Answer keys for the problems in Study Guide for Exam 2.

1.1

(i) $y = \sin(\sin(\sin x))$.

$$\begin{tikzcd}
x \arrow[r, mapsto] \arrow[rrrddd, bend right] & \sin x \arrow[d, equal] & & \\
& u \arrow[r, mapsto] & \sin u \arrow[d, equal] & \\
& & v \arrow[r, mapsto] & \sin v \arrow[d, equal] \\
& & & y
\end{tikzcd}$$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= \cos v \cdot \cos u \cdot \cos x$$

$$= \cos(\sin(\sin x)) \cdot \cos(\sin x) \cdot \cos x$$

(ii) $y = \left(\frac{t-2}{2t+1}\right)^9$.

$$\begin{tikzcd}
t \arrow[r, mapsto] \arrow[rrdd, bend right] & \left(\frac{t-2}{2t+1}\right) \arrow[d, equal] & \\
& u \arrow[r, mapsto] & u^9 \arrow[d, equal] \\
& & y
\end{tikzcd}$$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} = 9u^8 \cdot \frac{1 \cdot (2t+1) - (t-2) \cdot 2}{(2t+1)^2}$$

$$= 9\left(\frac{t-2}{2t+1}\right)^8 \cdot \frac{5}{(2t+1)^2}$$

(iii) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

Note that

$$\frac{d}{dx}\left(\sqrt{x+u}\right) = \frac{1 + \frac{du}{dx}}{2\sqrt{x+u}}$$

Using this formula

first with $u = \sqrt{x + \sqrt{x}}$

secondly with $u = \sqrt{x}$,

one obtains

$$\frac{dy}{dx} = \frac{1 + \frac{d}{dx}\left(\sqrt{x+\sqrt{x}}\right)}{2\sqrt{x + \sqrt{x + \sqrt{x}}}}$$

$$= \frac{1 + \dfrac{1 + \frac{1}{\sqrt{x}}}{2\sqrt{x + \sqrt{x}}}}{2\sqrt{x + \sqrt{x + \sqrt{x}}}}$$

(iv) $y = e^{\sec 3\theta}$

$$\begin{tikzcd}
\theta \arrow[r, mapsto] \arrow[rrrddd, bend right] & 3\theta \arrow[d, equal] & & \\
& u \arrow[r, mapsto] & \sec u \arrow[d, equal] & \\
& & v \arrow[r] & e^{v} \arrow[d, equal] \\
& & & y
\end{tikzcd}$$

$$\frac{dy}{d\theta} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{d\theta}$$

$$= e^{v} \cdot \sec u \tan u \cdot 3$$

$$= e^{\sec 3\theta} \cdot \sec 3\theta \tan 3\theta \cdot 3$$

(v) $y = e^{2^{x^3}}$

$$\begin{tikzcd}
x \arrow[r, mapsto] \arrow[rrrddd, bend right, dashed] & x^3 \arrow[d, equal] & & \\
& u \arrow[r, mapsto] & 2^u \arrow[d, equal] & \\
& & v \arrow[r, mapsto] & e^v \arrow[d, equal] \\
& & & y
\end{tikzcd}$$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= e^{v} \cdot \ln 2 \cdot 2^{u} \cdot 3x^2$$

$$= e^{2^{x^3}} \cdot \ln 2 \cdot 2^{x^3} \cdot 3x^2$$

(vi) $y = \sin^{-1}(\sqrt{1-x^2})$

$$\begin{tikzcd}
x \arrow[r, mapsto] \arrow[rrrddd, bend right, mapsto] & 1-x^2 \arrow[d, phantom, "\shortparallel"] & & \\
 & u \arrow[r, mapsto] & \sqrt{u} \arrow[d, phantom, "\shortparallel"] & \\
 & & v \arrow[r, mapsto] & \sin^{-1} v \arrow[d, phantom, "\shortparallel"] \\
 & & & y
\end{tikzcd}$$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{\sqrt{1-v^2}} \cdot \frac{1}{2\sqrt{u}} \cdot (-2x)$$

$$= \frac{1}{\sqrt{1-(1-x^2)}} \cdot \frac{1}{2\sqrt{1-x^2}} \cdot -2x$$

$$= \frac{-x}{\sqrt{x^2}\sqrt{1-x^2}}$$

$$= \frac{-x}{|x|\sqrt{1-x^2}}.$$

Warning: The answer is NOT $\frac{-1}{\sqrt{1-x^2}}$, since $\sqrt{x^2} = |x|$ and $\sqrt{x^2} \neq x$.

Actually $\frac{dy}{dx}$ does NOT exist at $x = 0$.

(vii.) $y = \sin^{-1}(\sqrt{1-x})$

Note: The domain of this function has to satisfy the condition

- $1 - x \geq 0$
- $\sqrt{1-x} \leq 1$

i.e. the domain is $[0, 1]$

$$\begin{tikzcd}
x \arrow[r, mapsto] \arrow[rrrddd, bend right, leftrightarrow] & 1-x \arrow[d, equal] & & \\
& u \arrow[r, mapsto] & \sqrt{u} \arrow[d, equal] & \\
& & v \arrow[r, mapsto] & \sin^{-1} v \arrow[d, equal] \\
& & & y
\end{tikzcd}$$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{\sqrt{1-v^2}} \cdot \frac{1}{2\sqrt{u}} \cdot (-1)$$

$$= \frac{1}{\sqrt{1-(1-x)}} \cdot \frac{1}{2\sqrt{1-x}} (-1) = -\frac{1}{\sqrt{x(1-x)}}$$

Warning: The domain for $\frac{dy}{dx}$ is $(0, 1)$, and NOT $[0, 1]$.

(vii) $y = \ln|\sec(3\theta) + \tan(3\theta)|$

$$\begin{tikzcd}
\theta \arrow[r] \arrow[rrrddd, bend right] & 3\theta \arrow[d, equal] & & \\
& u \arrow[r, mapsto] & \sec u + \tan u \arrow[d, equal] & \\
& & v \arrow[r, mapsto] & \ln|v| \arrow[d, equal] \\
& & & y.
\end{tikzcd}$$

$$\frac{dy}{d\theta} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{d\theta}$$

$$= \frac{1}{v} \cdot (\sec u \tan u + \sec^2 u) \cdot 3$$

$$= \cancel{\frac{1}{\sec(3\theta) + \tan(3\theta)}} \cdot \sec(3\theta) \cancel{\{\tan(3\theta) + \sec(3\theta)\}}\, 3$$

$$= 3 \sec(3\theta)$$

1.2.

$$F(x) = f(x)^2 \cdot f(g(x))$$

$$F'(x) = 2\,f(x)\,f'(x) \cdot f(g(x))$$
$$+ \; f(x)^2 \cdot f'(g(x)) \cdot g'(x)$$

$$F'(1) = 2\,f(1)\,f'(1) \cdot f(\underbrace{g(1)}_{3})$$
$$+ \; f(1)^2 \cdot f'(\underbrace{g(1)}_{3}) \cdot g'(1)$$

