5.1.

(18)

(a) $\displaystyle \lim_{x\to 0^+} \underbrace{\sin x}_{\downarrow \atop 0} \cdot \underbrace{\ln(2x)}_{\downarrow \atop -\infty} \quad (0\times(-\infty))$

$\displaystyle = \lim_{x\to 0} \frac{\ln(2x)}{1/\sin x} \quad \left(\frac{-\infty}{+\infty}\right)$

$\displaystyle = \lim_{x\to 0} \frac{\frac{\cancel{2}}{\cancel{2}x}}{-\frac{\cos x}{\sin^2 x}}$

$\displaystyle = \lim_{x\to 0} -\frac{\sin^2 x}{x\cdot\cos x} \quad \left(\frac{0}{0}\right)$

$\displaystyle = \lim_{x\to 0} -\frac{2\sin x\cos x}{1\cdot\cos x + x(-\sin x)} = 0.$

(b) $\displaystyle \lim_{x\to\infty} \underbrace{2x}_{\downarrow \atop \infty} \cdot \underbrace{\tan\left(\frac{1}{3x}\right)}_{\downarrow \atop 0} \quad (\infty\times 0)$

$\displaystyle = \lim_{x\to\infty} \frac{\tan\left(\frac{1}{3x}\right)}{\frac{1}{2x}}$

$\displaystyle = \lim_{x\to\infty} \frac{\sec^2\left(\frac{1}{3x}\right)\frac{1}{3}\left(\cancel{-\frac{1}{x^2}}\right)}{\frac{1}{2}\left(\cancel{-\frac{1}{x^2}}\right)} = \frac{2}{3}$

(d) $\displaystyle\lim_{x\to 0^+} \tan(5x)^{\sin x}$ $\quad (0^0)$

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Set $y = \tan(5x)^{\sin x}$

$$\ln y = \sin x \ln\{\tan(5x)\}.$$

Compute

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \underbrace{\sin x}_{\downarrow\, 0^+} \ \underbrace{\ln\{\overbrace{\tan(5x)}^{0^+}\}}_{\downarrow\, -\infty}$$

$$= \lim_{x\to 0^+} \frac{\ln\{\tan(5x)\}}{1/\sin x} \quad \left(\frac{-\infty}{+\infty}\right)$$

$$= \lim_{x\to 0^+} \frac{\dfrac{1}{\tan(5x)} \cdot \sec^2(5x) \cdot 5}{-\dfrac{\cos x}{\sin^2 x}}$$

$$= \lim_{x\to 0^+} \frac{\dfrac{\cos(5x)}{\sin(5x)} \cdot \sin^2 x \cdot 5}{-\cos x \cdot \cos^2(5x)}$$

$$= \lim_{x\to 0^+} \overbrace{\frac{\cos(5x)\cdot 5}{-\cos x \cdot \cos^2(5x)}}^{\to -5} \cdot \overbrace{\frac{\sin^2 x}{\sin(5x)}}^{\to 0} = 0$$

10.2

$$y = f(x) = \sqrt{x^2+2x-3}$$ 56

$$= \sqrt{(x+1)^2-4}$$

$$= \sqrt{(x+1)^2\left\{1-\frac{4}{(x+1)^2}\right\}}$$

$(x \gg 0)$

$$= (x+1)\sqrt{1-\frac{4}{(x+1)^2}}$$

$(x \ll 0)$

$$= -(x+1)\sqrt{1-\frac{4}{(x+1)^2}}$$

Slant asymptotes

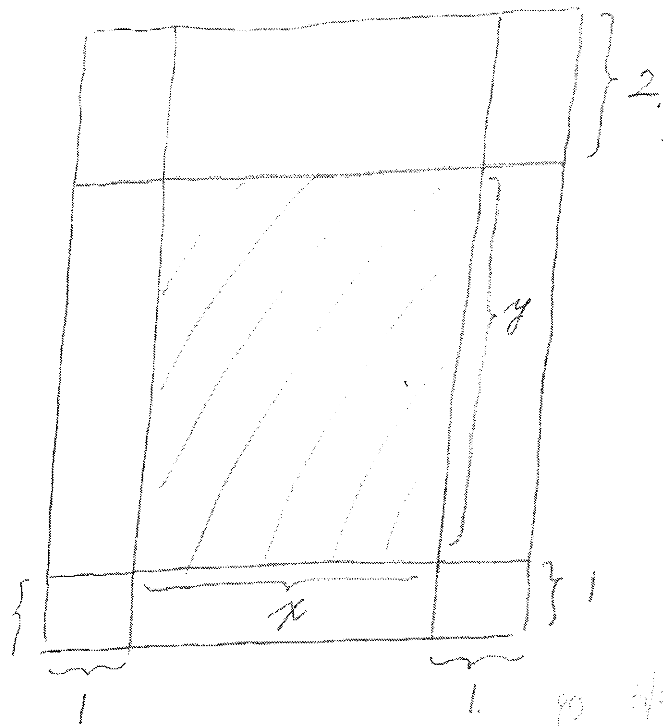
$$y = x+1$$

&

$$y = -(x+1) = -x \cancel{+} 1$$

11. 7.

Picture



Condition

$$(x+2)(y+3) = 180$$

Objective

Maximize

$$A = xy$$

$$A(x) = x\left(\frac{180}{x+2} - 3\right)$$

$$0 < x < \frac{180}{3} = 60 - 2$$

Solution

$$A'(x) = 1 \cdot \left(\frac{180}{x+2} - 3\right) + x \cdot \left\{ -\frac{180}{(x+2)^2} \right\}$$

$$= \frac{180(x+2) - 3(x+2)^2 - 180x}{(x+2)^2}$$

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$$= \frac{-3x^2 - 12x + 348}{(x+2)^2}$$

$$= \frac{-3(x^2 + 4x - 116)}{(x+2)^2}$$

$$= \frac{-3\{x-(-2+2\sqrt{30})\}\{x-(-2-2\sqrt{30})\}}{(x+2)^2}$$

x	0		$-2+2\sqrt{30}$		58
$A'(x)$		+	0	−	
$A(x)$		↗	max	↘	

When $x = -2 + 2\sqrt{30}$

$\rightarrow y = 3\sqrt{30} - 3$,

the printing area

is the largest