

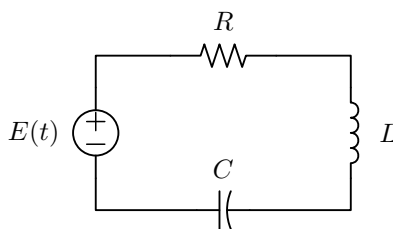
Computer Project 2. RLC Circuits

Goal: Investigate the charge on a capacitor in an RLC circuit with varying voltage.

Tools needed: ode45, plot

Description: If $Q(t)$ = charge on a capacitor at time t in an RLC circuit (with R , L and C being the resistance, inductance and capacitance, respectively) and $E(t)$ = applied voltage, then Kirchhoff's Laws give the following 2nd order differential equation for $Q(t)$:

$$LQ''(t) + RQ'(t) + \frac{1}{C}Q(t) = E(t) \quad (*)$$



Questions: Assume $L = 1$, $C = 1/5$, $R = 4$ and $E(t) = 10 \cos \omega t$.

1. Use `ode45` (and plot routines) to plot the solution of (*) with $Q(0) = 0$ and $Q'(0) = 0$ over the interval $0 \leq t \leq 80$ for $\omega = 0, 0.5, 1, 2, 4, 8, 16$.
2. Let $A(\omega) = \text{maximum of } |Q(t)| \text{ over the interval } 30 \leq t \leq 80$ (this approximates the amplitude of the steady-stat solution). Experiment with various values of ω and discuss what appears to happen to $A(\omega)$ as $\omega \rightarrow \infty$ and as $\omega \rightarrow 0$. Also, interpret your findings in terms of an equivalent spring-mass system.

Remark: There is an analogy between spring-mass system and RLC circuits given by:

Spring-mass system	RLC circuit
$mu'' + cu' + ku = F(t)$	$LQ'' + RQ' + \frac{1}{C}Q = E(t)$
u = Displacement	Q = Charge
u' = Velocity	$Q' = I$ = Current
m = Mass	L = Inductance
c = Damping constant	R = Resistance
k = Spring constant	$1/C = (\text{Capacitance})^{-1}$
$F(t)$ = External force	$E(t)$ = Voltage