Answer Keys for Study Guide

1.1

(i) $y = f(x) = \sqrt{\ln x}$

Conditions :

- $x > 0$
- $\ln x > 0 \iff x > 1.$

domain $(1, \infty)$

(ii) $y = \ln \left[\dfrac{1}{\sqrt{t-3}} - \dfrac{1}{t} \right]$

Conditions

- $t - 3 \geqq 0 \iff t \geqq 3$
- $\sqrt{t-3} \neq 0 \iff t > 3$
- $t \neq 0$
- $\dfrac{1}{\sqrt{t-3}} - \dfrac{1}{t} > 0$

$$\iff \frac{1}{\sqrt{t-3}} > \frac{1}{t}.$$

$$\overset{t>3}{\iff} \frac{1}{t-3} > \frac{1}{t^2}. \quad \text{i.e. } t^2 > t - 3$$

$$\overset{t>3}{\iff} t^2 - t + 3 > 0 \quad \left(t^2 - t + 3 = \left(t - \tfrac{1}{2}\right)^2 + \tfrac{11}{4} > 0 \right)$$

always holds.

domain $(3, \infty)$

(iii) $y = \dfrac{1}{x-1} + \dfrac{x}{\sqrt{x^2-9}}$

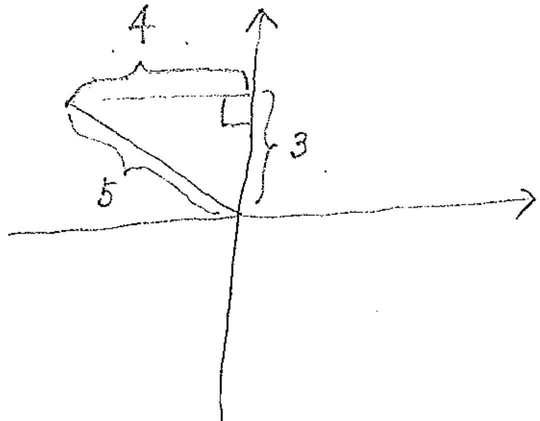
Condition

- $x - 1 \neq 0 \iff x \neq 1$
- $x^2 - 9 \geqq 0 \iff x \leqq -3$ or $3 \leqq x$
- $\sqrt{x^2-9} \neq 0 \iff x < -3$ or $3 < x$

domain

$(-\infty, -3) \cup (3, \infty)$

2.1 $\sin\theta = \dfrac{3}{5} \qquad \dfrac{\pi}{2} < \theta < \pi$

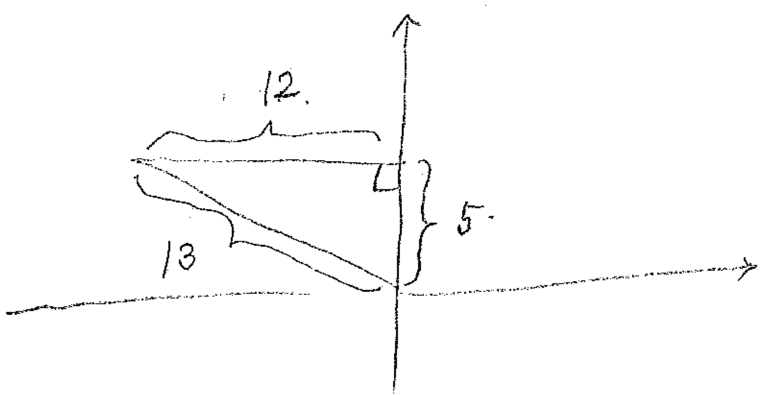


$\cos\theta = -\dfrac{4}{5}, \quad \tan\theta = -\dfrac{3}{4}$

$\csc\theta = \dfrac{5}{3}, \quad \sec\theta = \dfrac{5}{4}$

$\cot\theta = -\dfrac{4}{3}$

2.2. $\tan\theta = -\dfrac{5}{12}, \quad \cos\theta < 0, \quad \sin\theta > 0$



$\sin\theta = \dfrac{5}{13} \qquad \cos\theta = -\dfrac{12}{13}$

$\csc\theta = \dfrac{13}{5} \qquad \sec\theta = -\dfrac{13}{12}$

$\cot\theta = -\dfrac{12}{5}$

3. 1.

$$\cos x = \cos(2x) \text{ on } [0, 2\pi]$$

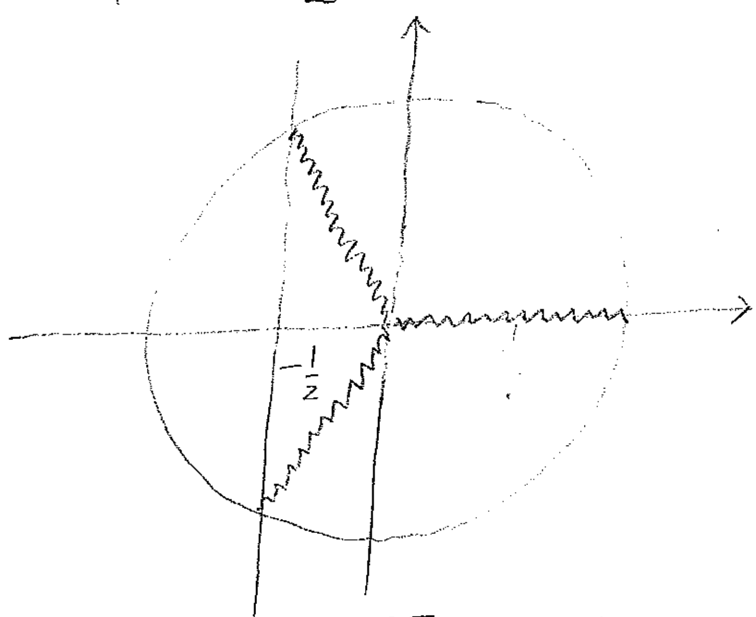
$$\Leftrightarrow \cos x = 2\cos^2 x - 1.$$