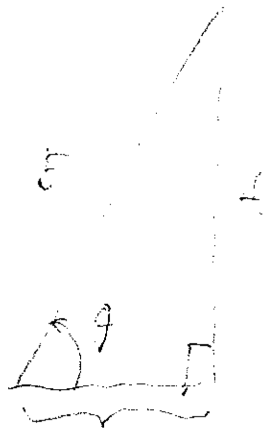
$$= 2 \cdot 5 \cdot 4 \cdot (-1)$$
$$+ \; 5^2 \cdot (-2) \cdot 2$$

$$= -40 - 100 = -140$$

2.1 (i) $\tan\left(\sin^{-1}\left(\frac{4}{5}\right)\right)$

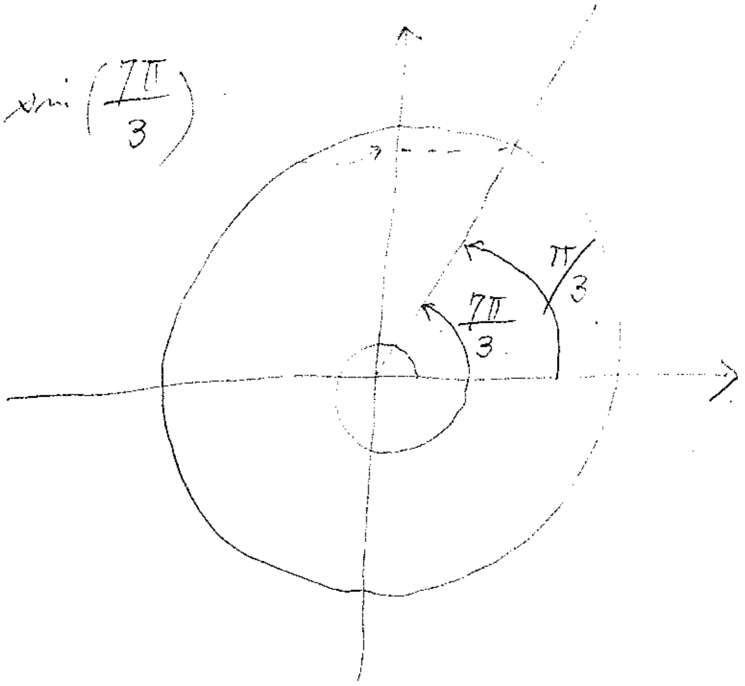
let $\theta = \sin^{-1}\left(\frac{4}{5}\right)$

$\tan\theta = \frac{4}{3}$



$\sqrt{5^2-4^2} = 3$

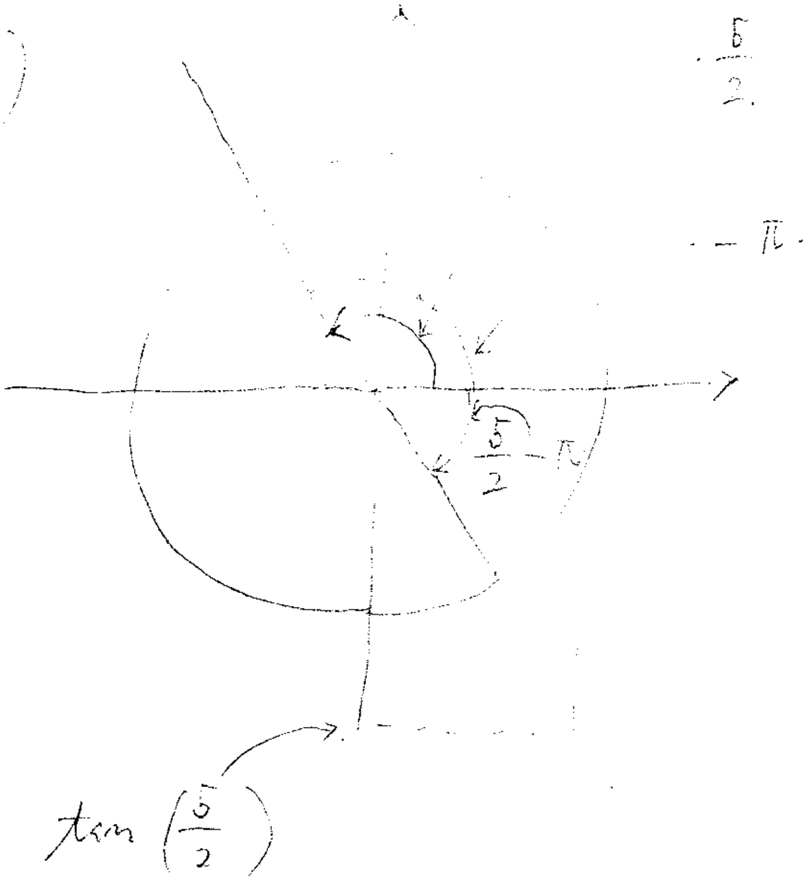
(ii) $\sin^{-1}\left(\sin\left(\frac{7\pi}{3}\right)\right) = \frac{\pi}{3}.$



(iii) $\frac{\pi}{2} < \frac{5}{2} < \pi$

$\tan^{-1}\left(\tan\left(\frac{5}{2}\right)\right)$

$= \frac{5}{2} - \pi$

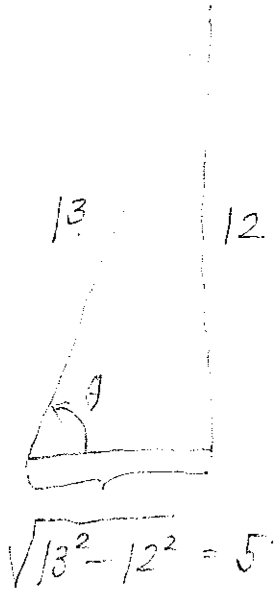


(iv) $\sin\left(2\sin^{-1}\left(\frac{12}{13}\right)\right)$

Let $\theta = \sin^{-1}\left(\frac{12}{13}\right)$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2 \cdot \frac{12}{13} \cdot \frac{5}{13} = \frac{120}{169}$



3.1 i) $y = x^{x}$

$$\ln y = \ln(x^{x})$$

$$= x \ln x$$

$$\frac{d}{dx}(\quad) = \frac{d}{dx}(\quad)$$

$$\frac{1}{y}\frac{dy}{dx} = \underline{1 \cdot \ln x} + x\frac{1}{x}$$

$$\frac{dy}{dx} = y \cdot (\quad)$$

$$= x^{x}(\ln x + 1)$$

ii) $y = (\ln x)^{\tan 3x}$

$$\ln y = \tan 3x \cdot \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \sec^{2} 3x \cdot 3 \cdot \ln x + \tan 3x \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = (\ln x)^{\tan 3x}\left\{3\sec^{2}3x \cdot \ln x + \frac{\tan 3x}{x}\right\}$$

