Study Guide for Final Exam

(1) You are supposed to be able to calculate the cross product $\vec{a}\times\vec{b}$ of two vectors $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^3$, and understand its geometric meaning. As an application, you should be able to compute the area of a parallelogram (or a triangle) in 3-space.

(2) You are supposed to be able to calculate the dot product $\vec{a}\cdot\vec{b}$ of two vectors $\vec{a}$ and $\vec{b}$, and understand its geometrical meaning. As an application, you should be able to use the dot product for judging if two given vectors are orthogonal (peerpendicular) to each other.

(3) You are supposed to be able to compute the vector projection $\text{proj}_{\vec{a}}\vec{b}$ and scalar projection $\text{comp}_{\vec{a}}\vec{b}$ of a vector $\vec{b}$ onto $\vec{a}$ by the formulas

$$\begin{aligned}\text{proj}_{\vec{a}}\vec{b} &= \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a}\\ \text{comp}_{\vec{a}}\vec{b} &= \frac{\vec{a}\cdot\vec{b}}{\sqrt{\vec{a}\cdot\vec{a}}}\end{aligned}$$

(4) You are supposed to be able to compute the scalar triple product of the vectors $\vec{a},\vec{b}$, and $\vec{c}$ as the determinant of the 3×3 matrix formed by these vectors, and to understand its geometrical meaning as the volume of the parallelepiped formed by these vectors.

(5) You are supposed to be able to calculate the area of the region bounded by two curves $y=f(x)$ and $y=g(x)$ between $x=a$ and $x=b$ by the formula

$$\int_a^b |f(x)-g(x)|\;dx.$$

(6) You are supposed to be able to calculate the volume of the solid obtained by rotation, using
- Washer method, and
- Cylindrical Shell method.

(7) You are supposed to be able to compute the volume of a solid when the description of its base and the perpendicular cross section is given.

(8) You are supposed to be able to calculate the average f_{ave} of a function $y = f(x)$ over the interval $[a, b]$ by the formula

$$f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\ dx.$$

(9) You are supposed to be able to calculate the work required to empty the tank, stretch the spring, and lift the chain.

(10) You are supposed to be able to calculate the arclength L of a curve $y = f(x), a \leq x \leq b$ by the formula

$$L = \int_a^b \sqrt{1 + \{f'(x)\}^2}\ dx.$$

(11) You are supposed to be able to calculate the Maclaurin series and Taylor series centerd at $x = a$ by the formulas

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \\ f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n \end{aligned}$$

(12) You are supposed to be able to compute the integration of the form

$$\int \sin^m(x)\cos^n(x)\ dx$$
$$\int \tan^m(x)\sec^n(x)\ dx.$$

(13) You are supposed to be able to evaluate the integral using Integration by Parts

$$\int f(x)g'(x)\ dx = f(x)g(x) - \int f'(x)g(x)\ dx$$

or equivalently

$$\int u dv = uv - \int v du.$$

(14) You are supposed to be able to compute the integration of the form

$$\int \frac{P(x)}{Q(x)}\ dx$$

with $P(x), Q(x)$ polynomials, using Partial Fractions.

(15) You are supposed to be able to evaluate the integral using Trigonometric Substitution

$$\begin{cases} \sqrt{a^2 - x^2}, & x = a\sin\theta, & dx = a\cos\theta d\theta, & \sqrt{a^2 - x^2} = a\cos\theta \\ \sqrt{x^2 + a^2}, & x = a\tan\theta, & dx = a\sec^2\theta d\theta, & \sqrt{x^2 + a^2} = a\sec\theta \\ \sqrt{x^2 - a^2}, & x = a\sec\theta, & dx = a\sec\theta\tan\theta d\theta, & \sqrt{x^2 - a^2} = a\tan\theta \end{cases}$$

(16) You are suposed to be able to approximate the given integral using
- ∘ Midpoint rule,
- ∘ Trapezoidal rule,
- ∘ Simpson's rule.

(17) You are supposed to be able to judge whether the given geometric series converges or diverges, and when it converges, to be evaluate its value.

(18) You are supposed to be able to judge whether the given series (absolutely, conditionally) converges or diverges by using various tests.

(19) You are supposed to be able to give an estimate of the alternating series within a given error using the Estimation Theorem for the Alternating Series. Sometimes this method is used to give an estimate of the integral in terms of the power series.

(20) You are supposed to be able to calculate the area of the surface obtained by rotating the curve $y = f(x), a \leq x \leq b$ about the x-axis by the formula

$$A = \int_a^b 2\pi f(x)\sqrt{1 + \{f'(x)\}^2}\, dx.$$

(21) You are supposed to be able to determine the radius of convergence and the interval of convergence for the given power series.

(22) You are supposed to be able to compute $\dfrac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the curve defined by the parametric equations

$$\begin{cases} x &= f(t) \\ y &= g(t) \end{cases}$$

by the formulas

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{f'(t)}{g'(t)} \\ \dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}(dy/dt)}{dx/dt} = \dfrac{\frac{d}{dt}(f'(t)/g'(t))}{f'(t)}. \end{cases}$$

(23) You are supposed to know how to add, subtract, multiply, and divide complex numbers. Especially the division is carried out by multiplying the complex conjugate of the denominator, i.e.,

$$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}i.$$

(24) You are supposed to tranform the equation in polar coordinates into the one in Cartesian coordinates.

(25) You are supposed to know how to express a complex number in polar coordinates, and how the multiplication and division are carried out in polar coordinates.

(26) You are supposed to know how to use De Moivre's Theorem to comoute the pwers of complex numbers, and solve the equation of the form $z^n = a + bi$.

(27) You are supposed to be able to draw the picture of a curve defined by the equation given in polar coordinates.

(28) You are supposed to be able to compute the differentiation and integration of the power series.

(29) You are supposec to be able to compute the coordinates of the centroid of the region enclosed by the curves in the xy-plane of uniform density.

(30) You are supposed to be able to compute some of the non-trivial and difficult limits, which often show up carrying out the Ratio Test and Root Test.

Examples

$$\lim_{n\to infty} \left(1 + \frac{1}{n}\right)^n$$

$$\lim_{n\to infty} \sqrt[n]{n}$$

$$\lim_{n\to infty} \frac{n^n}{1\cdot 3\cdot (2n-1)}$$