$$P(2, 1, -5)$$

1.1

$$x^2 + y^2 + z^2 + 2x - 6y + 9 = 0.$$

$$(x^2 + 2x + 1)$$

$$+ (y - 6y + 9)$$

$$+ z^2 = -9$$

$$+1.$$

$$+9$$

$$(x+1)^2 + (y-3)^2 + z^2 = 1$$

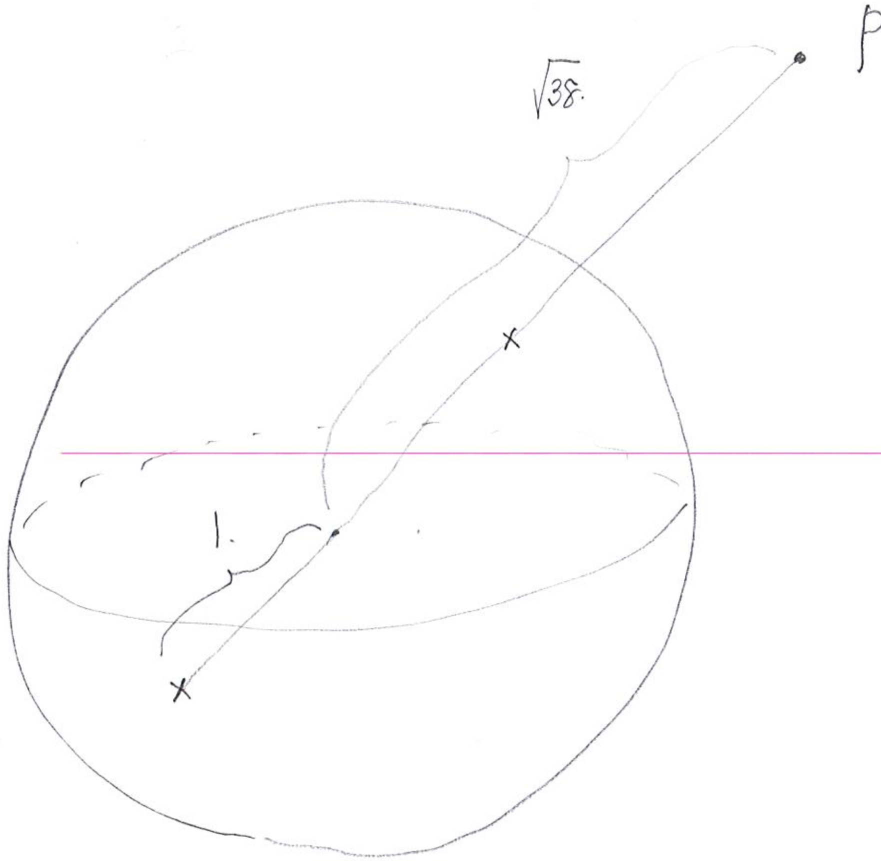
C : center $(-1, 3, 0)$.

$r = 1.$

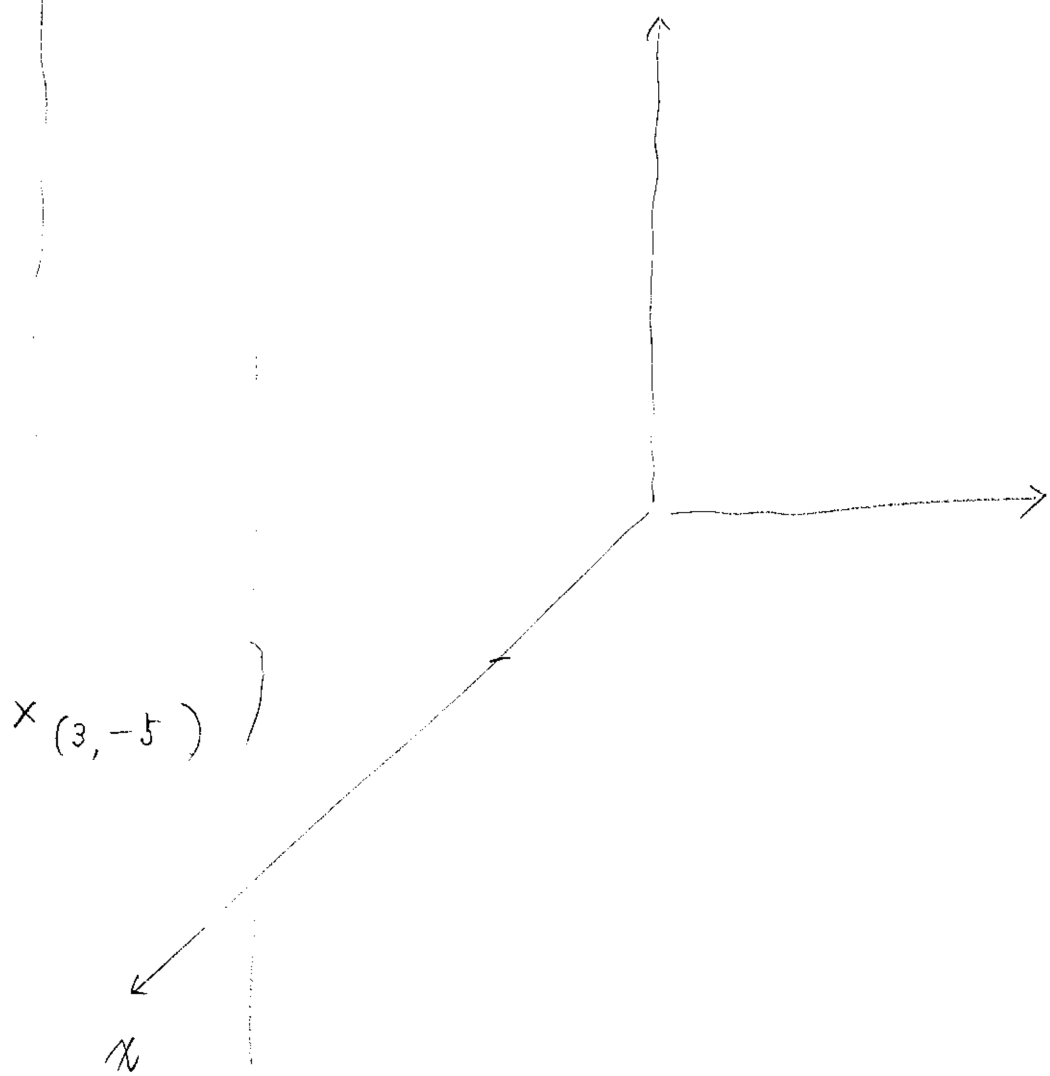
$$PC = \sqrt{(2+1)^2 + (1-3)^2 + (-5-0)^2}$$

