1.1.

(iii) $$y = \sqrt{x + \sqrt{x + \sqrt{x}}}$$

Note that

$$\frac{d}{dx}\left(\sqrt{x+u}\right) = \frac{1 + \frac{du}{dx}}{2\sqrt{x+u}}.$$

Using this formula

first with $u = \sqrt{x + \sqrt{x}}$

secondly with $u = \sqrt{x}$

one obtains

$$\frac{dy}{dx} = \frac{1 + \frac{d}{dx}\left(\sqrt{x+\sqrt{x}}\right)}{2\sqrt{x + \sqrt{x+\sqrt{x}}}}$$

$$= \frac{1 + \frac{1 + \frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}}{2\sqrt{x + \sqrt{x+\sqrt{x}}}}$$

1.1

(vii) $\quad y = \sin^{-1}\left(\sqrt{1-x}\right)$

Note: The domain of this function has to satisfy the conditions

- $1 - x \geq 0$
- $\sqrt{1-x} \leq 1$.

i.e. the domain is $[0, 1]$

$$x \longmapsto 1-x$$
$$\qquad\qquad \overset{\parallel}{u} \longmapsto \sqrt{u}$$
$$\qquad\qquad\qquad\qquad \overset{\parallel}{v} \longmapsto \sin^{-1} v$$
$$\qquad\qquad\qquad\qquad\qquad\qquad \overset{\parallel}{y}.$$

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{\sqrt{1-v^2}} \cdot \frac{1}{2\sqrt{u}} \cdot (-1)$$

$$= \frac{1}{\sqrt{1-(1-x)}} \cdot \frac{1}{2\sqrt{1-x}} \cdot (-1) = -\frac{1}{2\sqrt{x(1-x)}}$$

Warning: The domain for $\frac{dy}{dx}$ is $(0,1)$, and NOT $[0,1]$.

3 . 1 .

(ii) $$y = (\ln x)^{\tan 3x}$$

$$\ln y = \tan 3x \cdot \ln(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \sec^2 3x \cdot 3 \cdot \ln(\ln x) + \tan 3x \, \frac{1/x}{\ln x}$$

$$\frac{dy}{dx} = (\ln x)^{\tan 3x} \cdot \left\{ \sec^2 3x \cdot 3 \cdot \ln(\ln x) + \frac{\tan 3x}{x \ln x} \right\}$$

4.1.

$$f(x) + x^2 (f(x))^3 = 10$$

$$\frac{d}{dx}\left(\qquad \right) = \frac{d}{dx}\left(\quad \right)$$

$$f'(x) + 2x \cdot (f(x))^3$$

$$+ x^2 \cdot 3 (f(x))^2 \cdot f'(x) = 0.$$

$$f'(1) + 2 \cdot 1 \cdot (\underset{2}{f(1)})^3$$

$$+ 1^2 \cdot 3 (\overset{2}{\underset{2}{f(1)}})^2 \cdot f'(1) = 0$$

$$(1 + 12) f'(1) = -16$$

$$f'(1) = -\frac{16}{13}.$$

8.2.

$$f(x) = \sinh(\ln x)$$

$$f'(x) = \cosh(\ln x) \cdot \frac{1}{x}$$

$$f(2) = \sinh(\ln 2)$$

$$= \frac{e^{\ln 2} - e^{-\ln 2}}{2}$$

$$= \frac{2 - \frac{1}{2}}{2} = \frac{3}{4}$$

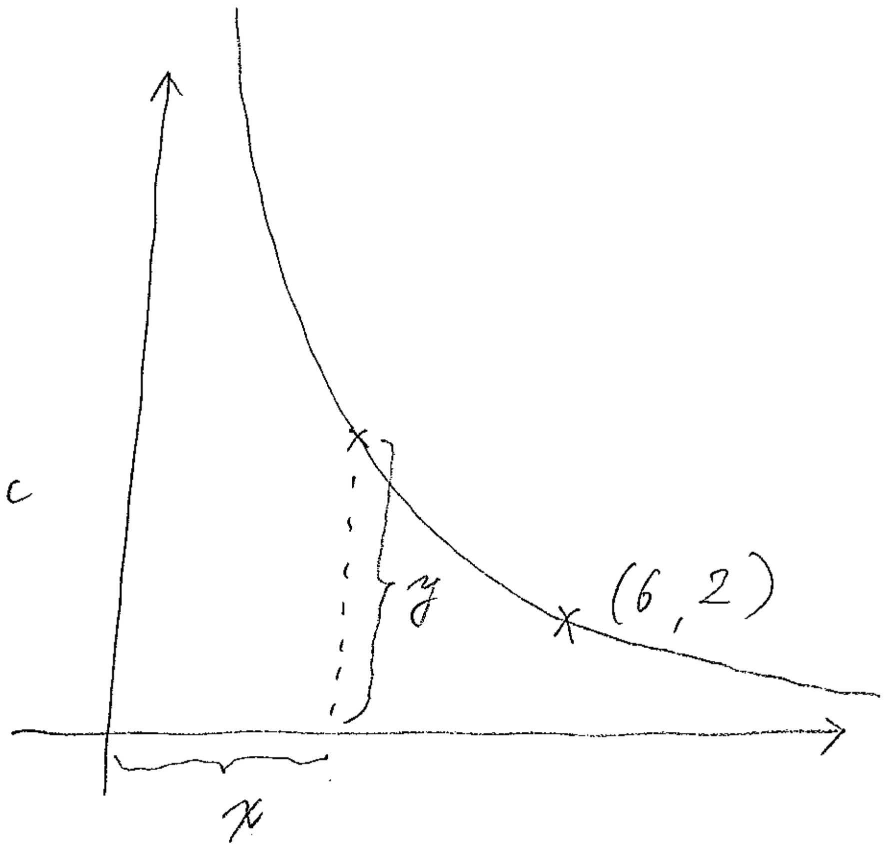
$$f'(2) = \cosh(\ln 2) \cdot \frac{1}{2}$$

$$= \frac{2 + \frac{1}{2}}{2} \cdot \frac{1}{2} = \frac{5}{8}$$

10.9.

Given

$$\frac{dx}{dt} = -5 \text{ cm/sec}$$



Unknown.

$$\frac{dy}{dt} = ?$$

when $(x, y) = (6, 2)$

Relation

$$xy = 12$$

Solution

$$\frac{dx}{dt} \cdot y + x \frac{dy}{dt} = 0$$

$$(-5) \cdot 2 + 6 \cdot \frac{dy}{dt} = 0.$$

$$\frac{dy}{dt} = \frac{5 \cdot 2}{6} = \frac{5}{3} \text{ cm/sec.}$$