Study Guide for Exam 3

1. You are supposed to know how to determine whether a geometric series converges or diverges, and when it converges, how to evaluate its value.

2. You are supposed to know how to the various tests to determine whether a given series is convergent or divergent.

- p-series
- geometric series
- (Limit) Comparison Test
- Test for Divergence
- Alternating Series Test
- Ration Test
- Root Test

Example Problems

○ Judge whether the following series converges or diverges

$$① . \sum_{n=1}^{\infty} \frac{n!}{n^n}$$

$$② \sum_{n=2} \frac{1}{\ln n}$$

$$③ . \sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

$$④ . \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

$$⑤ . \sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$$

$$⑥ . \sum_{n=1}^{\infty} (-1)^n \arctan\left(\frac{1}{n}\right)$$

$$⑦ . \sum_{n=1}^{\infty} (-1)^n \frac{\arctan(n)}{n}$$

$$⑧ . \sum_{n=1}^{\infty} (-1)^n \frac{\arctan(n)}{n^2}$$

$$⑨ . \sum_{n=1}^{\infty} \frac{e^n + 3n}{e^{5n}}$$

∘ (i) When $\alpha < 1$, which one is bigger, $\dfrac{1}{n^\alpha \ln n}$ or $\dfrac{1}{n \ln n}$?

Does the series $\displaystyle\sum_{n=2} \frac{1}{n^\alpha \ln n}$ converge or diverge ?

Which test are you using ?

(ii) When $\alpha > 1$, which one is bigger, $\dfrac{1}{n^\alpha \ln n}$ or $\dfrac{1}{n^\alpha}$ (for $n \geq 3$)?

Does the series $\displaystyle\sum_{n=2} \frac{1}{n^\alpha \ln n}$ converge or diverge ?

Which test are you using ?

3. You are supposed to know when $\displaystyle\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = 1$, Ratio Test is inconclusive (and hence in order to determine whether the series converges or diverges we have to apply some other methods or tests).

4. You are supposed to know various logical implications/connection to judge whether the series converges or diverges.

5. You are supposed to know how to use the Estimation Theorem for Alternating Series to find the minimum number of terms you should compute to have the error stay within the given value.

6. You are supposed to know how ot find the power series expression for a function, using the basic formula

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n.$$

$$e^x = \sum_{i=0}^{\infty} \frac{x^n}{n!}.$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

You are also supposed to know how to find the power series expression for its derivatives and integrations.

7. Given a power series, you are supposed to know how to determine the radius of convergence by fist using the Raation Test, and then to determine the interval of convergence by checking th behavior at the boundary points.

8. Given a function, you are supposed to be able to find its Maclaurin series (power series centered at 0) and Taylor series (power series centered at a) by the basic formulas

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \\ f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n. \end{aligned}$$

9. You are supposed to be able to evaluate the series, by looking at the Maclaurin series for a function and by plugging in the appropriate value.

Example Problems

∘ Evaluate the following series

①. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)! 6^{2n}}$

②. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!}$

③. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)! 4^{2n+1}}$

④. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)! 2^{2n+1}}$

⑤. $\displaystyle\sum_{n=0}^{\infty} \frac{(\ln 3)^n}{n!}$

⑥. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1}$