$$= \sqrt{\underset{9}{3^2} + \underset{4}{2^2} + \underset{25}{(-5)^2}}$$

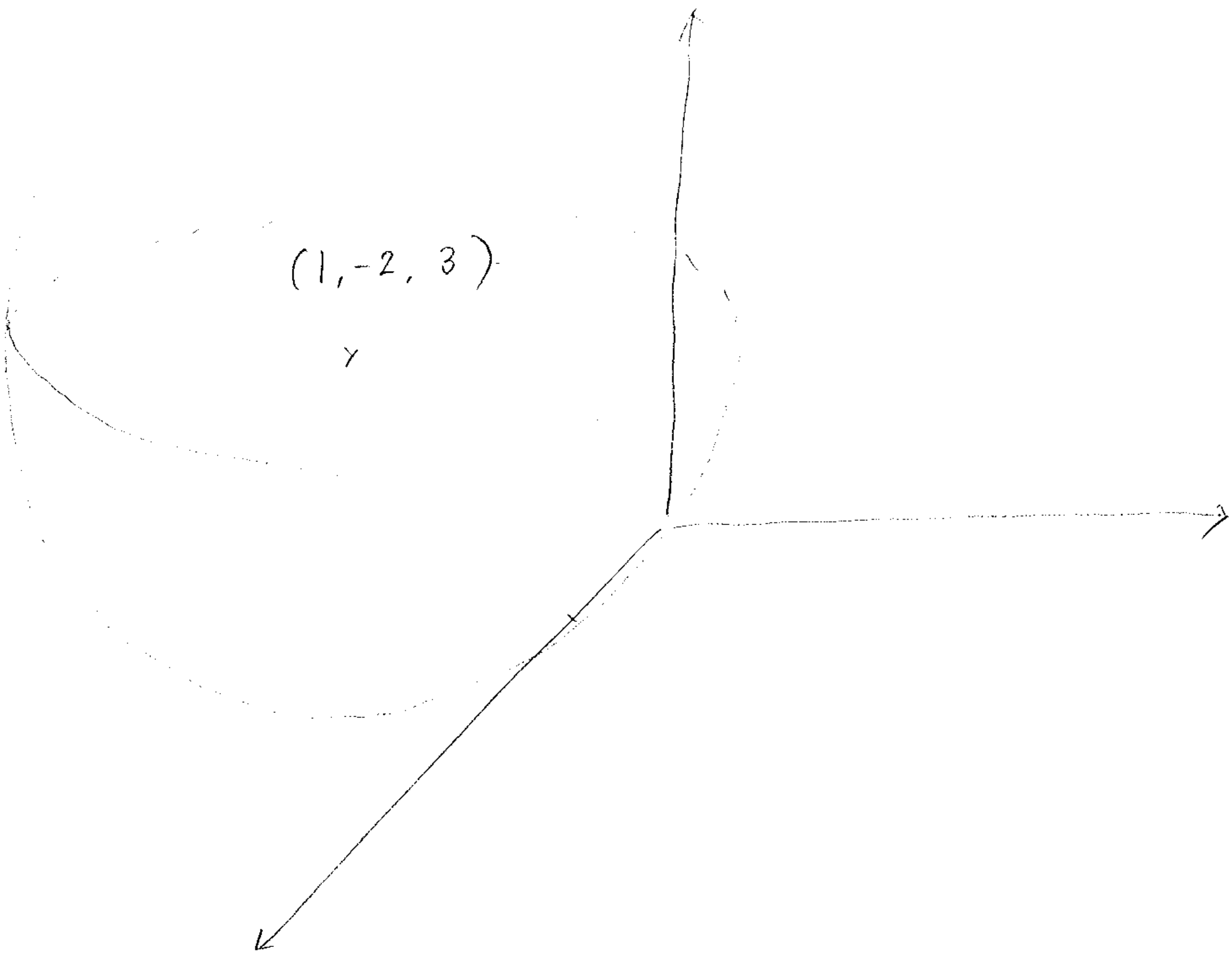
$$= \sqrt{38}.$$



P
√38.
1.



