Answers for Example Problems in Study Guide for Exam 1. ①

1.1.

(i) $\underbrace{\ln x + \ln(x-1)}_{\ln x(x-1)} = \underbrace{0}_{\ln 1.}$

$\therefore \quad x(x-1) = 1.$

i.e. $\quad x^2 - x - 1 = 0$

$$x = \frac{1 \pm \sqrt{5}}{2}.$$

We also have the condition

$$x > 0 \ , \quad x - 1 > 0.$$

$$\therefore \quad x = \frac{1+\sqrt{5}}{2}.$$

Answer $\quad x = \dfrac{1+\sqrt{5}}{2}.$

(ii) $\log_5 x^2 + 2\log_5 x = \log_5 81$

$$\underbrace{\log_5 x^2 + 2\log_5 x}_{\displaystyle \| }$$

$$\underbrace{\log_5 x^2 + \log_5 x^2}_{\displaystyle \| }$$

$$\log_5 x^4$$

$$\therefore \quad x^4 = 81.$$

$$\therefore \quad x = \pm 3$$

We also have the conditions

$$x^2 > 0 \quad , \quad x > 0.$$

$$\therefore \quad x = 3$$

Answer. $\quad x = 3$

(iii) $\underbrace{\log_2 5}_{=} \cdot \ln 2 = \ln x$

$$\frac{\ln 5}{\ln 2} \cdot \ln 2 = \ln x$$

$$\therefore \ln 5 = \ln x$$

$$\therefore 5 = x$$

Answer $x = 5$

(iv) $e^{x^2-3x+2} = 1 = e^0$

$$\therefore \underbrace{x^2 - 3x + 2}_{=} = 0.$$

$$(x-1)(x-2)$$

$$\therefore x = 1,\ 2.$$

Answer. $x = 1, 2$

2.1 $\sin\theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$

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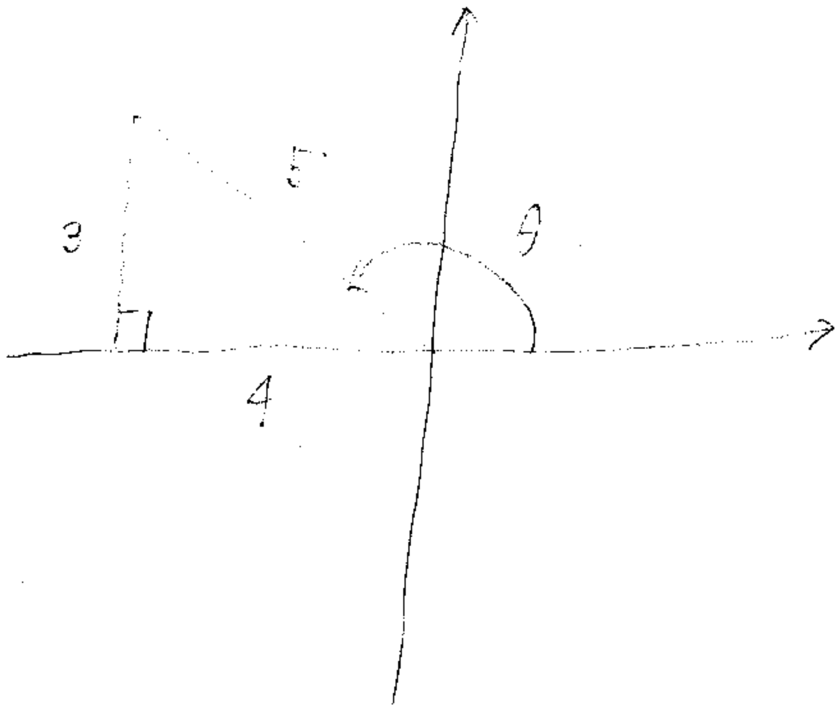

$\cos\theta = -\frac{4}{5}$

$\tan\theta = -\frac{3}{4}$

$\csc\theta = \frac{5}{3}$

$\sec\theta = -\frac{5}{4}$

$\cot\theta = -\frac{4}{3}$

2.2

$\tan\theta = -\dfrac{5}{12}$, $\sin\theta > 0$

$= \dfrac{\sin\theta}{\cos\theta}$.