$$\Leftrightarrow 2\cos^2 x - \cos x - 1 = 0.$$

$$\Leftrightarrow (2\cos x + 1)(\cos x - 1) = 0.$$

$$\Leftrightarrow 2\cos x + 1 = 0 \text{ or } \cos x - 1 = 0.$$

$$\Leftrightarrow \cos x = -\frac{1}{2} \text{ or } \cos x = 1.$$



$$\Leftrightarrow x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi.$$

3.2

$$3\cot x = 2\sin 2x \quad \text{on } [0, 2\pi].$$

$\iff$ $$3\frac{\cos x}{\sin x} = 2\cdot 2\sin x\cos x$$

$\iff$ $\sin x \neq 0.$

& $3\cos x = 4\sin^2 x\cos x$

$\iff$ $\sin x \neq 0.$

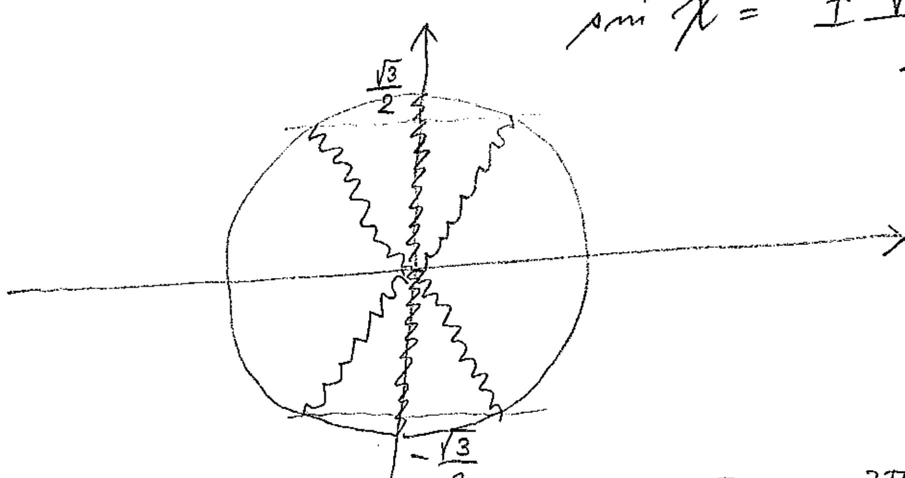
& $3\cos x - 4\sin^2 x\cos x = 0$

$= \cos x(3 - 4\sin^2 x).$

$\iff$ $\sin x \neq 0$

& $\left\{\cos x = 0 \text{ or } \sin^2 x = \frac{3}{4}\right.$

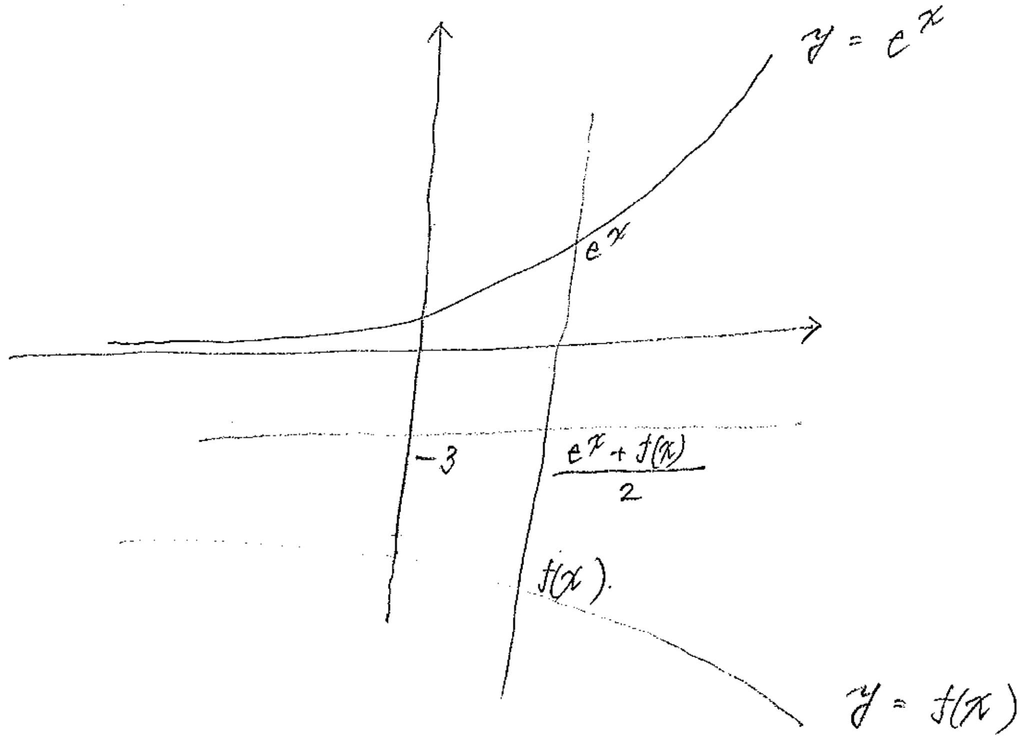
i.e. $\sin x = \pm\frac{\sqrt{3}}{2}.$



$\iff$ $$x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}.$$

4.1.