$$(x-3)^2 + (y+5)^2 \le 2^2$$



$$(x-1)^2 + (y+2)^2 + (z-3)^2 \le 7^2$$

&

$$z \ge 3$$

2.1.

$$P = (3, -1, 4)$$
$$Q = (7, 2, -5)$$

$$\overrightarrow{PQ} = \langle 7-3,\ 2-(-1),\ -5-4 \rangle$$
$$= \langle 4, 3, -9 \rangle$$

$$u = \frac{1}{|\overrightarrow{PQ}|}\overrightarrow{PQ} = \frac{1}{\sqrt{4^2+3^2+(-9)^2}} \langle 4, 3, -9 \rangle$$

2.2.

$$\vec{u} = \langle 1, 2 \rangle$$
$$\vec{v} = \langle 3, -4 \rangle$$

$$\vec{w} = \langle 1, 0 \rangle$$
$$= \alpha \vec{u} + \beta \vec{v}$$
$$= \alpha \langle 1, 2 \rangle + \beta \langle 3, -4 \rangle$$
$$= \langle 1 \cdot \alpha + 3 \cdot \beta,\ 2\alpha - 4\beta \rangle$$
$$= \langle 1, 0 \rangle$$

$$2\alpha + 6\beta = 2$$

$$\begin{cases} 1 \cdot \alpha + 3 \cdot \beta = 1 \\ 2\alpha - 4\beta = 0. \end{cases}$$

$$10\beta = 2$$

$$\beta = \frac{1}{5}$$

$$\alpha = \frac{2}{5}$$

$$4\alpha + 12\beta = 4$$

$$6\alpha - 12\beta = 0$$

$$10\alpha = 4$$

$$\alpha = \frac{2}{5}$$

3.1. $\vec{a} = \langle \frac{1}{3}, \frac{1}{3}, \alpha \rangle$

$\vec{b} = \langle \beta, 0, \sqrt{2} \rangle$

① $\vec{a}$ is a unit vector

$\Leftrightarrow$

$|\vec{a}| = 1$

i.e.

$$\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \alpha^2 = 1^2$$

$$\alpha^2 = \frac{7}{9}$$

$$\alpha = \pm \frac{\sqrt{7}}{3}$$

② $\vec{a} \perp \vec{b}$

$\Leftrightarrow$

$\vec{a} \cdot \vec{b} = 0$

$\parallel$

$$\frac{1}{3} \cdot \beta + \frac{1}{3} \cdot 0 + \alpha \cdot \sqrt{2} = 0$$

$$\frac{1}{3}\beta = -\alpha \cdot \sqrt{2} = -\left(\pm \frac{\sqrt{7}}{3}\right)\sqrt{2}$$

$$= \mp \frac{\sqrt{14}}{3}$$

$$\beta = \mp\sqrt{14}$$

$$\left(\beta = \pm\sqrt{14} \right)$$

3.2.

