

Study Guide for Exam 1

1. You are supposed to be able to determine the center and radius of a sphere by “completing the square”, given the equation of the form

$$x^2 + y^2 + z^2 + ax + by + cz + d = 0.$$

You are also supposed to be able to compute the distance between two given points.

2. You are supposed to be able to compute the dot product $\vec{a} \cdot \vec{b}$ of two vectors \vec{a} and \vec{b} . You are supposed to understand the geometrical interpretation of the dot product $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$, where θ is the angle between the two vectors.

You should be able to use the orthogonality criterion in terms of the dot product

$$\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0.$$

3. You are supposed to be able to compute the cross product $\vec{a} \times \vec{b}$ of two vectors \vec{a} and \vec{b} . You are supposed to understand the geometrical interpretation of the cross product $\vec{a} \times \vec{b}$ as the vector orthogonal to both \vec{a} and \vec{b} , where the direction is determined by the right hand rule, with the magnitude being equal to the area of the parallelogram formed by the two vectors \vec{a} and \vec{b} . (As an application, if you want to compute the area of the parallelogram formed by \vec{a} and \vec{b} , then you can just compute the cross product and its magnitude.)

4. You are supposed to be able to compute the vector projection $\text{proj}_{\vec{a}} \vec{b}$ of a vector \vec{b} onto \vec{a} , and scalar projection $\text{comp}_{\vec{a}} \vec{b}$ by the formulas

$$\begin{cases} \text{proj}_{\vec{a}} \vec{b} &= \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}} \vec{a} \\ \text{comp}_{\vec{a}} \vec{b} &= \frac{\vec{a} \cdot \vec{b}}{\sqrt{\vec{a} \cdot \vec{a}}} \end{cases}$$

WARNING: Make a clear distinction between $\text{proj}_{\vec{a}} \vec{b}$ and $\text{proj}_{\vec{b}} \vec{a}$.

5. You are supposed to be able to compute the area of the region bounded by two curves $y = f(x)$ and $y = g(x)$ between $x = a$ and $x = b$ by the formula

$$\int_a^b |f(x) - g(x)| dx.$$

6. You are supposed to be able to compute the volume of a solid obtained by rotation using **the washer method**.

7. You are supposed to be able to compute the volume of a solid obtained by rotation using **the method of cylindrical shells**. (Look at Example 1 in 6.3 on Page 451 of the textbook.)

8. You are supposed to be able to compute the volume of a solid, given the description of its base and its cross sections.

9. You are supposed to be able to compute the amount of work needed to carry out a task. Typical examples are:

- work needed to empty the water from a tank in the shape of a surface of revolution (Look at Example 5 in 6.4 on Page 457 of the textbook.),
- work needed to stretch a spring (Look at Example 3 in 6.4 on Page 457 of the textbook.),
- work needed to lift a chain (Look at Problem 19 in 6.4 on Page 459).

10. You are supposed to be able to compute the average value f_{ave} of a function $y = f(x)$ on the interval $[a, b]$ by the formula

$$f_{\text{ave}} = \frac{\int_a^b f(x) dx}{b - a}.$$

11. You are supposed to be able to evaluate the integral using integration by parts.

12. You are supposed to know how to compute the integration of the form

$$(1) \int \sin^m x \cos^n x \, dx$$

$$(2) \int \tan^m x \sec^n x \, dx$$