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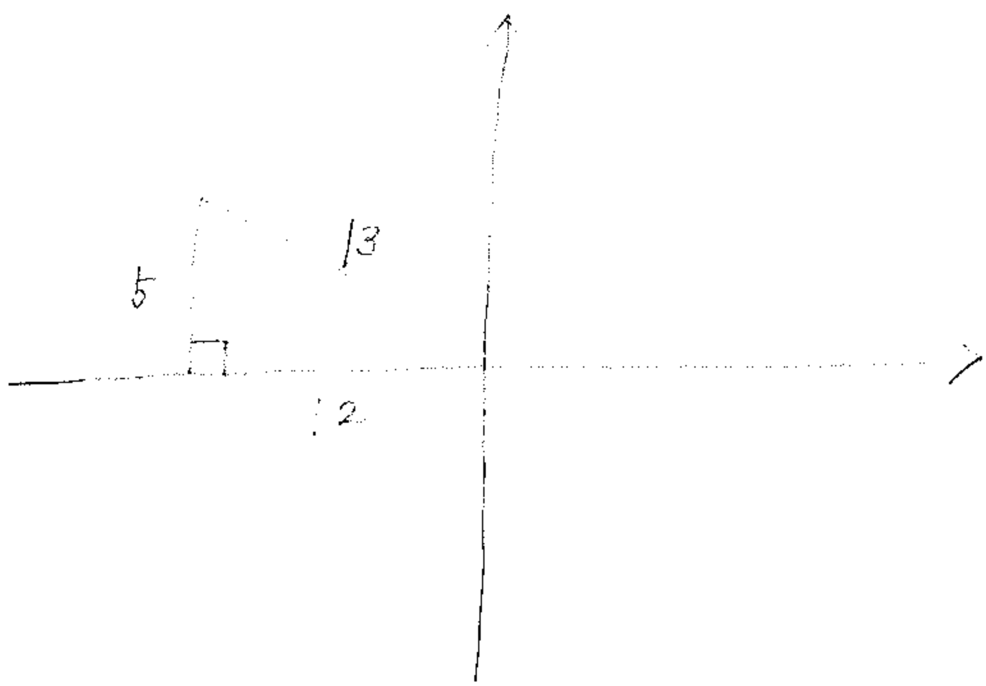
$\therefore \cos\theta < 0$



$\sin\theta = \dfrac{5}{13}$

$\cos\theta = -\dfrac{12}{13}$

$\csc\theta = \dfrac{13}{5}$

$\sec\theta = -\dfrac{13}{12}$

$\cot\theta = -\dfrac{12}{5}$

3.1

$$\cos x = \cos(2x) \qquad x \in [0, 2\pi]$$

$$= \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

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$$\therefore \; 2\cos^2 x - \cos x - 1 = 0$$

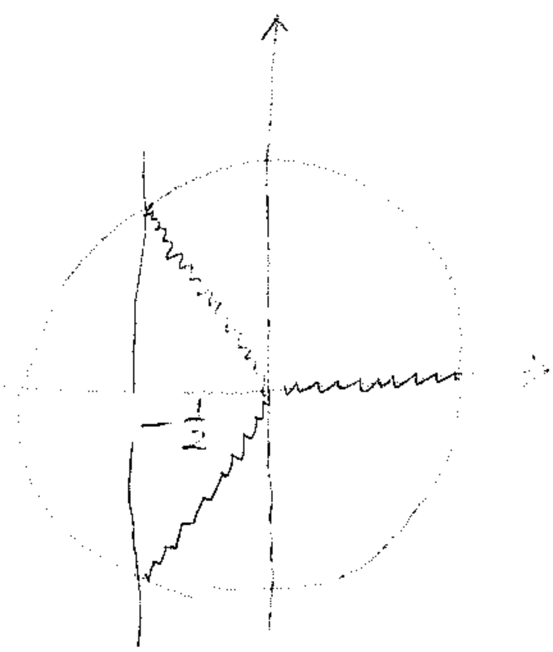
$$= (2\cos x + 1)(\cos x - 1)$$

$$\therefore \begin{cases} 2\cos x + 1 = 0 \\ \text{or} \\ \cos x - 1 = 0 \end{cases}$$

$$\rightarrow \begin{cases} \cos x = -\frac{1}{2} \\ \text{or} \\ \cos x = 1 \end{cases}$$

Answer.

$$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$$



3.2.