$y = 3x$

tan. at $(1, 3)$

slope 3

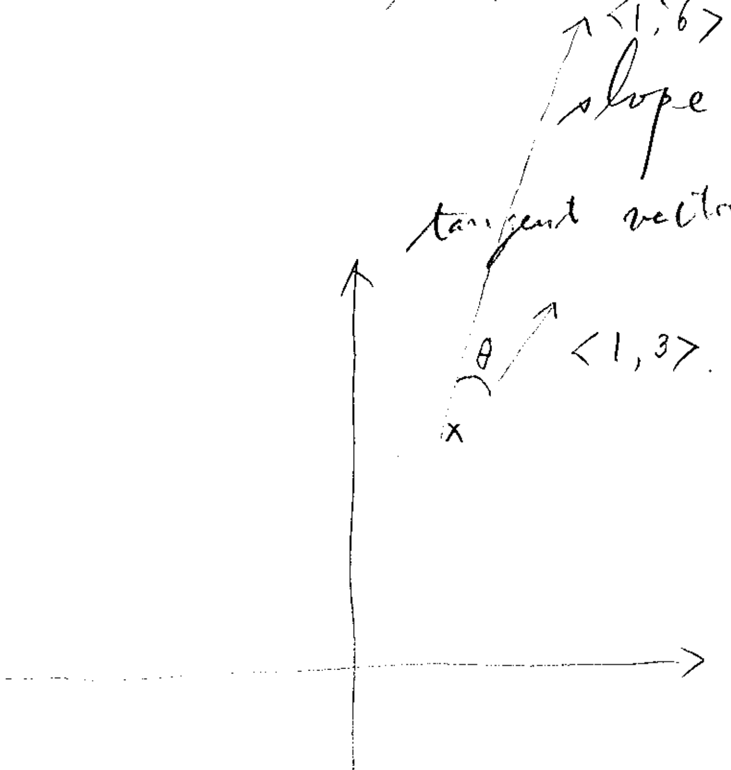
tangent vector $\langle 1, 3 \rangle$

$y = 3x^2$

tan. at $(1, 3)$ $= \vec{a}$

slope 6

tangent vector $= \langle 1, 6 \rangle = b$



$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$= \frac{1 \cdot 1 + 3 \cdot 6}{\sqrt{1^2 + 3^2}\,\sqrt{1^2 + 6^2}}$$

$$\theta = \cos^{-1}\left(\qquad \right)$$

4.1

$$\vec{a} = \langle 2, 1, -1 \rangle$$

$$\vec{b} = \langle 1, 3, 2 \rangle$$

$$\vec{a} \times \vec{b}$$

$$= \left\langle \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix}, -\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \right\rangle$$

$$= \langle 5, -5, 5 \rangle$$

4.2. $\vec{a} = \langle 3, -1, 5 \rangle$

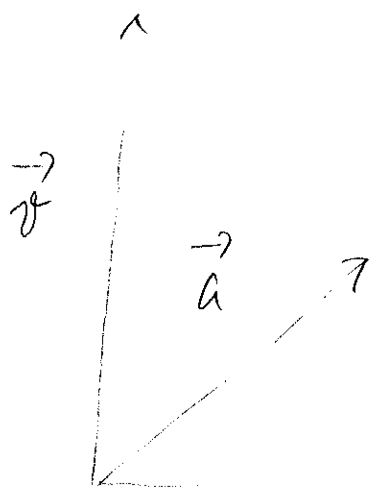
Find $\vec{v}$ s.t.

(i) $\vec{v} \perp \vec{a}$

(ii) $\vec{a} \times \vec{v} = \langle -1, 2, 1 \rangle$

Conclusion

$\vec{v}$ is $\perp$ to $\vec{a}$ & $\langle -1, 2, 1 \rangle$



$\langle -1, 2, 1 \rangle$

$\rightarrow$ $\vec{v}$ is a scalar multiple of

$\vec{a} \times \langle -1, 2, 1 \rangle$

$\vec{a} \times \langle -1, 2, 1 \rangle$

$= \langle 3, -1, 5 \rangle \times \langle -1, 2, 1 \rangle$

$\quad -1 \quad 2 \quad 1$

$= \left\langle \begin{vmatrix} -1 & 5 \\ 2 & 1 \end{vmatrix}, -\begin{vmatrix} 3 & 5 \\ -1 & 1 \end{vmatrix}, \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} \right\rangle$

$= \langle -11, -8, 5 \rangle$

$\vec{v} = c \langle -11, -8, 5 \rangle$

(i) ✓

(ii) $\langle 3, -1, 5 \rangle$
$\times$
$c \langle -11, -8, 5 \rangle = \langle -1, 2, 1 \rangle$

$c \left\langle \begin{vmatrix} -1 & 5 \\ -8 & 5 \end{vmatrix}, -\begin{vmatrix} 3 & 5 \\ -11 & 5 \end{vmatrix}, \begin{vmatrix} 3 & -1 \\ -11 & -8 \end{vmatrix} \right\rangle \qquad c = -\frac{1}{35}$

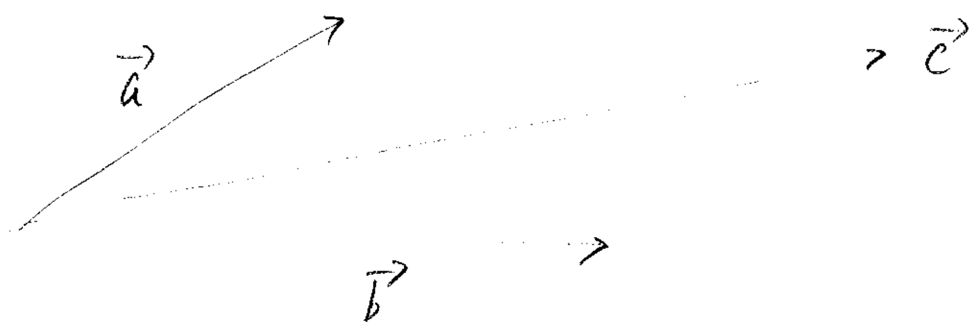
$= c \langle 35, -70, -35 \rangle$

$$\vec{v} = -\frac{1}{35}\langle -11, -8, 5\rangle$$

4.3

$\vec{a}, \vec{b}, \vec{c}$ coplanar

$\Longleftrightarrow$



on the same plane

$\Longleftrightarrow$ volume of parallelepiped $= 0$

$\Longleftrightarrow$

$$\begin{vmatrix} 1 & 4 & k \\ 2 & -1 & 4 \\ 0 & -9 & 18 \end{vmatrix} = 0$$

$$\| $$

$$1 \cdot \begin{vmatrix} -1 & 4 \\ -9 & 18 \end{vmatrix} - 4 \begin{vmatrix} 2 & 4 \\ 0 & 18 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ 0 & -9 \end{vmatrix}$$