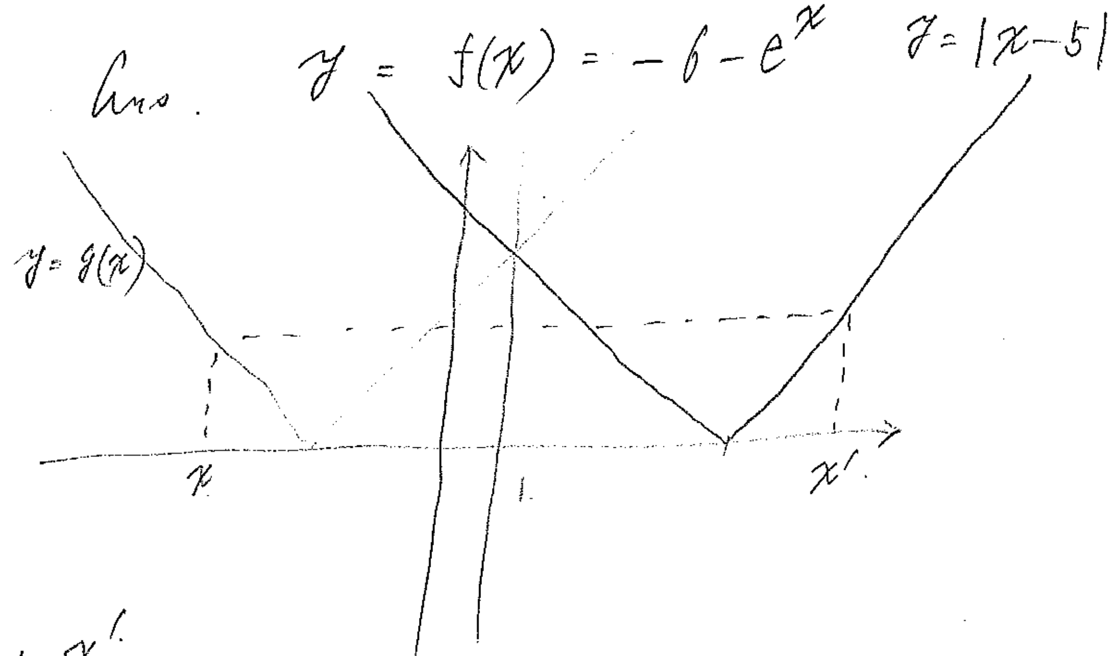
(i)



$$\frac{e^x + f(x)}{2} = -3$$

Ans. $y = f(x) = -6 - e^x$

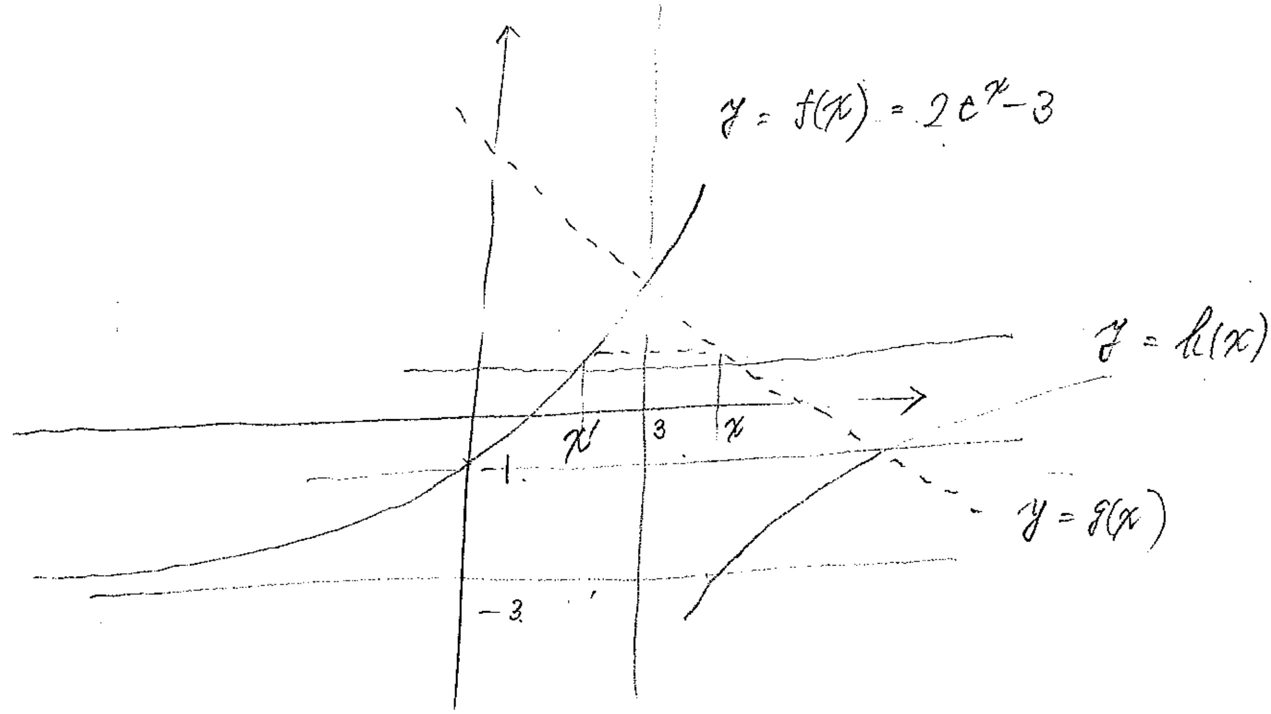
(ii)



$$\begin{cases} \dfrac{x + x'}{2} = 1 \\ g(x) = |x' - 5| \end{cases}$$

Ans: $y = g(x) = |(2-x) - 5|$.

4.2.