(iii) $y = (\sqrt{x})^{\sin x}$

$$\ln y = \sin x \cdot \ln\sqrt{x}$$

$$= \frac{1}{2}\sin x \cdot \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left\{\cos x \cdot \ln x + \sin x \cdot \frac{1}{x}\right\}$$

$$\frac{dy}{dx} = (\sqrt{x})^{\sin x} \cdot \frac{1}{2}\left\{\cos x \cdot \ln x + \frac{\sin x}{x}\right\}$$

(iv) $y = x^{\frac{1}{x}}$

$$\ln y = \frac{1}{x}\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x^2}\cdot\ln x + \frac{1}{x}\cdot\frac{1}{x}$$

$$\frac{dy}{dx} = x^{\frac{1}{x}}\cdot\left\{-\frac{1}{x^2}\cdot\ln x + \frac{1}{x^2}\right\}$$

$$= x^{\frac{1}{2}-2}\left\{1 - \ln x\right\}$$

4.1

$$f(x) + x^2 \cdot (f(x))^3 = 10$$

$$\frac{d}{dx}\left(\qquad \downarrow \qquad \right) = \frac{d}{dx}\left(\downarrow \right)$$

$$f'(x) + 2x \cdot (f(x))^3$$

$$+ x^2 \cdot 3 (f(x))^2 \cdot f'(x) = 0$$

$$f'(1) + 2 \cdot 1 \, (f(1))^3$$

$$+ 1^2 \cdot 3 (f(1))^2 \cdot f'(1) = 0$$

$$f'(1) \cdot \{ 2 \cdot 1 \cdot 2^3 + 1^2 \cdot 3 \cdot 2^2 \} = 0$$

$$\therefore \quad f'(1) = 0$$

4.2

$$x^2 + 2xy - y^2 + x = 2$$

$$\frac{d}{dx}\left(\qquad \right) = \frac{d}{dx}\left(\downarrow \right)$$

$$2x + 2y + 2x \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} + 1 = 0$$

$$2 \cdot 1 + 2 \cdot 2 + 2 \cdot 1 \frac{dy}{dx} - 2 \cdot 2 \frac{dy}{dx} + 1 = 0$$

$$7 - 2 \frac{dy}{dx} = 0$$

$$\therefore \quad \frac{dy}{dx} = \frac{7}{2}.$$

4.3 $y^2(\ln x) + y = 3x$

point $(1, 3)$

$$2y\frac{dy}{dx}\cdot\ln x + y^2\cdot\frac{1}{x} + \frac{dy}{dx} = 3$$

$$2\cdot 1\cdot\frac{dy}{dx}\cdot\underbrace{\ln 1}_{0} + 3^2\cdot\frac{1}{1}\cdot\frac{dy}{dx} = 3$$

$$\therefore \frac{dy}{dx} = \frac{1}{3}$$

Eq. of tan.

$$y - 3 = \frac{1}{3}(x - 1)$$

4.4. $e^{\frac{x}{y}} = 7x - y$

$$e^{\frac{x}{y}}\left\{\frac{1\cdot y - x\cdot\frac{dy}{dx}}{y^2}\right\} = 7 - \frac{dy}{dx}$$

$$e^{\frac{x}{y}}\cdot\frac{1}{y} - 7 = \left(e^{\frac{x}{y}}\cdot x - 1\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{e^{\frac{x}{y}}\cdot\frac{1}{y} - 7}{e^{\frac{x}{y}}\cdot x - 1}$$

5.1. Linear approx. of $f(x) = e^x$ at $a = 0$

$$L(x) = f(a) + f'(a)(x-a)$$
$$= e^0 + e^0 \cdot (x - 0)$$
$$= 1 + x$$

$$e^{0.01} = f(0.01)$$
$$\approx L(0.01) = 1 + 0.01 = 1.01$$

5.2. Linear approx. of $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$ at $a = 27$

$$L(x) = f(a) + f'(a)(x-a)$$
$$= \sqrt[3]{27} + \frac{1}{3(\sqrt[3]{27})^2}(x - 27)$$
$$= 3 + \frac{1}{27}(x - 27)$$

$$\sqrt[3]{26.8} = f(26.8)$$
$$\approx L(26.8)$$
$$= 3 + \frac{1}{27}(26.8 - 27)$$
$$= 3 - \frac{0.2}{27} \doteq 2.9826.$$

0.0074

5.3. Linear approx. of $f(x) = x^{30}$ at $a = 1$.

$$
\begin{aligned}
L(x) &= f(a) + f'(a)(x-a) \\
&= 1^{30} + 30 \cdot 1^{29}(x-1) \\
&= 1 + 30(x-1)
\end{aligned}
$$

$$
\begin{aligned}
(1.035)^{30} &= f(1.035) \\
&\approx L(1.035) \\
&= 1 + 30(1.035 - 1) \\
&= 2.05
\end{aligned}
$$

$$
\begin{aligned}
\$100,000 \times (1.035)^{30} &\approx \$100,000 \times 2.05 \\
&= \$205,000
\end{aligned}
$$

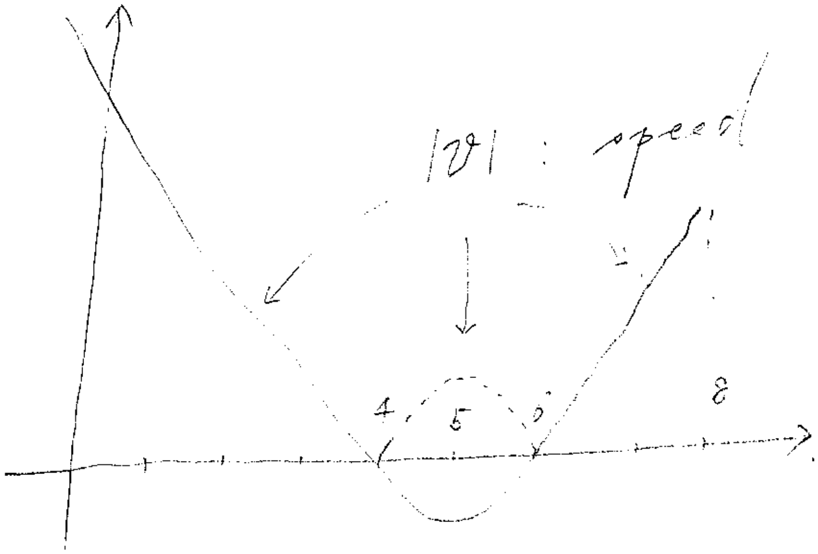
6.1.