$$1 \cdot 18 - 4 \cdot 36 + k(-18) = 0$$

$\Longleftrightarrow k = -7$ $\quad -7 \cdot 18 + k(-18) = 0$

4. 4

$$P(1, 0, 1)$$
$$Q(-2, 1, 3)$$
$$R(4, 2, 5)$$

$$\overrightarrow{PQ} = \langle -3, 1, 2 \rangle$$
$$\overrightarrow{PR} = \langle 3, 2, 4 \rangle$$

$$\triangle = \frac{1}{2}\,\text{▱}$$

$$= \frac{1}{2} \left| \overrightarrow{PQ} \times \overrightarrow{PR} \right|$$

$$= \frac{1}{2} \left| \left\langle \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}, -\begin{vmatrix} -3 & 2 \\ 3 & 4 \end{vmatrix}, \begin{vmatrix} -3 & 1 \\ 3 & 2 \end{vmatrix} \right\rangle \right|$$

$$= \frac{1}{2} \left| \langle 0, 18, -9 \rangle \right|$$

$$= \frac{1}{2} \sqrt{18^2 + (-9)^2}$$

$$(9 \times 2)^2 + 9^2$$

$$= \frac{1}{2}\, 9 \sqrt{2^2 + 1^2} = \frac{9}{2}\sqrt{5}$$

5.1

$$\vec{a} = \langle -2, 3, 1 \rangle$$

$$\vec{b} = \langle 1, 1, 2 \rangle$$

$$\mathrm{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}} \vec{a}$$

$$\mathrm{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\sqrt{\vec{a} \cdot \vec{a}}}$$

6.1

$$\vec{a} = \langle 2, -1, 3 \rangle$$

$$\cos\alpha = \frac{2}{|\vec{a}|}$$

$$\cos\beta = \frac{-1}{|\vec{a}|}$$

$$\cos\gamma = \frac{3}{|\vec{a}|}$$

6.2.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\frac{a_1}{\sqrt{a_1^2+a_2^2+a_3^2}}\right)^2 + \left(\frac{a_2}{\sqrt{a_1^2+a_2^2+a_3^2}}\right)^2 + \left(\frac{a_3}{\sqrt{a_1^2+a_2^2+a_3^2}}\right)^2$$

$$\frac{a_1^2 + a_2^2 + a_3^2}{\left(\sqrt{a_1^2+a_2^2+a_3^2}\right)^2} = 1$$

$$\cos^2\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{3}\right) + \cos^2\gamma = 1$$

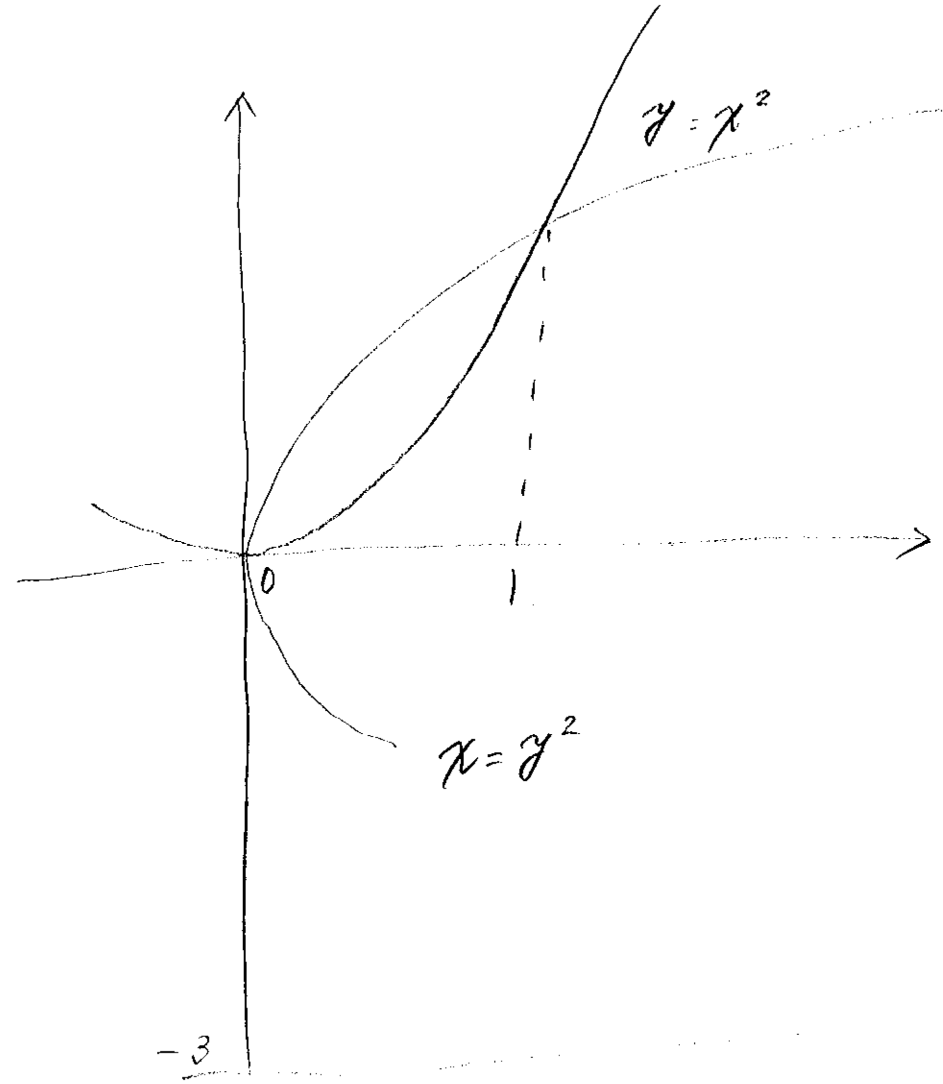
$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$$

