivative, and being able to determine the values of the related limits.

Example: Suppose we have a function $f(x)$ with $f'(2) = 5$.

Determine the following values:

(i) $g'(1)$ where $g(x) = f(2x)$.

(ii) $\lim_{h\to 0} \dfrac{f(2 + 4h) - f(2)}{3h}$.

(iii) $\lim_{h\to 0} \dfrac{f(2 + 4h) - f(2 + 5h)}{9h}$.

9. When a function is defined piecewise and depending on some variables, you are supposed to know how to determine those variables so that the function becomes continuous entirely over its domain.

Example:
(i) Look at Webassign HW 4 Problem 18.
(ii) Find the values of a and b so that the function

$$f(x) = \begin{cases} x^2 - a & \text{if} \quad x \leq 1 \\ \dfrac{3x^2 + 12x - b}{x^2 + 2x - 3} & \text{if} \quad x > 1 \end{cases}$$

is continuous on $(-\infty, \infty)$.

10. You are supposed to be able to find the horizontal/vertical asymptote(s) of a given function.

Example: Find the horizontal/vertical asymptote(s) of a function

$$y = f(x) = \frac{x^3 + 4x^2 + x - 6}{x(x^2 - 1)}.$$

11. You are supposed to be able to compute the limits, using the Squeeze Theorem and some basic limits such as $\lim\limits_{\theta \to 0} \dfrac{\sin \theta}{\theta}$.

Example: Compute the following limits:
(i) $\lim_{x \to 0} \dfrac{\sin(3x)}{5x}$
(ii) $\lim_{x \to \infty} \dfrac{\sin(3x)}{5x}$
(iii) $\lim_{x \to 0} \dfrac{\sin(1/x)}{1/x}$

12. You are supposed to be able to compute the derivative of a function, and understand that its value represents the slope of the tangent line to the graph of the function.

Example:
(i) Look at Problems 22, 23 on Page 149 of the textbook.
(ii) Find the equation of the line that is tangent to the curve $y = x^3$ and is also parallel to the line $y = 3x$.

13. You are supposed to be able to compute the derivative using the power, product, and quotient rules. You are also supposed to be able to compute the derivatives of the trigonometric functions.

Challenge Problem: Consider the function described below:

$$g(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$$

Choose the right statement about the continuity and differentiability of the function $y = g(x)$ at 0.

A. The function g is continuous at 0 and differentiable at 0.
B. The function g is continuous at 0 but not differentiable at 0.
C. The function g is not continuous at 0 but differentiable at 0.
D. The function g is not continuous at 0 and not differentiable at 0.
E. The above description is not sufficient to judge the continuity or the differentiability of the function g.