$$3\cot x = 2\sin(2x) \qquad x \in [0, 2\pi]$$

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$$3\,\frac{\cos x}{\sin x} = 2 \cdot 2\sin x\cos x$$

$$3\cos x = 4\sin^2 x\cos x$$

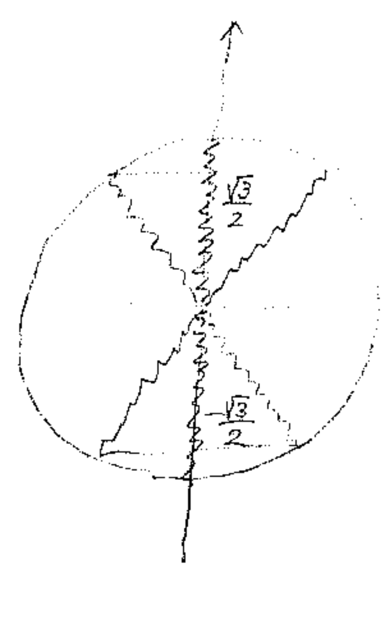
$$3\cos x - 4\sin^2 x\cos x = 0$$

$$\cos x\,(3 - 4\sin^2 x)$$

$$\begin{cases} \cos x = 0 \\ \text{or} \\ 3 - 4\sin^2 x = 0, \text{ i.e. } \frac{3}{4} = \sin^2 x \end{cases}$$

i.e.

$$\begin{cases} \cos x = 0 \\ \sin x = \pm\frac{\sqrt{3}}{2} \end{cases}$$



Answer:

$$x = \frac{\pi}{3},\ \frac{\pi}{2},\ \frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{5\pi}{3}.$$

3.3

$$\tan^2 x - 3 = 0 \qquad x \in [0, 2\pi]$$

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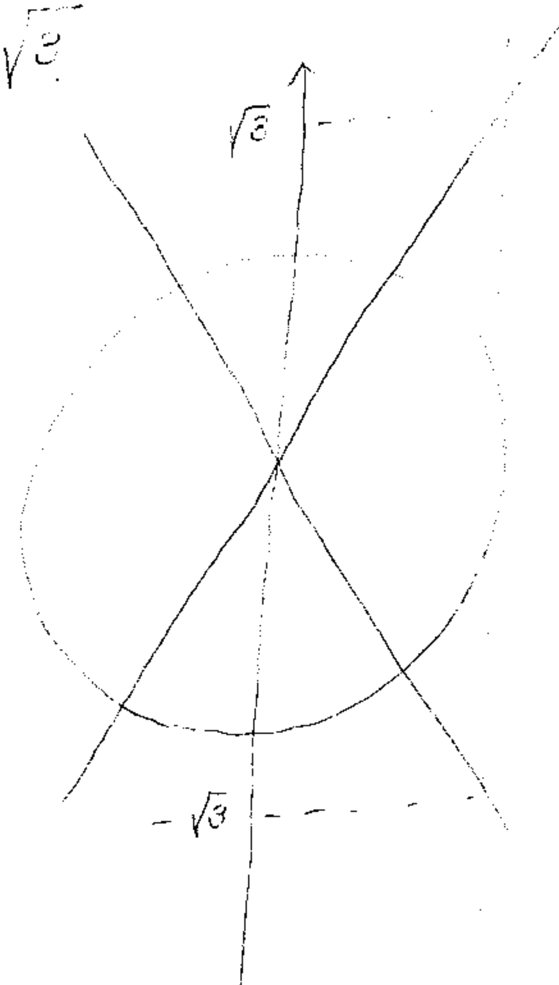
$$\tan^2 x = 3$$

$$\tan x = \pm\sqrt{3}$$



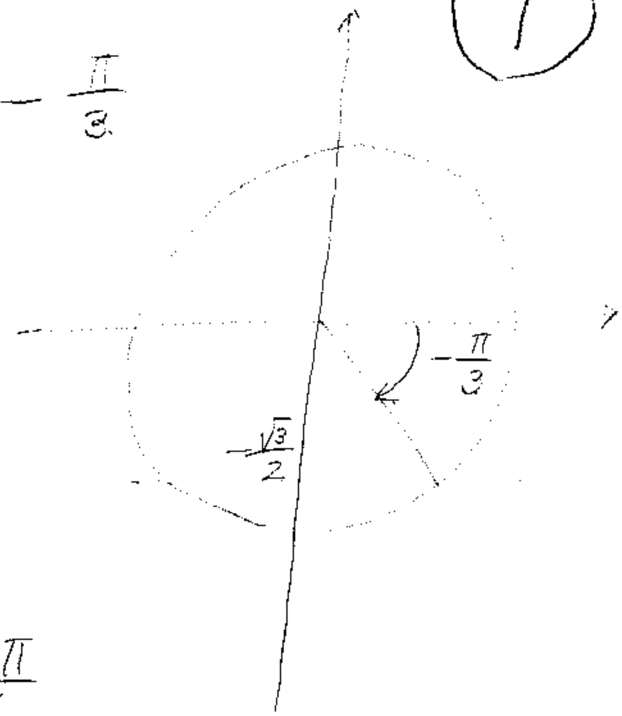
Answer.