$$\cos^2\gamma = \frac{1}{4}$$

$$\cos\gamma = \pm\frac{1}{2}$$

$$\gamma = \cos^{-1}\left(\pm\frac{1}{2}\right) = \frac{\pi}{3}, \frac{2\pi}{3}$$

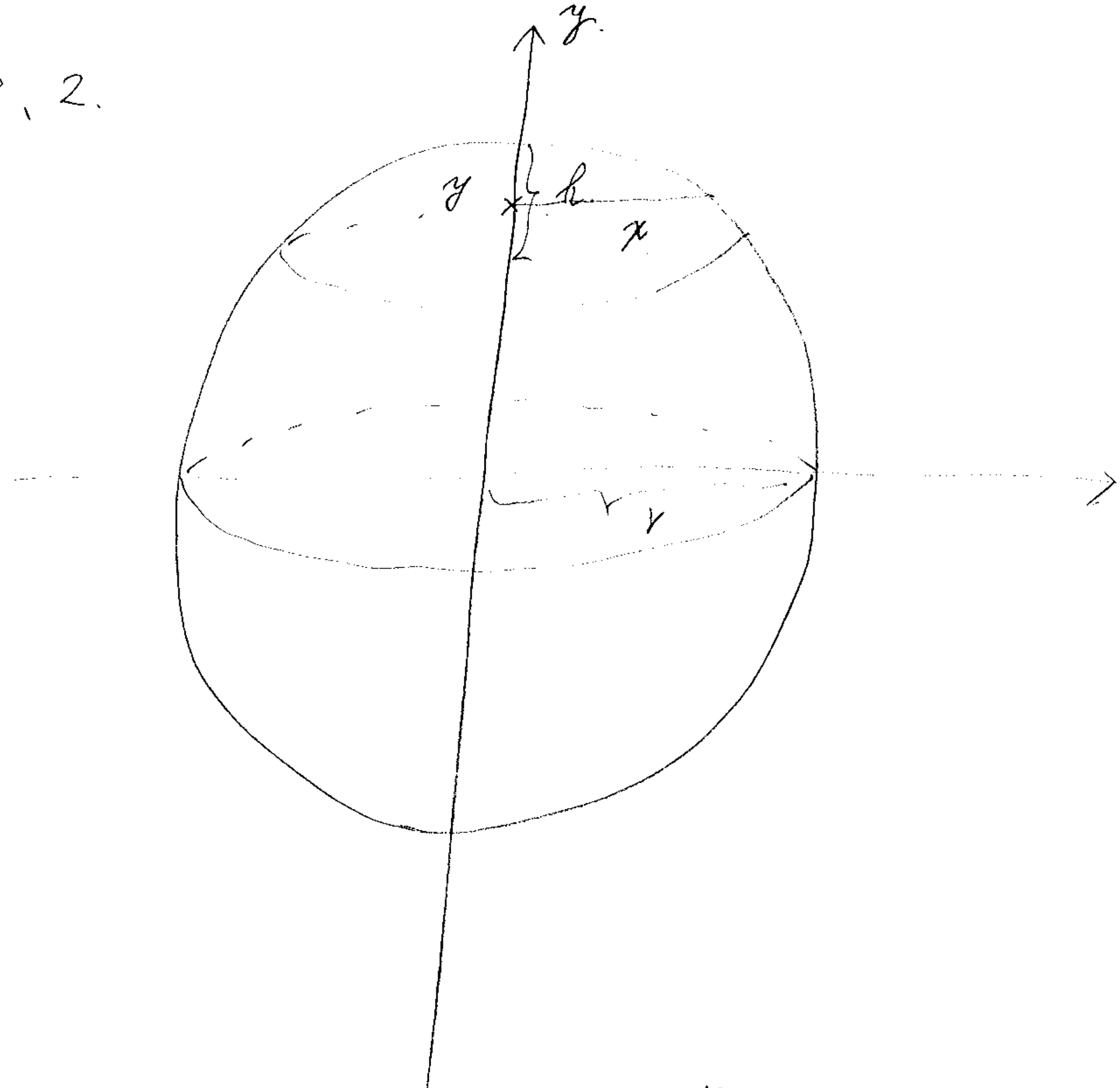
8.1.