1st

$$\begin{cases} \dfrac{x + x'}{2} = 3 \\ g(x) = f(x') = 2e^{x'} - 3 \end{cases}$$

$$g(x) = 2e^{6-x} - 3$$

2nd.

$$\frac{h(x) + g(x)}{2} = -1$$

$$\begin{aligned} h(x) &= -2 - g(x) \\ &= -2 - (2e^{6-x} - 3) \\ &= 1 - 2e^{6-x} \end{aligned}$$

Ans. $y = 1 - 2e^{6-x}$, range $y < 1$.

5.1.

<u>original</u> $y = f(x) = \dfrac{1-e^{-x}}{1+e^{-x}}$

domain $(-\infty, \infty)$

range $(-1, 1)$

Note: $-1 < \dfrac{1-e^{-x}}{1+e^{-x}} < 1$

$\lim_{x\to\infty} \dfrac{1-e^{-x}}{1+e^{-x}} = 1$

$\lim_{x\to-\infty} \dfrac{1-e^{-x}}{1+e^{-x}} = -1$.

<u>inverse</u>

Step 1. $y = \dfrac{1-e^{-x}}{1+e^{-x}}$

Step 2. Solve for x

$$(1+e^{-x})\,y = 1-e^{-x}$$

$$e^{-x}(y+1) = 1-y$$

$$e^{-x} = \frac{1-y}{1+y}$$

$$-x = \ln\frac{1-y}{1+y}$$

$$x = -\ln\frac{1-y}{1+y} = \ln\frac{1+y}{1-y}$$

Step 3 Switch x & y

$$y = \ln\frac{1+x}{1-x}$$

Ans. $f^{-1}(x) = \ln \frac{1+x}{1-x}$

domain $(-1, 1)$

range $(-\infty, \infty)$

5.2.

(i)

Step 1.

$y = x^2 - 1$

Step 2. Solve for x

$y = x^2 - 1$

$y + 1 = x^2$

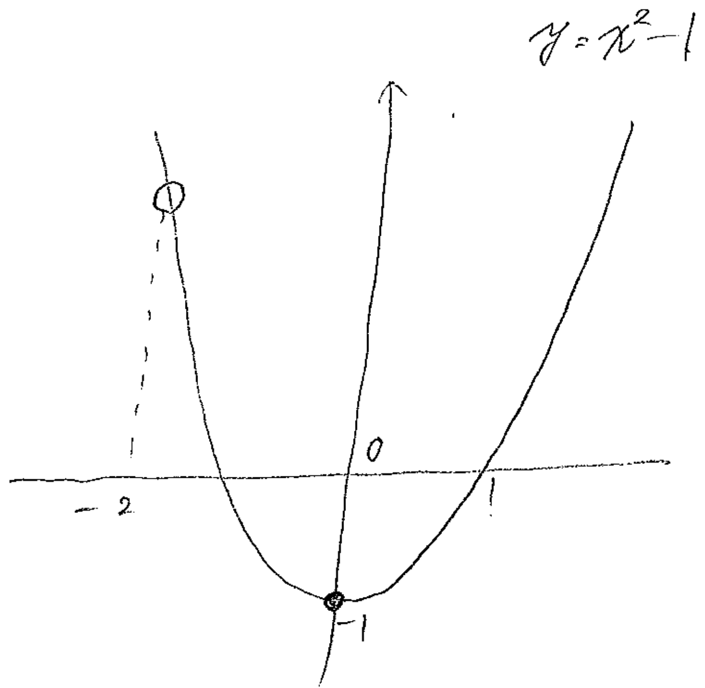
$-\sqrt{y+1} = x$

minus since

$x \in (-2, 0]$

Step 3. Switch x & y.

$y = -\sqrt{x+1}$



domain $(-2, 0]$

range $[-1, 3)$

Ans. $f^{-1}(x) = -\sqrt{x+1}$

domain $[-1, 3)$

range $(-2, 0]$

(ii)

Step 1.

$$y = x^2 - 1$$

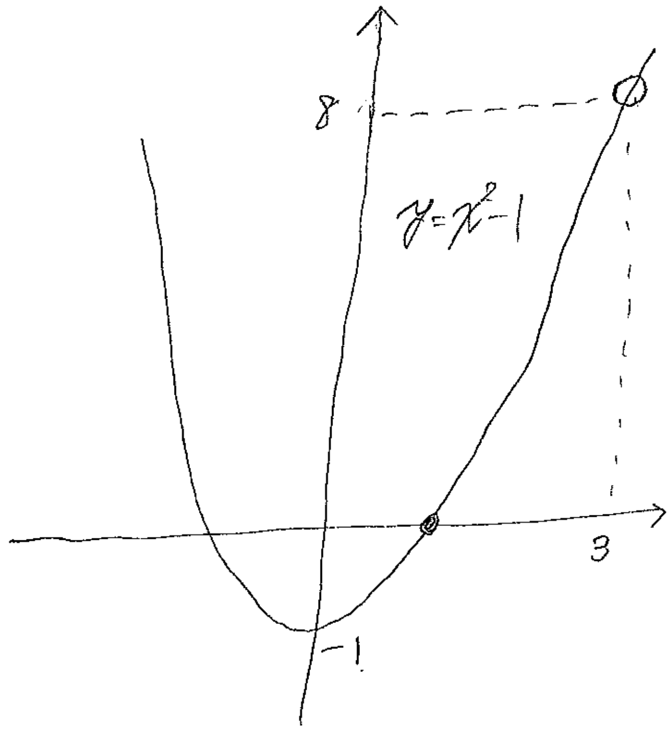
Step 2. Solve for x

$$y = x^2 - 1$$

$$y + 1 = x^2$$

$$+\sqrt{y+1} = x$$

plus since $x \in [1, 3)$



domain $[1, 3)$
range $[0, 8)$

Step 3. Switch x & y

$$y = +\sqrt{x+1}$$

Ans. $f^{-1}(x) = +\sqrt{x+1}$.