$$x = \frac{\pi}{3}, \quad \frac{2\pi}{3}$$

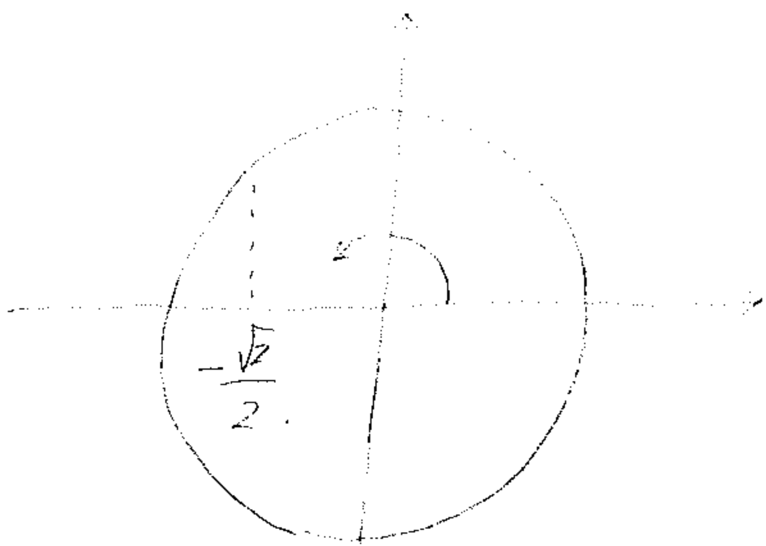
$$\frac{4\pi}{3}, \quad \frac{5\pi}{3}$$

4.1

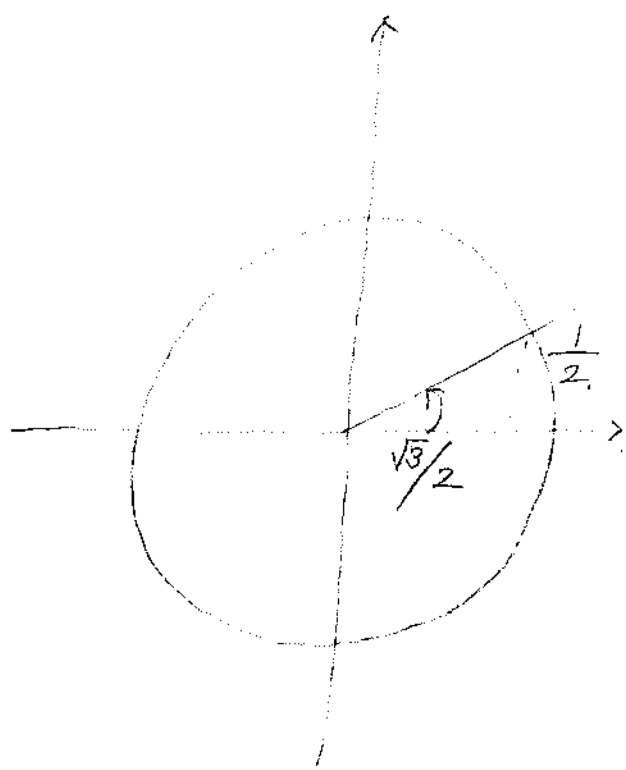
(i) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$



(ii) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$

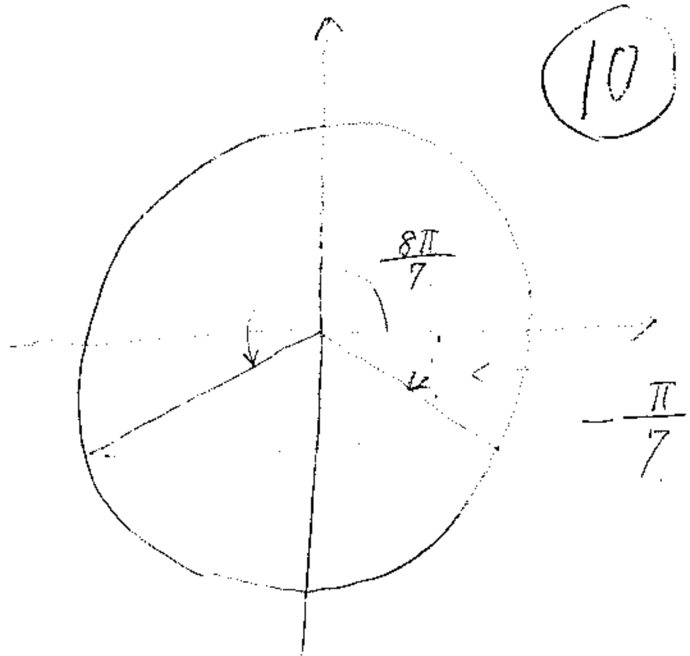


(iii) $\tan^{-1}\underbrace{\left(\frac{\sqrt{3}}{3}\right)}_{=\frac{1}{\sqrt{3}}} = \frac{\pi}{6}$



