

### Study Guide for Final Exam

- (1) You are supposed to be able to calculate the cross product  $\vec{a} \times \vec{b}$  of two vectors  $\vec{a}$  and  $\vec{b}$  in  $\mathbb{R}^3$ , and understand its geometric meaning. As an application, you should be able to compute the area of a parallelogram (or a triangle) in 3-space.
- (2) You are supposed to be able to calculate the dot product  $\vec{a} \cdot \vec{b}$  of two vectors  $\vec{a}$  and  $\vec{b}$ , and understand its geometrical meaning. As an application, you should be able to use the dot product for judging if two given vectors are orthogonal (perpendicular) to each other.
- (3) You are supposed to be able to compute the vector projection  $\text{proj}_{\vec{a}}\vec{b}$  and scalar projection  $\text{comp}_{\vec{a}}\vec{b}$  of a vector  $\vec{b}$  onto  $\vec{a}$  by the formulas

$$\begin{aligned}\text{proj}_{\vec{a}}\vec{b} &= \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}}\vec{a} \\ \text{comp}_{\vec{a}}\vec{b} &= \frac{\vec{a} \cdot \vec{b}}{\sqrt{\vec{a} \cdot \vec{a}}}\end{aligned}$$

- (4) You are supposed to be able to compute the scalar triple product of the vectors  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$  as the determinant of the  $3 \times 3$  matrix formed by these vectors, and to understand its geometrical meaning as the volume of the parallelepiped formed by these vectors.
- (5) You are supposed to be able to calculate the area of the region bounded by two curves  $y = f(x)$  and  $y = g(x)$  between  $x = a$  and  $x = b$  by the formula

$$\int_a^b |f(x) - g(x)| dx.$$

- (6) You are supposed to be able to calculate the volume of the solid obtained by rotation, using
  - Washer method, and
  - Cylindrical Shell method.
- (7) You are supposed to be able to compute the volume of a solid when the description of its base and the perpendicular cross section is given.

- (8) You are supposed to be able to calculate the average  $f_{\text{ave}}$  of a function  $y = f(x)$  over the interval  $[a, b]$  by the formula

$$f_{\text{ave}} = \frac{1}{b-a} \int_a^b f(x) dx.$$

- (9) You are supposed to be able to calculate the work required to empty the tank, stretch the spring, and lift the chain.
- (10) You are supposed to be able to calculate the arclength  $L$  of a curve  $y = f(x)$ ,  $a \leq x \leq b$  by the formula

$$L = \int_a^b \sqrt{1 + \{f'(x)\}^2} dx.$$

- (11) You are supposed to be able to calculate the Maclaurin series and Taylor series centered at  $x = a$  by the formulas

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

- (12) You are supposed to be able to compute the integration of the form

$$\int \sin^m(x) \cos^n(x) dx$$

$$\int \tan^m(x) \sec^n(x) dx.$$

- (13) You are supposed to be able to evaluate the integral using Integration by Parts

$$\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx$$

or equivalently

$$\int u dv = uv - \int v du.$$

- (14) You are supposed to be able to compute the integration of the form

$$\int \frac{P(x)}{Q(x)} dx$$

with  $P(x), Q(x)$  polynomials, using Partial Fractions.

- (15) You are supposed to be able to evaluate the integral using Trigonometric Substitution

$$\left\{ \begin{array}{lll} \sqrt{a^2 - x^2}, & x = a \sin \theta, & dx = a \cos \theta d\theta, & \sqrt{a^2 - x^2} = a \cos \theta \\ \sqrt{x^2 + a^2}, & x = a \tan \theta, & dx = a \sec^2 \theta d\theta, & \sqrt{x^2 + a^2} = a \sec \theta \\ \sqrt{x^2 - a^2}, & x = a \sec \theta, & dx = a \sec \theta \tan \theta d\theta, & \sqrt{x^2 - a^2} = a \tan \theta \end{array} \right.$$

- (16) You are supposed to be able to approximate the given integral using
- Midpoint rule,
  - Trapezoidal rule,
  - Simpson's rule.
- (17) You are supposed to be able to judge whether the given geometric series converges or diverges, and when it converges, to be evaluate its value.
- (18) You are supposed to be able to judge whether the given series (absolutely, conditionally) converges or diverges by using various tests.
- (19) You are supposed to be able to give an estimate of the alternating series within a given error using the Estimation Theorem for the Alternating Series. Sometimes this method is used to give an estimate of the integral in terms of the power series.
- (20) You are supposed to be able to calculate the area of the surface obtained by rotating the curve  $y = f(x)$ ,  $a \leq x \leq b$  about the  $x$ -axis by the formula

$$A = \int_a^b 2\pi f(x) \sqrt{1 + \{f'(x)\}^2} dx.$$

- (21) You are supposed to be able to determine the radius of convergence and the interval of convergence for the given power series.
- (22) You are supposed to be able to compute  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for the curve defined by the parametric equations

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$$

by the formulas

$$\begin{cases} \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{f'(t)}{g'(t)} \\ \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(dy/dt)}{dx/dt} = \frac{\frac{d}{dt}(f'(t)/g'(t))}{f'(t)}. \end{cases}$$

- (23) You are supposed to know how to add, subtract, multiply, and divide complex numbers. Especially the division is carried out by multiplying the complex conjugate of the denominator, i.e.,

$$\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i.$$

- (24) You are supposed to transform the equation in polar coordinates into the one in Cartesian coordinates.
- (25) You are supposed to know how to express a complex number in polar coordinates, and how the multiplication and division are carried out in polar coordinates.
- (26) You are supposed to know how to use De Moivre's Theorem to compute the powers of complex numbers, and solve the equation of the form  $z^n = a + bi$ .
- (27) You are supposed to be able to draw the picture of a curve defined by the equation given in polar coordinates.
- (28) You are supposed to be able to compute the differentiation and integration of the power series.
- (29) You are supposed to be able to compute the coordinates of the centroid of the region enclosed by the curves in the  $xy$ -plane of uniform density.
- (30) You are supposed to be able to compute some of the non-trivial and difficult limits, which often show up carrying out the Ratio Test and Root Test.

### Examples

$$\begin{aligned} & \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \\ & \lim_{n \rightarrow \infty} \sqrt[n]{n} \\ & \lim_{n \rightarrow \infty} \frac{n^n}{1 \cdot 3 \cdot (2n - 1)} \end{aligned}$$