(i) $$\int_0^1 \left\{ \pi \left(\sqrt{x}+3\right)^2 - \pi (x^2)^2 \right\} dx$$

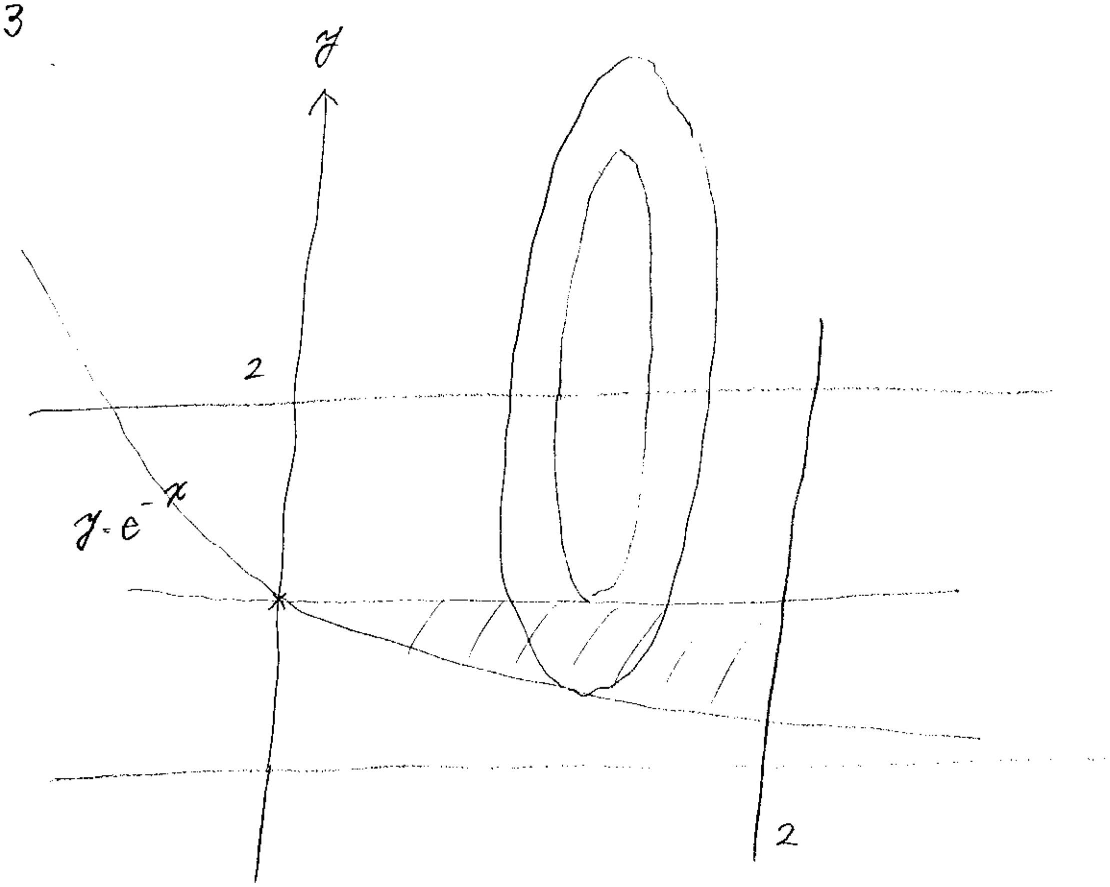
(ii) $$\int_0^1 2\pi (y+3) \left\{ \sqrt{y} - y^2 \right\} dy$$

8、2.