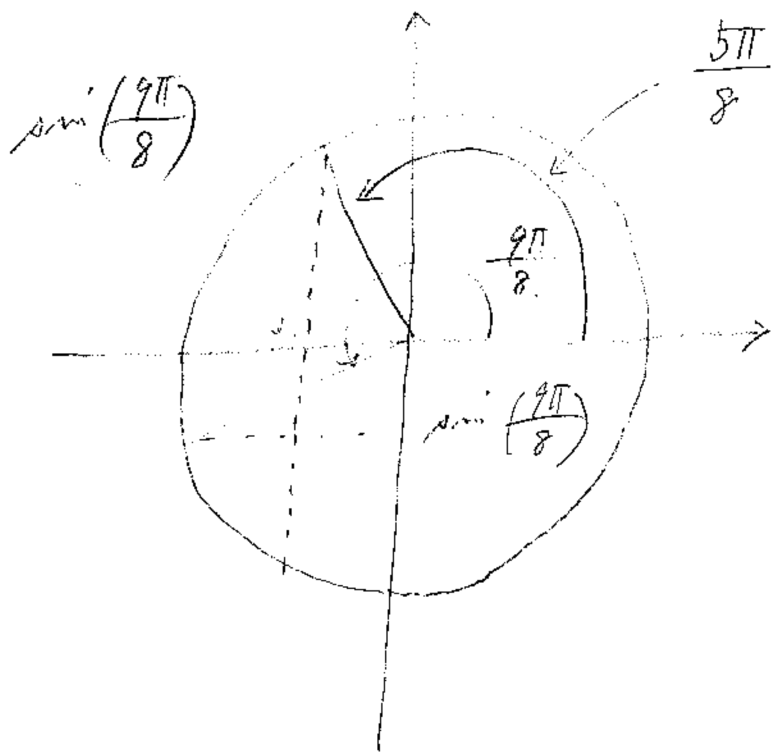
(iv)

$$\sin^{-1}\left(\sin\left(\frac{8\pi}{7}\right)\right) = -\frac{\pi}{7}$$



(v)

$$\cos^{-1}\left(\sin\left(\frac{9\pi}{8}\right)\right) = \frac{5\pi}{8}$$



(vi) $\sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right)$

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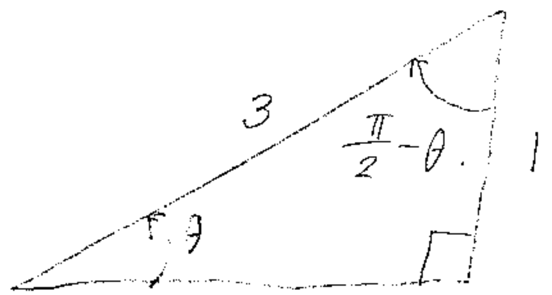
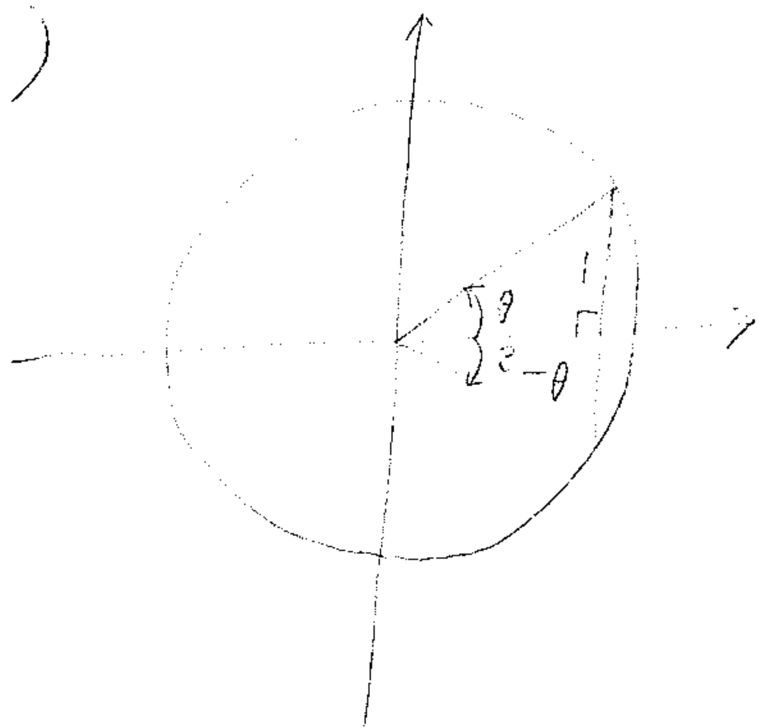