domain $[0, 8)$
range $[1, 3)$

5.3

symmetric w.r.t. $y = x$

$\Leftrightarrow$ the graph of the inverse

the function we are looking for is the inverse

Step 1. $y = \dfrac{3x+1}{-5x+3}$

Step 2 Solve for x.

$$(-5x+3)y = 3x+1$$

$$(-5y-3)x = 1-3y$$

$$x = \frac{1-3y}{-5y-3} = \frac{3y-1}{5y+3}$$

Switch x & y.

$$y = \frac{3x-1}{5x+3}$$

Ans. $y = \dfrac{3x-1}{5x+3}$

6.1

51. (a) $e^{7-4x} = 6$

$7 - 4x = \ln 6$

$7 - \ln 6 = 4x \qquad x = \dfrac{7 - \ln 6}{4}$

(b) $\ln(3x - 10) = 2$

$3x - 10 = e^2$

$3x = e^2 + 10$

$x = \dfrac{e^2 + 10}{3}$

52 (a) $\ln(x^2 - 1) = 3$

$x^2 - 1 = e^3$

$x^2 = e^3 + 1$

$x = \pm\sqrt{e^3 + 1}$

(b) $e^{2x} - 3e^x + 2 = 0$

Set $T = e^x$

$T^2 - 3T + 2 = 0$

$(T-1)(T-2)$

$T = 1, 2 \qquad e^x = 1, 2$

$x = \ln 1, \ln 2 = 0, \ln 2.$

53. (a)

$$2^{x-5} = 3$$

$$(x-5)\ln 2 = \ln 3$$

$$x - 5 = \frac{\ln 3}{\ln 2}$$

$$x = \frac{\ln 3}{\ln 2} + 5$$

(b)

$$\ln x + \ln(x-1) = 1$$

$$e \qquad\qquad e$$

$$x(x-1) = e$$

$$x^2 - x - e = 0$$

$$x = \frac{1 \pm \sqrt{1+4e}}{2}$$

$$(x > 0, \quad x - 1 > 0)$$

$$x = \frac{1 + \sqrt{1+4e}}{2}$$

54. (a) $\ln(\ln x) = 1.$

$$\ln x = e.$$

$$x = e^{e}.$$

(b) $e^{ax} = C e^{bx} \qquad a \neq b.$

$$ax = \ln C + bx.$$

$$(a-b)\, x = \ln C.$$

$$x = \frac{\ln C}{a-b}$$

7.

(i) $\lim_{x \to 5^-} \dfrac{x^2 - 3x - 5}{|x-5|}$

$= \lim_{x \to 5^-} \dfrac{x^2 - 3x - 5}{-(x-5)}$ $\rightarrow 5$, $\rightarrow 0^+$