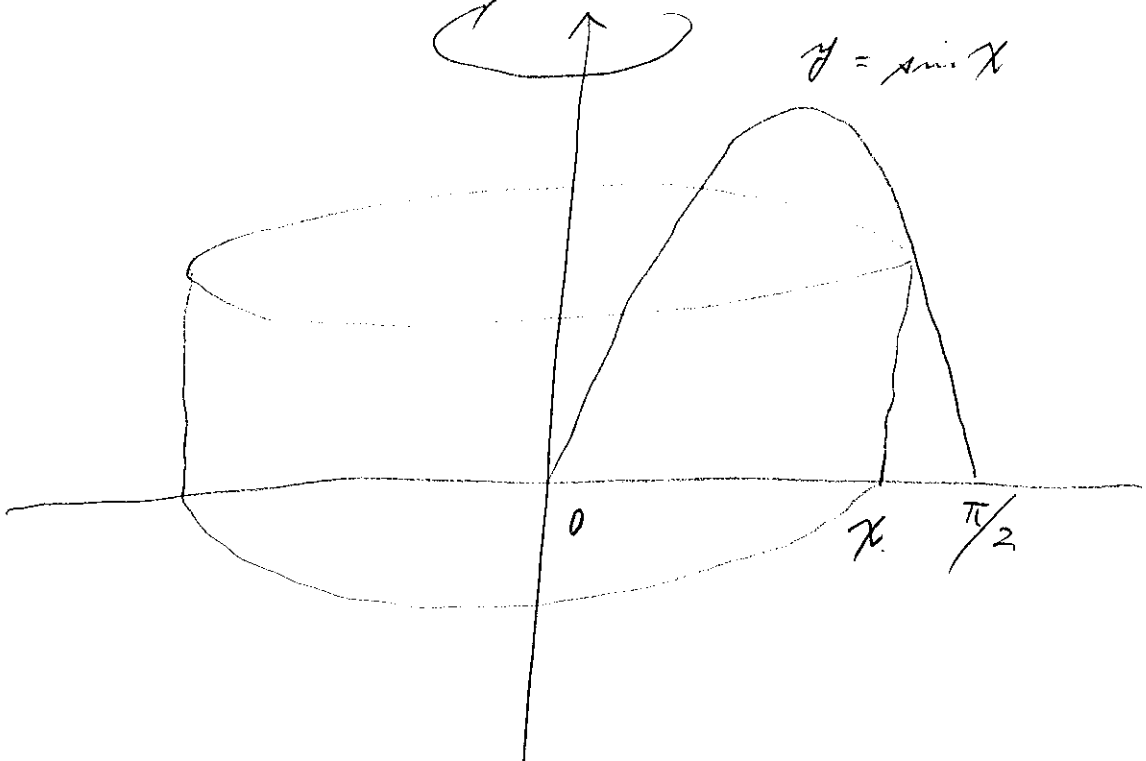
$$\int_{r-h}^{r} \pi x^2 \, dy = \int_{r-h}^{r} \pi (r^2 - y^2) \, dy$$

8、3



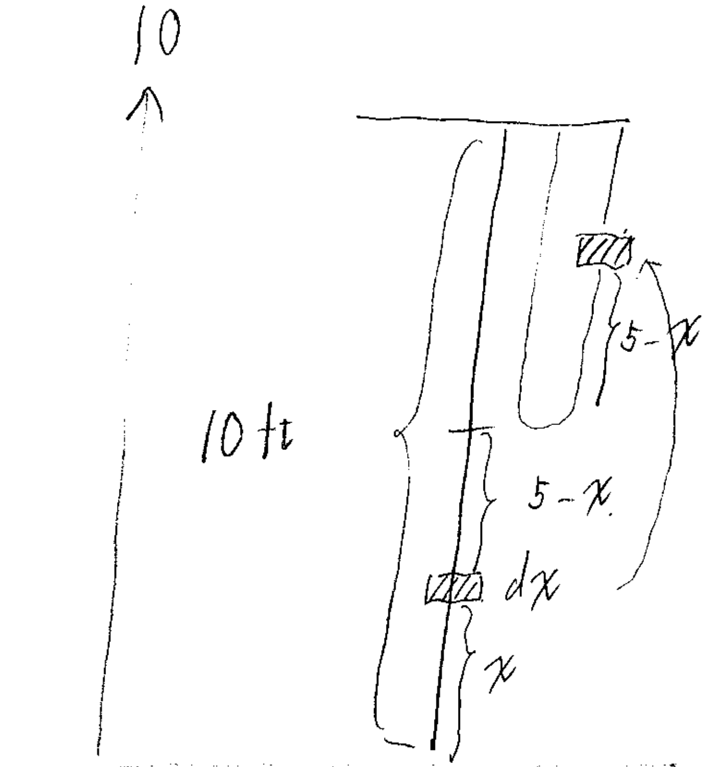
$$V = \int_0^2 \left\{ \pi \left(2 - e^{-x}\right)^2 - \pi \left(2-1\right)^2 \right\} dx$$

8.5



$\times \ 1 \ \times \ dx$

$$V = \int_0^{\pi/2} 2\pi x \sin x \, dx$$



25 lb

10 ft

density $\frac{25}{10}$ lb/ft

weight of a small piece

$$\frac{25}{10}\,dx$$

work

$$2(5-x)\frac{25}{10}\,dx$$

total work

$$\int_0^5 2(5-x)\frac{25}{10}\,dx$$

11. 1.

$$\frac{1}{b-0}\int_0^b (2+6x-3x^2)\,dx = 3$$

$$\parallel$$

$$\frac{1}{b}\left[2x+3x^2-x^3\right]_0^b$$

$$\parallel$$

$$\frac{1}{b}\left[2b+3b^2-b^3\right]$$

$$\parallel$$

$$2+3b-b^2$$

$$2+3b-b^2 = 3$$

$$b^2-3b+1 = 0$$

$$b = \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

11.2

$$\frac{1}{\pi - 0} \int_0^{\pi} \sin^4 x \cos x \, dx$$

x	$u = \sin x$	$du = \cos x \, dx$
π	0	
0	0	

$$= \frac{1}{\pi} \int_0^0 u^4 \, du = 0$$