$= \theta + \left(\frac{\pi}{2} - \theta\right) = \frac{\pi}{2}$

(vii) $\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(-\frac{1}{3}\right)$

$= \theta + (-\theta)$

$= 0$

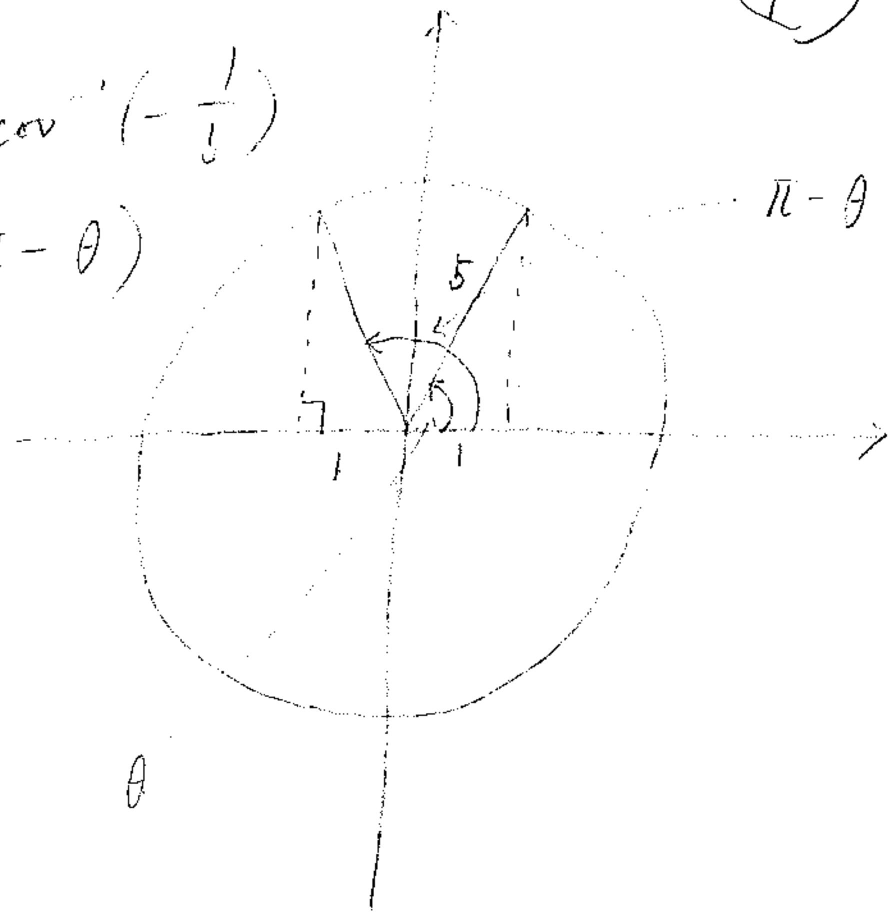


(viii)