since $x \to 5^- \rightarrow x < 5$

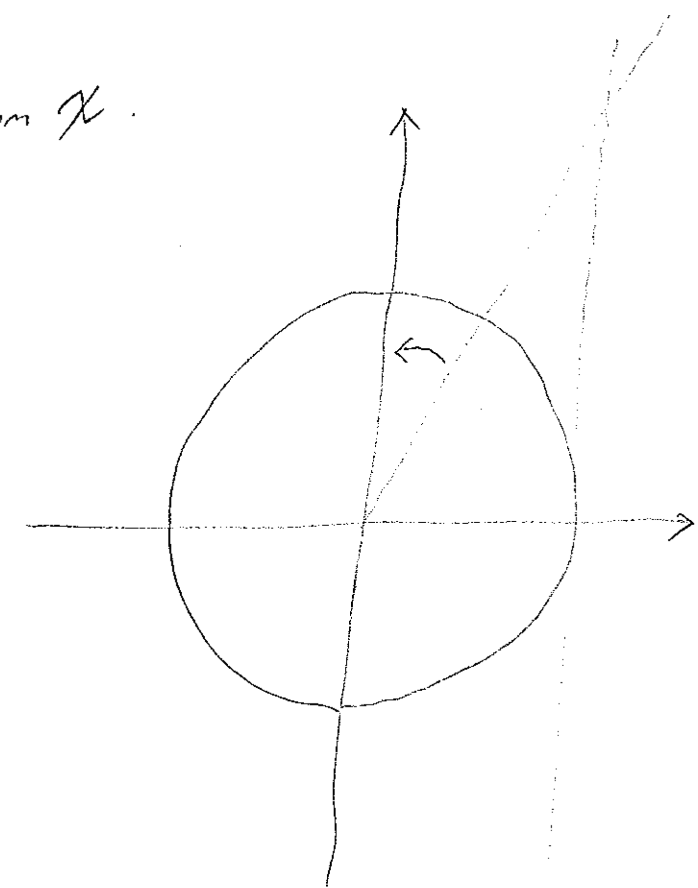
$\rightarrow x - 5 < 0.$

$\rightarrow |x-5| = -(x-5)$

$= +\infty$

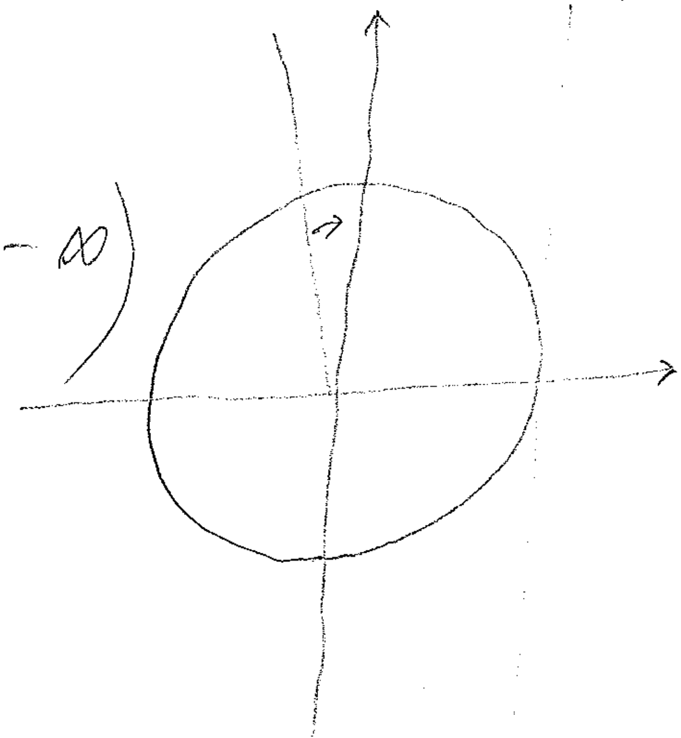
(ii) $\lim_{x \to \left(\frac{\pi}{2}\right)^-} \tan x.$

$= +\infty$



(iii) $\displaystyle\lim_{x \to \left(\frac{\pi}{2}\right)^+} e^{\tan x}$

$$\left(\lim_{x \to \left(\frac{\pi}{2}\right)^+} \tan x = -\infty \right)$$



$= 0$

(iv) $\displaystyle\lim_{x \to 0} \left(\frac{5}{x^2 - x} + \frac{5}{x} \right)$

$= 5\cancel{x}$

$$= \lim_{x \to 0} \frac{5 + 5(x-1)}{\cancel{x}(x-1)}$$

$$= \lim_{x \to 0} \frac{5}{x-1} = -5$$

(v) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + 3x + 1} - x \right)$

$$= \lim_{x \to \infty} \frac{\left(\sqrt{x^2+3x+1} - x \right)\left(\sqrt{x^2+3x+1} + x \right)}{\left(\sqrt{x^2+3x+1} + x \right)}$$

$$= \lim_{x \to \infty} \frac{(\cancel{x^2} + 3x + 1) - \cancel{x^2}}{\sqrt{x^2+3x+1} + x}$$

$$= \lim_{x \to \infty} \frac{(3x + 1)/x}{\left(\sqrt{x^2+3x+1} + x \right)/x}$$

$$= \lim_{x \to \infty} \frac{3 + \frac{1}{x}}{\sqrt{1 + 3 \cdot \frac{1}{x} + \frac{1}{x^2}} + 1}$$

$$= \frac{3}{2}.$$

(vi) $\lim_{x\to 0} \dfrac{|3x-4| - |5x+4|}{x}$

$= \lim_{x\to 0} \dfrac{-(3x-4) - (5x+4)}{x}$

$\left(\begin{array}{l} x \to 0 \to 3x-4 < 0 \\ \quad \to |3x-4| = -(3x-4) \\ x \to 0 \to 5x+4 > 0 \\ \quad |5x+4| = 5x+4 \end{array} \right)$

$= \lim_{x\to 0} \dfrac{-8\cancel{x}}{\cancel{x}} = -8$

(vii) $\lim_{x\to -3} \dfrac{2|x| - 6}{x+3}$

$= \lim_{x\to -3} \dfrac{2(-x) - 6}{x+3}$

$\left(\begin{array}{l} x \to -3 \to x < 0 \\ \quad \to |x| = -x \end{array} \right)$

$= \lim_{x\to -3} \dfrac{-2\cancel{(x+3)}}{\cancel{x+3}} = -2.$

8.1.

Set $\quad 64 - 12$

$$f(x) = x^3 - 3x - 5$$

x	0	1	2	3	4
$f(x)$	-5	-7.	-3	13	47.

$f(x)$ cont. over $[2, 3]$

$f(2) = -3 < 0$.

$f(3) = 13 > 0$.

$f(2) < N = 0 < f(3)$

$\overset{\text{I.V.Th.}}{\Longrightarrow} \quad \exists\, c \in (2, 3)$

s.t.

$f(c) = N = 0$.

$\parallel$

$c^3 - 3c - 5$.

$c^3 - 3c - 5 = 0$.

$\Longleftrightarrow \quad c^3 - 3c = 5$. $\qquad$ Ans. $(2, 3)$

8.2

$\sin x = x^3$

$x = 0$

$\sin x = 0$

$x^3 = 0$

Done!

$\sin x = x^3 + 1$

Set $f(x) = \sin x - (x^3 + 1)$

x	-2	-1	0	1
$f(x)$	$\sin(-2) + 7$	$\sin(-1)$	-1	$\sin 1 - 2$
	> 0		< 0	

f cont. over $[-2, 0]$

$f(-2) > 0$

$f(0) < 0$

$f(-2) > N = 0 > f(0)$

I.V.Th.

$\Longrightarrow$

$$\exists\ c \in (-2, 0)$$

s.t.

$$\underset{\substack{\| \\ \sin c - (c^3+1)}}{f(c)} = N = 0$$

i.e.

$$\sin c - (c^3+1) = 0.$$

$$\Longleftrightarrow \quad \sin c = c^3 + 1$$

$\square$

9.1. $f(x)$ with $f'(2) = 5$.

