Computer Project 1. Nonlinear Springs

**Goal:** Investigate the behavior of nonlinear springs.

**Tools needed:** `ode45`, `plot`

**Description:** For certain (nonlinear) spring-mass systems, the spring force is not given by Hooke’s Law but instead satisfies

\[ F_{\text{spring}} = ku + \epsilon u^3, \]

where \( k > 0 \) is the spring constant and \( \epsilon \) is small but may be positive or negative and represents the “strength” of the spring (\( \epsilon = 0 \) gives Hooke’s Law). The spring is called a *hard spring* if \( \epsilon > 0 \) and a *soft spring* if \( \epsilon < 0 \).

**Questions:** Suppose a nonlinear spring-mass system satisfies the initial value problem

\[
\begin{align*}
    u'' + u + \epsilon u^3 &= 0 \\
    u(0) &= 0, \quad u'(0) = 1
\end{align*}
\]

Use `ode45` and `plot` to answer the following:

1. Let \( \epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 \) and plot the solutions of the above initial value problem for \( 0 \leq t \leq 20 \). Estimate the amplitude of the spring. Experiment with various \( \epsilon > 0 \). What appears to happen to the amplitude as \( \epsilon \to \infty \)? Let \( \mu^+ \) denote the first time the mass reaches equilibrium after \( t = 0 \). Estimate \( \mu^+ \) when \( \epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0 \). What appears to happen to \( \mu^+ \) as \( \epsilon \to \infty \)?

2. Let \( \epsilon = -0.1, -0.2, -0.3, -0.4 \) and plot the solutions of the above initial value problem for \( 0 \leq t \leq 20 \). Estimate the amplitude of the spring. Experiment with various \( \epsilon < 0 \). What appears to happen to the amplitude as \( \epsilon \to -\infty \)? Let \( \mu^- \) denote the first time the mass reaches equilibrium after \( t = 0 \). Estimate \( \mu^- \) when \( \epsilon = -0.1, -0.2, -0.3, -0.4 \). What appears to happen to \( \mu^- \) as \( \epsilon \to -\infty \)?

3. Given that a certain nonlinear hard spring satisfies the initial value problem

\[
\begin{align*}
    u'' + \frac{1}{5} u' + (u + \frac{1}{5} u^3) &= \cos \omega t \\
    u(0) &= 0, \quad u'(0) = 0
\end{align*}
\]

plot the solution \( u(t) \) over the interval \( 0 \leq t \leq 60 \) for \( \omega = 0.5, 0.7, 1.0, 1.3, 2.0 \). Continue to experiment with various values of \( \omega \), where \( 0.5 \leq \omega \leq 2.0 \), to find a value \( \omega^* \) for which \( |u(t)| \) is largest over the interval \( 40 \leq t \leq 60 \).