$$\cos^{-1}\left(\frac{1}{5}\right) + \cos^{-1}\left(-\frac{1}{5}\right)$$

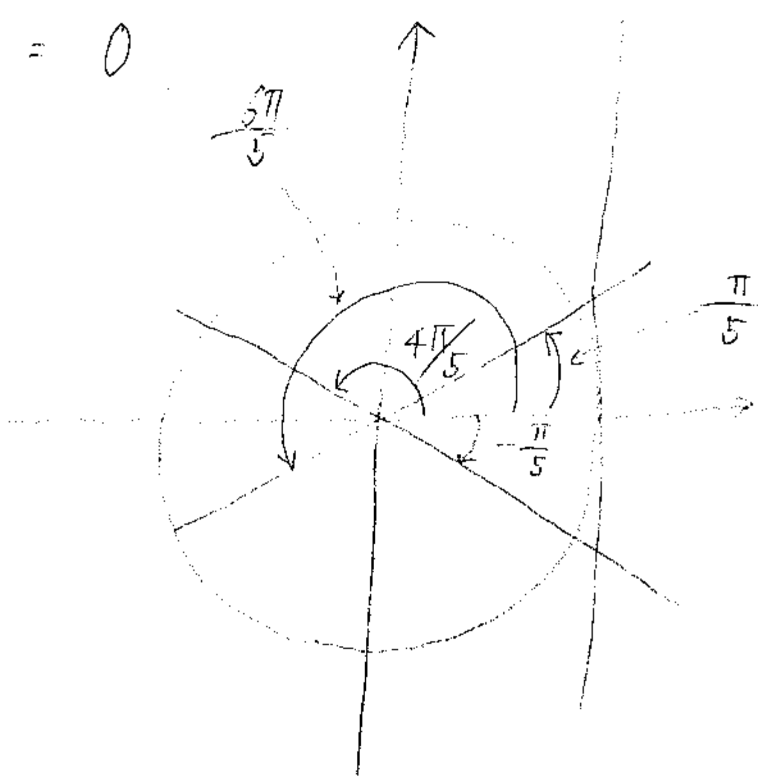
$$= \theta + (\pi - \theta)$$

$$= \pi.$$



(ix) $\tan^{-1}\left(\tan\left(\frac{4\pi}{5}\right)\right) + \tan^{-1}\left(\tan\left(\frac{6\pi}{5}\right)\right)$

$$= -\frac{\pi}{5} + \frac{\pi}{5} = 0$$



5.1

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Step 1 $y = f(x) = \dfrac{1-e^{-x}}{1+e^{-x}}$

Step 2 Solve for x.

$$y = \frac{1-e^{-x}}{1+e^{-x}}$$

$$(1+e^{-x})\,y = 1-e^{-x}$$

$$e^{-x}(y+1) = -y+1$$

$$e^{-x} = \frac{-y+1}{y+1}$$

$$-x = \ln\frac{-y+1}{y+1}$$

$$x = -\ln\frac{-y+1}{y+1}$$

$$= \ln\frac{y+1}{-y+1}$$

Step 3. Switch x & y

$$y = f^{-1}(x) = \ln\frac{x+1}{-x+1}$$

domain of $f^{-1}(x)$: $(0, 1)$ 14

$\because$) Condition $\dfrac{x+1}{-x+1} > 0$

Case : $-x+1 < 0$ i.e. $x > 1$.

$$\frac{x+1}{-x+1} > 0 \rightarrow x+1 < -x+1$$
$$\rightarrow 2x < 0$$
$$\rightarrow x < 0.$$

(No such x when $x > 1$)

Case : $-x+1 > 0$ i.e. $x < 1$

$$\frac{x+1}{-x+1} > 0 \rightarrow x+1 > -x+1$$
$$\rightarrow 2x > 0$$
$$\rightarrow x > 0$$

$\therefore$ $0 < x < 1$

i.e. $x \in (0, 1)$

range of $f^{-1}(x)$: $(-\infty, \infty)$ 15

$\cdot$ domain of $f(x)$

$\because$) $y = f(x) = \dfrac{1-e^{-x}}{1+e^{-x}}$

is defined over $(-\infty, \infty)$

5.2.

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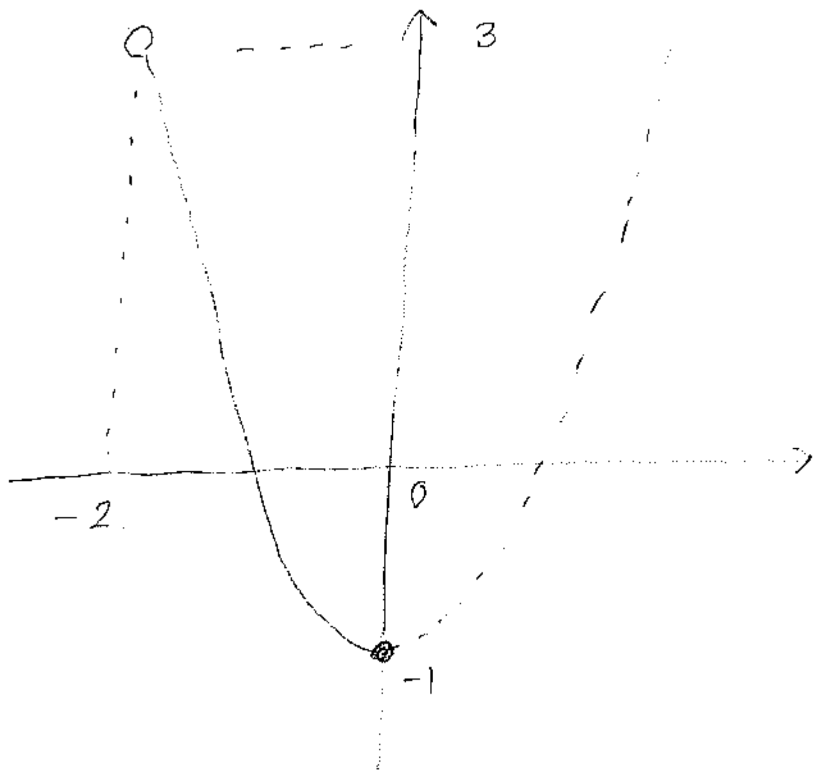
(i) $y = f(x) = x^2 - 1.$

over $(-2, 0]$



$y = x^2 - 1$

$y + 1 = x^2$

$x = \pm\sqrt{y+1}$ ($-$ since $x \in (-2, 0]$)