(i)

$$g'(1) = \lim_{h \to 0} \frac{g(1+h) - g(1)}{h}$$

$$= \lim_{h \to 0} \frac{f(2(1+h)) - f(2 \cdot 1)}{h}$$

$$= \lim_{h \to 0} \frac{f(2+2h) - f(2)}{h}$$

$$= \lim_{h \to 0} \underbrace{\frac{f(2+2h) - f(2)}{2h}}_{f'(2)} \cdot \frac{2h}{h} = 10.$$

(ii)

$$\lim_{h \to 0} \frac{f(2+4h) - f(2)}{3h}$$

$$= \lim_{h \to 0} \underbrace{\frac{f(2+4h) - f(h)}{4h}}_{f'(2)} \quad \frac{4h}{3h}$$

$$= 5 \cdot \frac{4}{3} = \frac{20}{3}.$$

(iii)

$$\lim_{h \to 0} \frac{f(2+4h) - f(2+5h)}{9h}$$

$$= \lim_{h \to 0} \underbrace{\frac{f(2+4h) - f(2+5h)}{\underbrace{(2+4h) - (2+5h)}_{-h}}}_{f'(2)} \cdot \frac{-h}{9h}$$

$$= 5 \cdot \left(-\frac{1}{9}\right) = -\frac{5}{9}$$

9.2 Set $g(x) = -(x-1)^2 + 3$

$k(x) = 2e^x$

$$\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^-} \frac{g(0+h) - 2}{h}$$

$$= \lim_{h \to 0^-} \frac{g(0+h) - g(0)}{h}$$

$$-2x+2$$

$$= g'(0) = (-x^2 + 2x - 1 + 3)' \Big|_{x=0}$$

$$= 2.$$

$$\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{k(0+h) - 2}{h}$$

$$= \lim_{h \to 0^+} \frac{k(0+h) - k(0)}{h}$$

$$= k'(0) = (2e^x)' \Big|_{x=0} = 2.$$

Since the above two are both equal to 2,

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = 2.$$

Thus. $f'(0)$ exists, and $f'(0) = 2$.

10.1.

$$f(x) = \begin{cases} x^2 - a & x \le 1. \\[1em] \dfrac{3x^2 + 12x - b.}{x^2 + 2x - 3} & x > 1. \end{cases}$$

cont. everywhere.

$\iff$

cont. at $x = 1$.

(cont. everywhere else is automatic)

$\iff$

$$\begin{array}{ccccc} \lim_{x\to 1^-} f(x) & = & f(1) & = & \lim_{x\to 1^+} f(x) \\ \| & & \| & & \\ 1-a. & & 1-a. & & \end{array}$$

$$\therefore \quad b = 15 \quad \left\{ \begin{array}{l} \text{for this to exist,} \\ \lim_{x\to 1} (3x^2 + 12x - b) = 0 \\ \qquad \| \\ \qquad 3 + 12 - b \\ \text{since} \\ \lim_{x\to 1} (x^2 + 2x - 3) = 0. \end{array} \right.$$

$$\lim_{x \to 1^+} \frac{3x^2 + 12x - 15}{x^2 + 2x - 3}$$

$$= \lim_{x \to 1^+} \frac{3\cancel{(x-1)}(x+5)}{\cancel{(x-1)}(x+3)} = 3 \cdot \frac{6}{4} = \frac{9}{2}$$

Finally,

$$1 - a = \frac{9}{2}$$

$$\therefore \quad a = 1 - \frac{9}{2} = -\frac{7}{2}$$

Ans. $a = -\frac{7}{2}$, $b = 15$

10.2.

$$f(x) = \begin{cases} 3x - 2c & \text{if } x \le c \\ 5x^2 - 4. & \text{if } x > 0. \end{cases}$$

f cont everywhere

$\iff$ f cont at $x = c$.

(cont everywhere else is automatic)

$\iff$

$$\lim_{x \to c^-} f(x) = f(c) = \lim_{x \to c^+} f(x)$$

$$\lim_{x \to c^-} f(x) = 3c - 2c = c$$

$$\lim_{x \to c^+} f(x) = 5c^2 - 4.$$

i.e.

$$c = 5c^2 - 4$$

i.e. $$5c^2 - c - 4 = 0.$$

$$5c^2 - c - 4 = (5c + 4.)(c - 1)$$

$$\therefore \quad c = -\frac{4}{5}, \ 1.$$

11.1.

$-1 + 4 - 1 - 6$

(i) $y = f(x) = \dfrac{x^3 + 4x^2 + x - 6}{x(x^2 - 1)}$

- denominator $= 0$ $\quad x = 0, \pm 1$.
- numerator $\quad -6, \; -4, \; 0$
  at $x = \quad 0, \; -1, \; +1$.

$$f(x) = \frac{\cancel{(x-1)}(x^2 + 5x + 6)}{x(x+1)\cancel{(x-1)}}$$

vertical asymptotes :

$x = 0, \quad x = -1$.

$$\lim_{x \to \infty} \frac{x^3 + 4x^2 + x - 6}{x(x^2 - 1)} = 1$$

$$\lim_{x \to -\infty} \frac{x^3 + 4x^2 + x - 6}{x(x^2 - 1)} = 1$$

horizontal asymptote(s) : $y = 1$

(ii) $\quad y = f(x) = \dfrac{x^2 - x}{x^2 - 4x + 3}$

- denominator $= 0 \quad x = 1, 3$
- numerator $\quad 0, \quad 6$

at $\quad x = 1, \quad 3$

$$f(x) = \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x-3)}$$

vertical asymptote(s) : $x = 3$

$$\lim_{x\to\infty} \frac{x^2 - x}{x^2 - 4x + 3} = 1$$

$$\lim_{x\to-\infty} \frac{x^2 - x}{x^2 - 4x + 3} = 1$$

horizontal asymptote(s) : $y = 1$

(iii) $y = f(x) = \dfrac{3e^x}{e^x - 1}$.

