Study Guide for Exam 2

1. You are supposed to be able to use the chain rule properly and precisely, even when the function is obtained as the composition of several functions.

Example problems:

1.1. Compute the derivative of the following function:

(i) $y = \sin(\sin(\sin x))$

(ii) $y = \left(\dfrac{t-2}{2t+1}\right)^9$

(iii) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

(iv) $y = e^{\sec 3\theta}$

(v) $y = e^{2^{x^3}}$

1.2. Suppose that $F(x) = f(x)^2 \cdot f(g(x))$ and that the functions f and g satisfy the following conditions:

$$\begin{cases} f(1) = 5, & f(2) = 3, & f(3) = -1 \\ f'(1) = 4, & f'(2) = 3, & f'(3) = -2 \\ g(1) = 3, & g(2) = 2, & g(3) = -1 \\ g'(1) = 2, & g'(2) = 3, & g'(3) = 4 \end{cases}$$

Find $F'(1)$.

2. You are supposed to be able to determine the exact values of the formulas involving the trigonometric and inverse trigonometric functions. You should pay special attention to the range of the inverse trigonometric function. You need to know the double angle formulas for sine, cosine, and tangent.

Example Problems:

2.1. Find the exact values of the following expression.

(i) $\tan(\sin^{-1}\left(\dfrac{4}{5}\right)$

(ii) $\sin^{-1}\left(\sin\left(\dfrac{7\pi}{3}\right)\right)$

(iii) $\sin\left(2\sin^{-1}\left(\dfrac{12}{13}\right)\right)$

3. You are supposed to know how to compute the derivative of a function of the form $y = f(x)^{g(x)}$.

Example Problems:

3.1. Find the derivative of the following function.
(i) $y = x^x$
(ii) $y = (\ln x)^{\tan 3x}$
(iii) $y = (\sqrt{x})^{\sin x}$

4. You are supposed to understand the method of implicit differentiation to compute the derivative. For example, you should be able to determine the equation of the tangent line to the graph of a function implicitly defined, computing the derivative using the implicit differentiation.

Example Problems:

4.1. Suppose that f is a differentiable function defined on $(-\infty, \infty)$ satisfying the following equation

$$f(x) + x^2 \left(f(x)\right)^3 = 10$$

and that $f(1) = 2$.
Find $f'(1)$.
4.2. Find the slope of the tangent to the curve given by the equation

$$x^2 + 2xy - y^2 + x = 2$$

at point $(x, y) = (1, 2)$
4.3. Find $\dfrac{dy}{dx}$ given $e^{x/y} = 7x - y$.

5. You are supposed to be able to provide an approximation of the value of a function, using the linear approximation.

Example Problems:

5.1. Find the linear approximation $L(x)$ of the function $f(x) = e^x$ at $a = 0$. Use this to estimate the value $e^{0.01}$.
5.2. Estimate the value of $\sqrt[3]{26.8}$ using a linear approximation.

6. You are supposed to be able to determine when a particle is speeding up or down, whether it is accelerating or decelerating, given its position function. You are also supposed to be able to compute the total distance travelled during the given period.

Example Problems:

6.1. Webassign HW 14 Problems 5, 6, 7, 8
6.2. The position of a particle is given by the function $s = f(t) = t^3 - 6t^2 + 9t$. Find the total distance traveled during the first 6 seconds.

7. You are supposed to know how to provide the formulas for the trigonometric functions when the angles are given in the form of $\sin^{-1}(x), \cos^{-1}(x), \tan^{-1}(x)$.

Example Problems:

7.1. Find the formula for the following expression.
(i) $\tan\left(\sin^{-1} x\right)$
(ii) $\cos\left(\tan^{-1}\left(\dfrac{x}{\sqrt{9-x^2}}\right)\right)$
(iii) $\csc\left(\cot^{-1}\left(\dfrac{\sqrt{25-x^2}}{x}\right)\right)$ when $x > 0$.
Note: $\csc\left(\cot^{-1}\left(\dfrac{\sqrt{25-x^2}}{x}\right)\right)$ when $x < 0$, is a tricky problem. (Pay attention to the range of $\cot^{-1}$ in Page 66 of the textbook.)

8. You are supposed to know the definitions of the hyperbolic functions and be able to compute their derivatives, determine their exact values.

Example Problems:

8.1. Find the exact value f the following expression.
(i) $\sinh(0)$
(ii) $\sinh(\ln 5)$
(iii) $\dfrac{1+\tanh(1/2)}{1-\tanh(1/2)}$.
8.2. Find the derivative of the function $f(x) = \sinh(\ln x)$, and then evaluate $f'(5)$.

9. FOUR "Related Rates" problems will be given in Exam 2. The problems are very similar to the ones given in the Webassign, and also to the examples discussed in the textbook. Of particular importance are:

- Snowball problem
- Light house problem
- Inverted circular conical tank problem (Gravel problem)
- Kite problem
- Ladder problem
- Man walking away from the light problem
- Two cars (ships) moving east-west, south-north problem