$$f(t) = t^3 - 15t^2 + 72t$$

$$v = f'(t) = 3t^2 - 30t + 72$$

$$= 3(t^2 - 10t + 24)$$

$$= 3(t-4)(t-6)$$



slowing down

$(0, 4) \cup (5, 6)$

6.2. $s = f(t) = t^3 - 6t^2 + 9t$

$$v = f'(t) = 3t^2 - 12t + 9$$

$$= 3(t^2 - 4t + 3)$$

$$= 3(t-1)(t-3)$$

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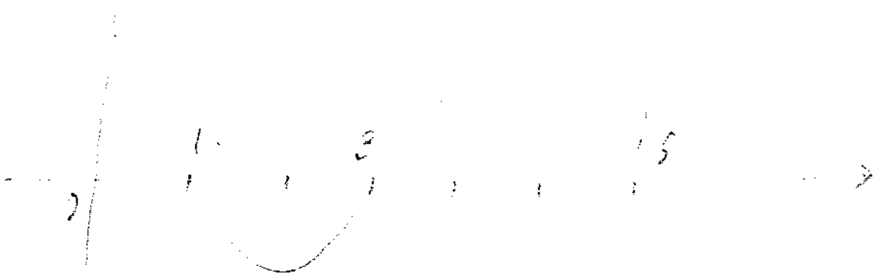


$t = 0 \qquad 1$

$f(0) = 0 \xrightarrow{4.} f(1) = 4.$

$t = 3 \qquad 1.$

$f(3) = 0 \xleftarrow{4} f(1) = 4$

$t = 3 \qquad 6.$

$f(3) = 0 \xrightarrow{54} f(6) = 54.$

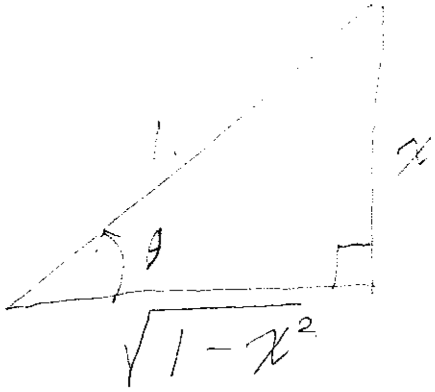
total distance traveled

$= 4 + 4 + 54 = 62.$

7.1

(i) $\tan(\sin^{-1} x)$

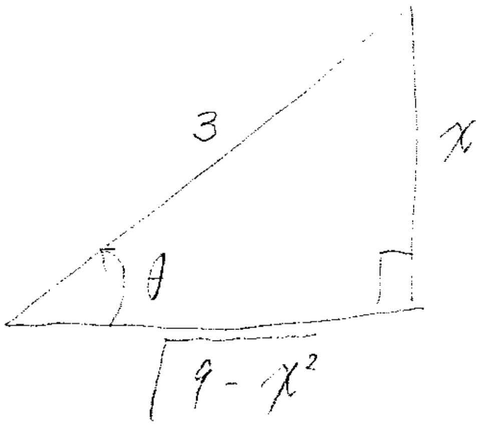
Set $\theta = \sin^{-1} x$



$$\tan(\sin^{-1} x) = \tan\theta = \frac{x}{\sqrt{1-x^2}}$$

(ii) $\cos\left(\tan^{-1}\left(\frac{x}{\sqrt{9-x^2}}\right)\right)$

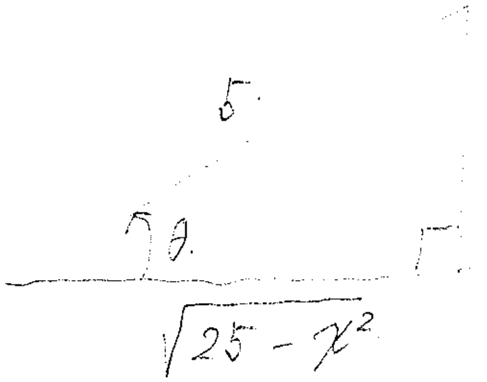
Set $\theta = \tan^{-1}\left(\frac{x}{\sqrt{9-x^2}}\right)$



$$\cos\left(\tan^{-1}\left(\frac{x}{\sqrt{9-x^2}}\right)\right) = \cos(\theta) = \frac{\sqrt{9-x^2}}{3}$$

(iii) $\csc\left(\cot^{-1}\left(\frac{\sqrt{25-x^2}}{x}\right)\right)$ when $x > 0$

Let $\theta = \cot^{-1}\left(\frac{\sqrt{25-x^2}}{x}\right)$

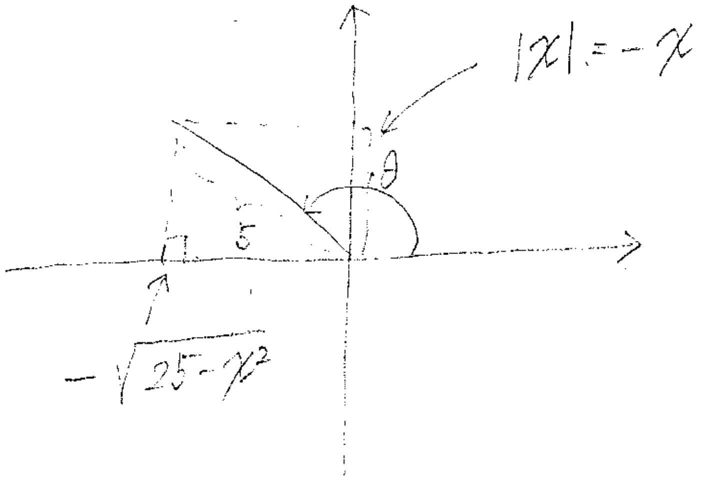


$\csc\left(\cot^{-1}\left(\frac{\sqrt{25-x^2}}{x}\right)\right)$

$\csc(\theta) = \frac{5}{x}$

Note: $\csc\left(\cot^{-1}\left(\frac{\sqrt{25-x^2}}{x}\right)\right)$ when $x < 0$

Let $\theta = \cot^{-1}\left(\frac{\sqrt{25-x^2}}{x}\right)$, $0 < \theta < \pi$



$\csc\theta = \frac{5}{-x}$

$= -\frac{5}{x}$

8.1

(i) $\sinh(0) = \dfrac{e^0 - e^{-0}}{2} = 0$

(ii) $\sinh(\ln 5)$

$$= \frac{e^{\ln 5} - e^{-\ln 5}}{2} = \frac{5 - \frac{1}{5}}{2} = \frac{12}{5}$$

(iii) $\cosh(\ln 5)$

$$= \frac{e^{\ln 5} + e^{-\ln 5}}{2} = \frac{5 + \frac{1}{5}}{2} = \frac{13}{5}$$