- denominator $= 0$ $\quad x = 0$.
- numerator $\quad 3$.

at $\quad x = 0$.

vertical asymptote(s) : $x = 0$

$$\lim_{x\to\infty} \frac{3e^x}{e^x - 1} = \lim_{x\to\infty} \frac{3}{1 - \frac{1}{e^x}} = 3$$

$$\lim_{x\to-\infty} \frac{3e^x}{e^x - 1} = 0.$$

horizontal asymptotes : $y = 3$, $y = 0$

12 . 1.

(i). $\displaystyle \lim_{x \to 0} \frac{\sin(3x)}{5x}$

$$= \lim_{x \to 0} \underbrace{\frac{\sin(3x)}{3x}}_{\to 1} \cdot \frac{3x}{5x} = \frac{3}{5}$$

(ii) $\displaystyle \lim_{x \to \infty} \frac{\sin(3x)}{5x} = 0$

$\because$) $-1 \leq \sin(3x) \leq 1$.

Multiply $\dfrac{1}{5x}$

$$-\frac{1}{5x} \leq \frac{\sin(3x)}{5x} \leq \frac{1}{5x}$$

$x \to \infty$: $-\dfrac{1}{5x} \to 0$, $\dfrac{1}{5x} \to 0$

$\therefore$ $\dfrac{\sin(3x)}{5x} \to 0$ Sq. Th.

(iii) $\displaystyle \lim_{x \to 0} \frac{\sin(1/x)}{1/x}$

$\displaystyle = \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$

$\because$) $-1 \leq \sin\left(\dfrac{1}{x}\right) \leq 1$

Multiply x

$(x > 0)$ $\quad -x \leq x \sin\left(\dfrac{1}{x}\right) \leq x$

$(x < 0)$

$$-x \geq x \sin\left(\frac{1}{x}\right) \geq x$$

$x \to 0$ in either case

$$\begin{array}{ccc} \text{L.H.S.} & \text{Middle} & \text{R.H.S} \\ \downarrow & \Big\downarrow \text{ Sq. Th} & \downarrow \\ 0 & 0 & 0 \end{array}$$

(iv) $\displaystyle \lim_{x\to 0} \frac{\sin(4x)^2}{2x^2 \cos x}$

$$= \lim_{x\to 0} \underbrace{\frac{\sin(16x^2)}{16x^2}}_{\to 1} \cdot \frac{16x^2}{2x^2} \cdot \frac{1}{\underbrace{\cos x}_{\to 1}}$$

$= 8.$

(v) $\displaystyle \lim_{x\to -\infty} \cos 3x \cdot e^x = 0.$

$\because)$ $\quad -1 \leq \cos 3x \leq 1.$

Multiply e^x

$$-e^x \leq e^x \cos 3x \leq e^x$$

$x \to -\infty$: $\quad -e^x \to 0$, $\quad e^x \to 0$, $\quad e^x\cos 3x \to 0$ (Sq. Th.)

13.1 $\quad y = f(x) = x^3$

(i) tan. parallel to $y = 3x$

$$f'(x) = 3x^2 = 3$$

$$x = \pm 1.$$

eq. of tan.

slope : 3

passing $(1, 1)$

or $(-1, -1)$

$$y - 1 = 3(x - 1)$$

or $y - (-1) = 3(x - (-1))$

(ii) tan. perpendicular to $y = 3x$

$$f'(x) = 3x^2 = -\frac{1}{3}$$

no such x.

13.2.

tan. to parabola $y = f(x) = x^2$

passing $(1, -3)$

Set (a, a^2) the point of tangency.

eq. of tan,

slope $2a$.

passing (a, a^2).

$$y - a^2 = 2a(x - a).$$

This has to pass $(1, -3)$.

$$\therefore (-3) - a^2 = 2a(1 - a)$$

i.e. $$a^2 - 2a - 3 = 0$$

$$\quad\quad\ \, \shortparallel$$

$$(a-3)(a+1)$$

$$a = -1, 3.$$

Ans. $$y - (-1)^2 = 2(-1)(x - (-1))$$

or $$y - 3^2 = 2 \cdot 3 (x - 3)$$

14.1.

(i) $f(x) = x^2 e^x$

$$f'(x) = 2x \cdot e^x + x^2 \cdot e^x$$

(ii) $f(x) = \dfrac{1+x}{1-x}$

$$f'(x) = \frac{(1+x)'(1-x) - (1+x)(1-x)'}{(1-x)^2}$$

$$= \frac{(1-x) - (1+x)(-1)}{(1-x)^2}$$

$$= \frac{2}{(1-x)^2}$$

14.2. $f(x) = e^x g(x)$

$$f'(x) = e^x \cdot g(x) + e^x \cdot g'(x)$$

$$f''(x) = \{e^x \cdot g(x) + e^x \cdot g'(x)\} + \{e^x \cdot g'(x) + e^x \cdot g''(x)\}.$$

$$f''(0) = \{\underset{1}{e^0} \cdot \underset{3}{g(0)} + \underset{1}{e^0} \cdot \underset{5}{g'(0)}\} + \{\underset{1}{e^0} \cdot \underset{5}{g'(0)} + \underset{1}{e^0} \cdot \underset{7}{g''(0)}\} = 20.$$

Challenge Problem.

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0. \\ 0 & \text{if } x = 0. \end{cases}$$

$$\lim_{h \to 0} \frac{g(0+h) - g(0)}{h}$$

$$= \lim_{h \to 0} \frac{h^2 \sin\left(\frac{1}{h}\right) - 0}{h}$$

$$= \lim_{h \to 0} h \sin\left(\frac{1}{h}\right) = 0.$$

$$\therefore \quad g'(0) = 0.$$

$\rightarrow$ g cont. and diff. at $x = 0$.

Ans. A.