(iv) $\dfrac{1 + \tanh(\ln 2)}{1 - \tanh(\ln 2)}$

$$= \frac{(e^{\ln 2} + e^{-\ln 2}) + (e^{\ln 2} - e^{-\ln 2})}{(e^{\ln 2} + e^{-\ln 2}) - (e^{\ln 2} - e^{-\ln 2})} = \frac{2 \cdot 2}{2 \cdot \frac{1}{2}}$$

$$= 4$$

8.2 $f(x) = \sinh(\ln x)$

$$f'(x) = \cosh(\ln x) \cdot \frac{1}{x}$$

$$f(2) = \sinh(\ln 2) = \frac{e^{\ln 2} - e^{-\ln 2}}{2}$$

$$= \frac{2 - \frac{1}{2}}{2} = \frac{3}{4}$$

$$f'(2) = \cosh(\ln 2) \cdot \frac{1}{2} = \frac{e^{\ln 2} + e^{-\ln 2}}{2} \cdot \frac{1}{2}$$

$$= \frac{2 + \frac{1}{2}}{2} \cdot \frac{1}{2} = \frac{5}{4}$$

9.1.

(i)
$$y = \ln\left(x\sqrt{x^2-10}\right)$$
$$= \ln x + \frac{1}{2}\ln(x^2-10)$$

$$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{x^2-10}.$$

$$= \frac{2x^2-10}{x(x^2-10}$$

(ii)
$$y = \ln(e^x + xe^x)$$
$$= \ln(e^x(1+x))$$
$$= \ln e^x + \ln(1+x)$$
$$= x + \ln(1+x)$$

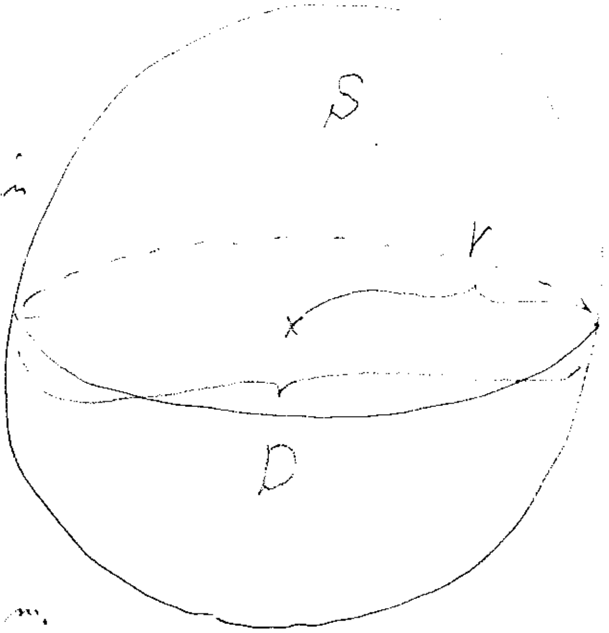
$$\frac{dy}{dx} = 1 + \frac{1}{1+x} = \frac{2+x}{1+x}.$$

10.1

Picture

Given

$$\frac{dS}{dt} = -3 \ \text{cm}^2/\text{min}$$



Unknown

$$\frac{dD}{dt} = ?$$

when $D = 11$ cm.

Relation

$$S = 4\pi r^2 = 4\pi\left(\frac{D}{2}\right)^2$$
$$= \pi D^2$$

Solution

$$\frac{dS}{dt} = \pi \cdot 2D \frac{dD}{dt}$$

$$-3 = \pi \cdot 2 \cdot 11 \cdot \frac{dD}{dt}$$

$$\therefore \frac{dD}{dt} = -\frac{3}{22\pi}$$

Ans. The diameter decreases at the rate of $\frac{3}{22\pi}$ cm/min

10.2

Given

$$\frac{d\theta}{dt} = 5 \cdot 2\pi.$$

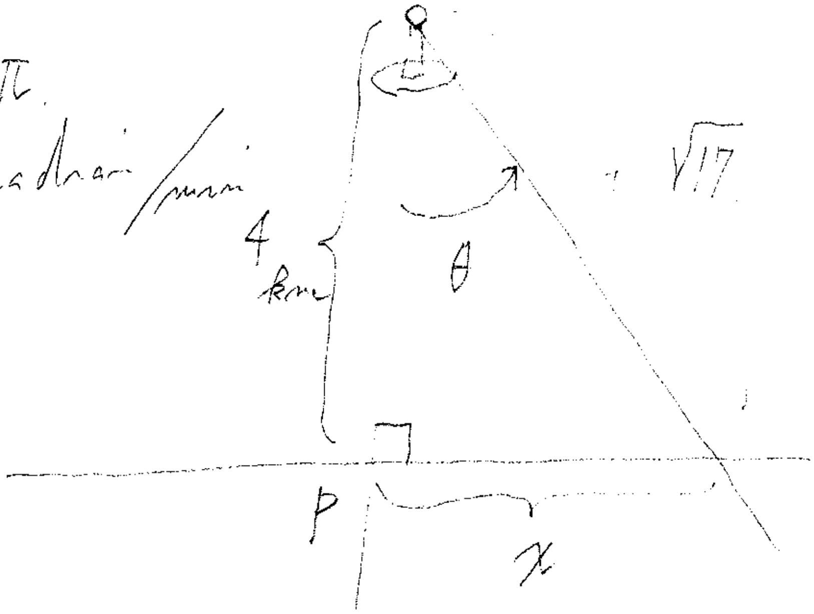
radian/min

Unknown

$$\frac{dx}{dt} = ?$$

when $x = 1$.

Picture



$x = 1$.

Relation

$$\frac{x}{4} = \tan\theta \quad \text{i.e.} \quad x = 4\tan\theta.$$

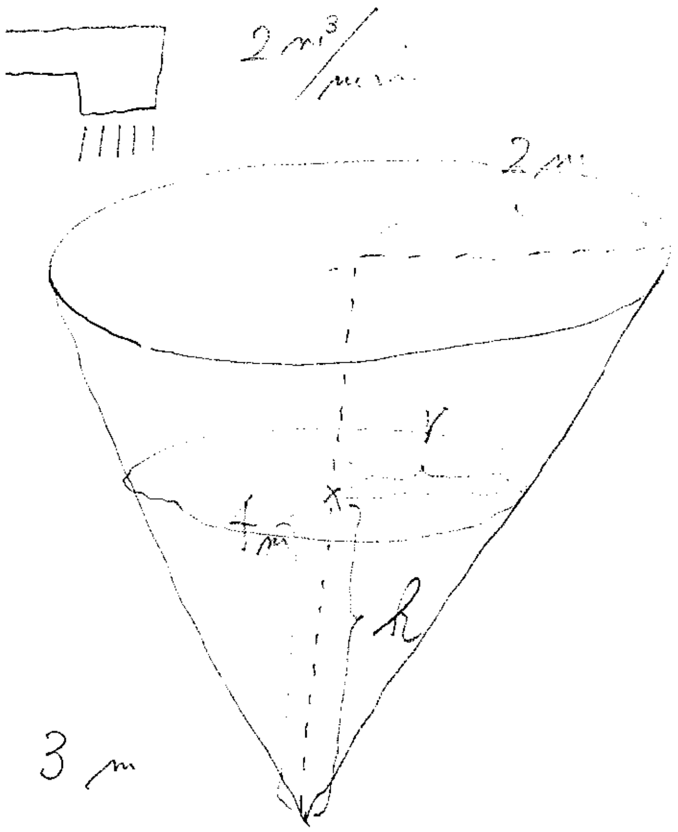
Solution

$$\frac{dx}{dt} = 4 \sec^2\theta \cdot \frac{d\theta}{dt}$$

$$= 4 \cdot \left(\frac{\sqrt{17}}{4}\right)^2 \cdot 10\pi$$

$$= \frac{85}{2}\pi \quad \text{km/min}.$$

10.3



Given

$$\frac{dV}{dt} = 2 \quad m^3/min$$

Unknown

$$\frac{dh}{dt} = ?$$

when $h = 3$ m

Relation

$$V = \frac{1}{3}\pi r^2 \cdot h$$

$$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h = \frac{\pi}{12} h^3$$

Solution

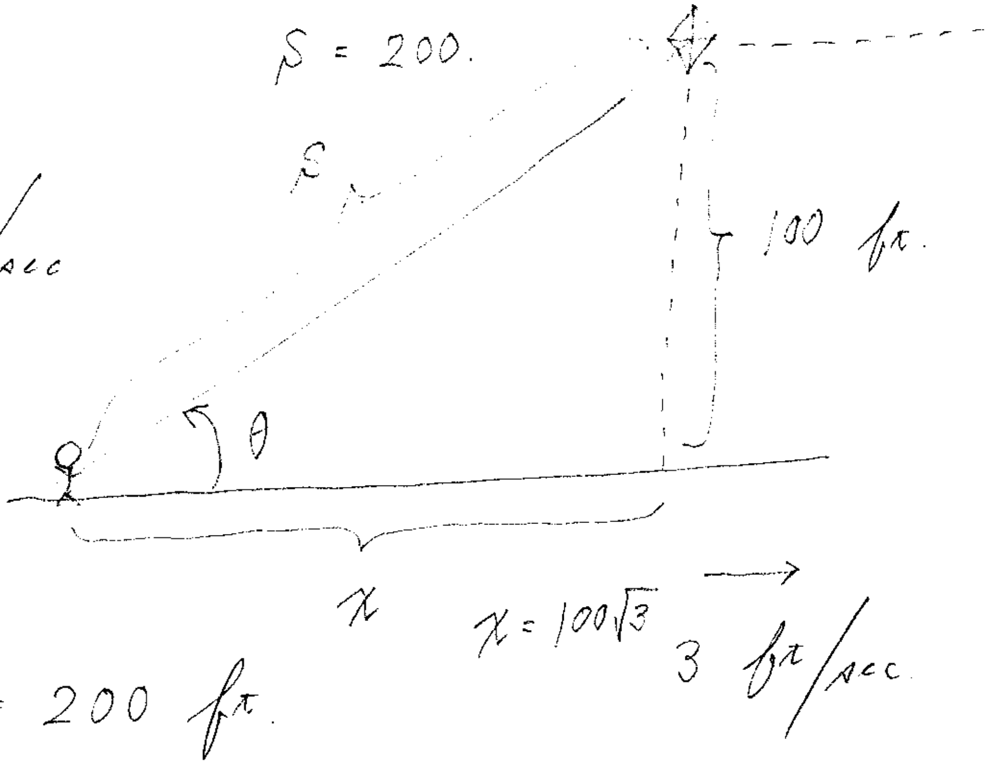
$$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$2 = \frac{\pi}{12} \cdot 3 \cdot 3^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2}{\frac{\pi}{12_4} \cdot 3 \cdot 3^2} = \frac{8}{9\pi} \quad m/min$$

10. 4

Picture



Given

$$\frac{dx}{dt} = 3 \text{ ft/sec}$$

Unknown

$$\frac{d\theta}{dt} = ?$$

when $S = 200$ ft.

Relation

$$\frac{100}{x} = \tan\theta \quad \text{i.e.} \quad 100 = x \cdot \tan\theta$$

Solution

$$0 = \frac{dx}{dt}\tan\theta + x \cdot \sec^2\theta \cdot \frac{d\theta}{dt}$$

$$0 = 3 \cdot \frac{1}{\sqrt{3}} + 100\sqrt{3} \cdot \left(\frac{2}{\sqrt{3}}\right)^2 \cdot \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{-\sqrt{3}}{\frac{400}{\sqrt{3}}} = -\frac{3}{400}.$$

Ans. The angle is decreasing at the rate of $\frac{3}{400}$ radian/sec.

10.5 (speed)

Picture ㉗

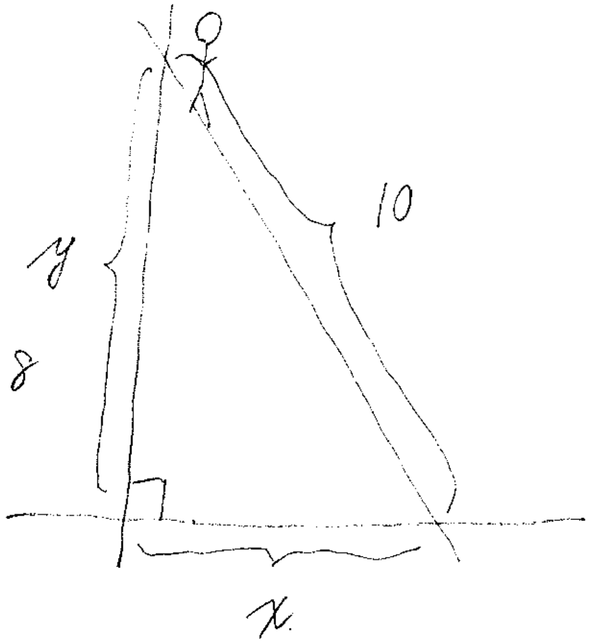
Given

$$\frac{dx}{dt} = 1 \text{ ft/sec}$$

Unknown

$$\frac{dy}{dt} = ?$$

when $x = 6$ ft.



$x = 6$.

Relation

$$x^2 + y^2 = 10^2$$

Solution

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

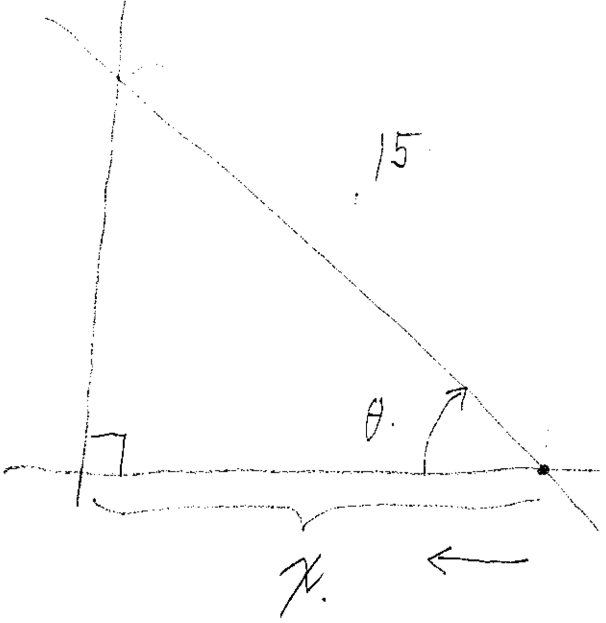
$$2\cdot 6\cdot 1 + 2\cdot 8\cdot \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{2\cdot 6}{2\cdot 8} = -\frac{3}{4}$$

Ans. The top of the ladder is sliding down at the speed of $\frac{3}{4}$ ft/sec.

10.5 (angle)

Picture



Given

$$\frac{dx}{dt} = -2 \text{ ft/sec}$$

Unknown

$$\frac{d\theta}{dt} = ? \quad \text{when } \theta = \frac{\pi}{4}$$

Relation

$$\frac{x}{15} = \cos\theta \quad \text{i.e.} \quad x = 15\cos\theta$$

Solution

$$\frac{dx}{dt} = 15\,(-\sin\theta)\,\frac{d\theta}{dt}$$

$$-2 = 15\,\underbrace{\left(-\sin\frac{\pi}{4}\right)}_{-\frac{\sqrt{2}}{2}}\,\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = -\frac{2\sqrt{2}}{15} \text{ radian/sec}$$

10. 6.

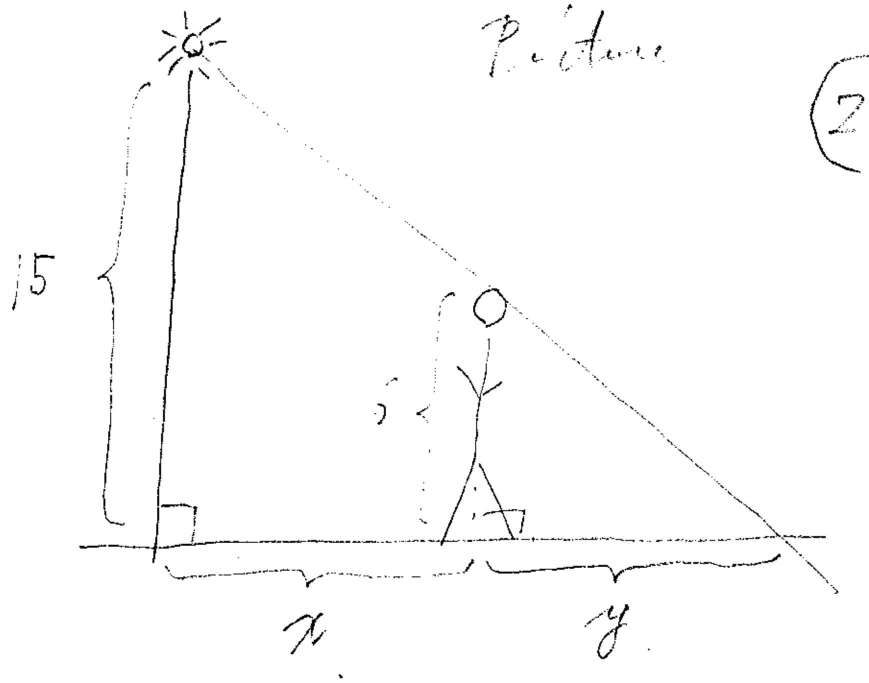
Given

$$\frac{dy}{dt} = 2 \text{ ft/sec}$$

Unknown

$$\frac{dx}{dt} = ?$$

Picture



Relation

$$\frac{x+y}{15} = \frac{y}{6}$$

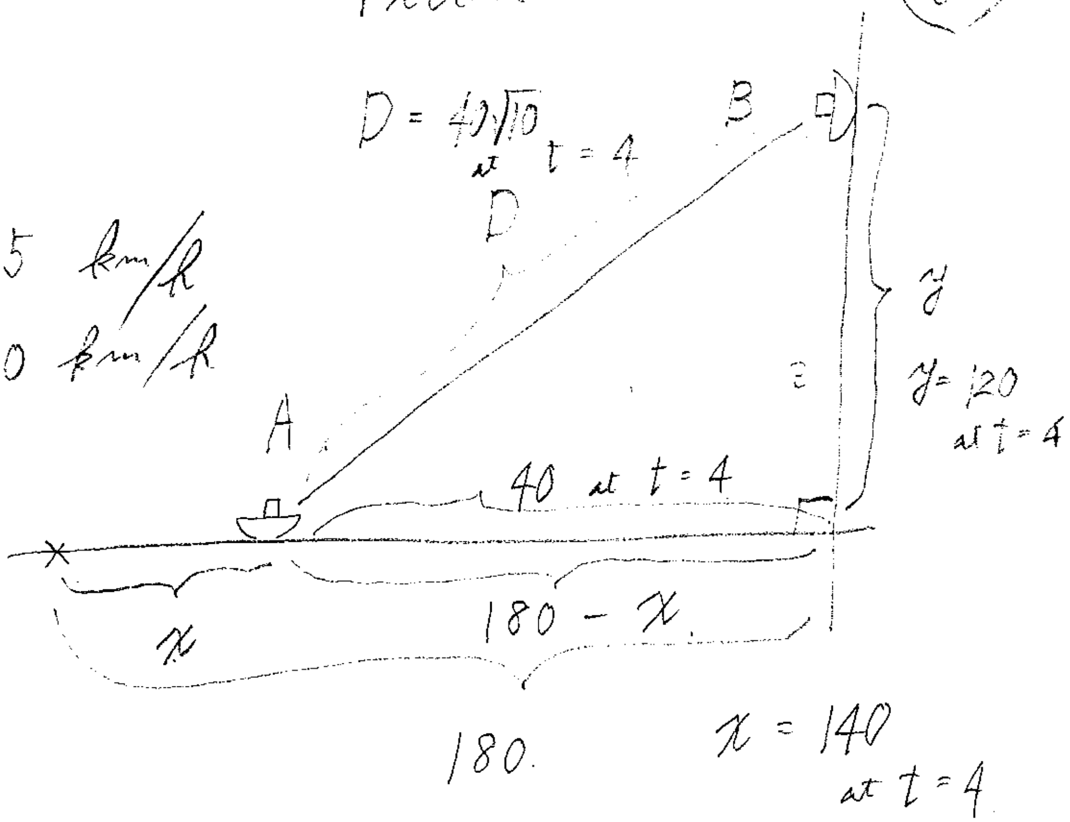
$$x = \frac{3}{2} y$$

Solution

$$\frac{dx}{dt} = \frac{3}{2} \frac{dy}{dt} = \frac{3}{2} \cdot 2 = 3 \text{ ft/sec}$$

10.7

Picture



Given

$$\frac{dx}{dt} = 35 \text{ km/h}$$

$$\frac{dy}{dt} = 30 \text{ km/h}$$

Unknown

$$\frac{dD}{dt} = ?$$

when $t = 4$

Relation

$$D^2 = (180 - x)^2 + y^2$$

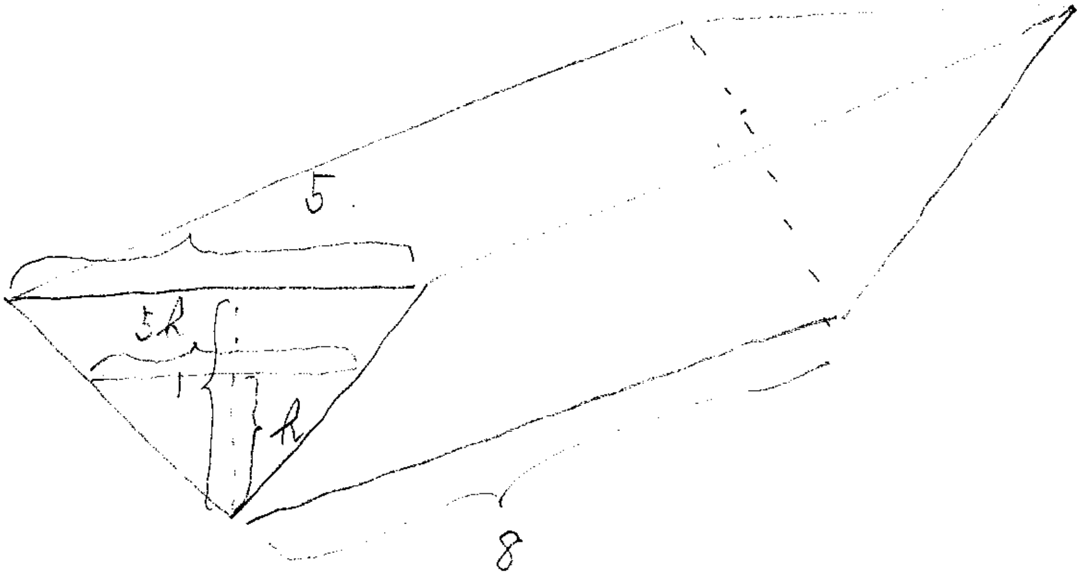
Solution

$$2D\frac{dD}{dt} = 2(180 - x)\left(-\frac{dx}{dt}\right) + 2y \cdot \frac{dy}{dt}$$

$$2 \cdot 40\sqrt{10}\,\frac{dD}{dt} = 2 \cdot 40 \cdot (-35) + 2 \cdot \overset{3}{\cancel{120}} \cdot 30$$

$$\frac{dD}{dt} = \frac{55}{\sqrt{10}} = \frac{11}{2}\sqrt{10} \text{ km/h}$$

10. 8 Picture



Given $\frac{dV}{dt} = 11$ ft³/min

Unknown $\frac{dh}{dt} = ?$ when $h = \frac{4}{12}$ ft

Relation

$$V = \frac{1}{2} 5h \cdot h \cdot 8 = 20 h^2$$

Solution

$$\frac{dV}{dt} = 20 \cdot 2h \cdot \frac{dh}{dt}$$

$$11 = 20 \cdot 2 \cdot \frac{4}{12} \cdot \frac{dh}{dt}$$

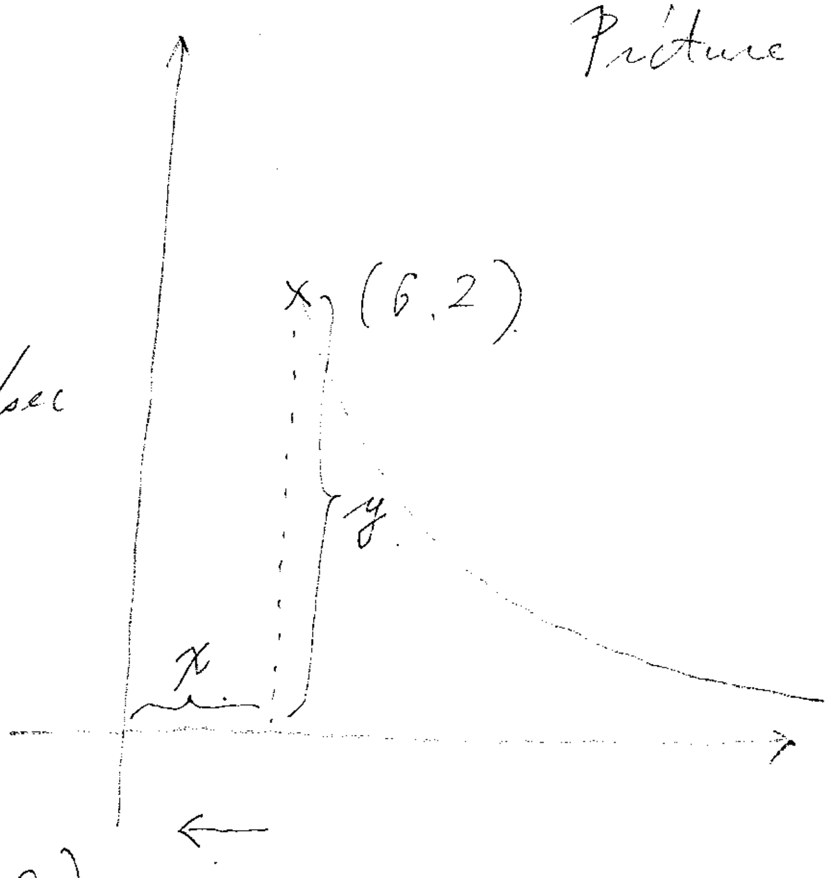
$$\frac{dh}{dt} = \frac{33}{40} \text{ ft/min}$$

10. 9.

Picture

Given

$$\frac{dx}{dt} = -5 \ \ cm/sec$$



Unknown

$$\frac{dy}{dt} = ?$$

when

$(x, y) = (6, 2)$

Relation $xy = 12$

Solution

$$\frac{dx}{dt} \cdot y + x \cdot \frac{dy}{dt} = 0$$

$$(-5) \cdot 6 + 2 \cdot \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{5 \cdot 6}{2} = 15 \ \ cm/sec$$