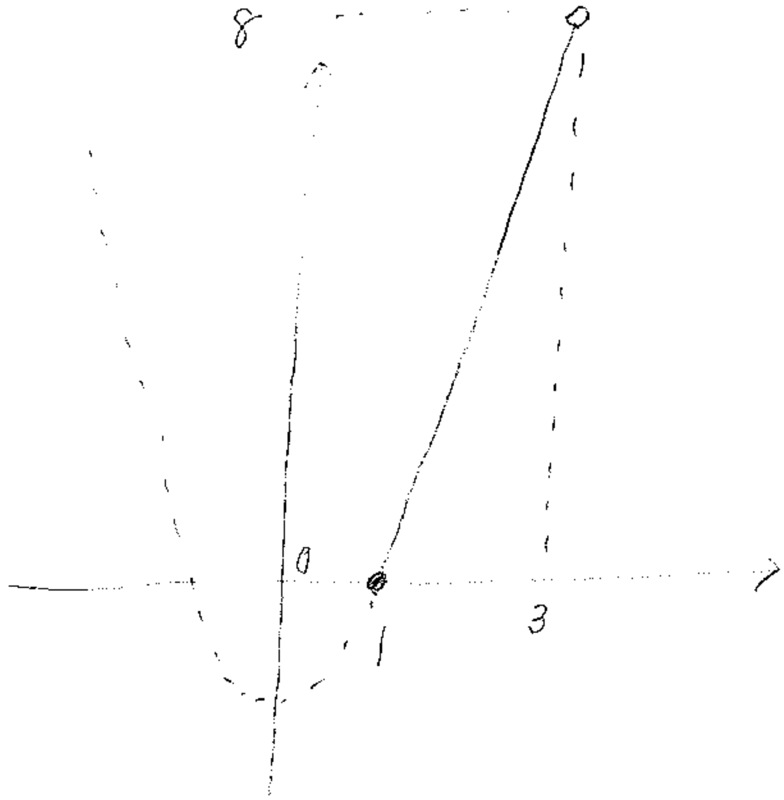
$x = -\sqrt{y+1}$

$y = f^{-1}(x) = -\sqrt{x+1}$

domain $[-1, 3)$ range $(-2, 0]$

(ii)

(17)



$$y = x^2 - 1$$

$$y + 1 = x^2$$

$$x = \cancel{\pm}\sqrt{y+1} \quad \left(\text{since } x \in [1, 3)\right)$$

$$= +\sqrt{y+1}$$

$$y = f^{-1}(x) = +\sqrt{x+1}$$

domain $[0, 8)$ range $[1, 3)$

5.3. Given $y = \dfrac{3x+1}{-5x+3}$, (18)

the function whose graph
is symmetric with respect to
$y = x$ to the one of the given
function
is its inverse.

Step 1 $\quad y = f(x) = \dfrac{3x+1}{-5x+3}$

Step 2 $\quad y = \dfrac{3x+1}{-5x+3}$

$$(-5x+3)y = 3x+1$$

$$3y-1 = (5y+3)x$$

$$x = \frac{3y-1}{5y+3}$$

Step 3 $\quad y = f^{-1}(x) = \dfrac{3x-1}{5x+3}$

5.4

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$$y = (e^{|x-5|})^2 - 1$$

$$= e^{2|x-5|} - 1 \quad \text{over } [1, 3]$$

$$= e^{2(-(x-5))} - 1 \quad \text{since}$$

($x - 5 < 0$ when $x \in [1, 3]$)

$$= e^{2(5-x)} - 1$$

one-to-one over $[1, 3]$

$\rightarrow$ inverse exists.

Step 1. $y = f(x) = e^{2(5-x)} - 1$

Step 2 $y = e^{2(5-x)} - 1.$

$$y + 1 = e^{2(5-x)}$$

$$\ln(y+1) = 2(5-x)$$

$$\frac{\ln(y+1)}{2} = 5 - x$$

$$x = -\frac{\ln(y+1)}{2} + 5$$

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Step 3 $\quad y = f^{-1}(x) = -\frac{\ln(x+1)}{2} + 5$

domain $[\, e^{4}-1 \;,\; e^{8}-1 \,]$

$\qquad\qquad \overset{\shortparallel}{f(3)} \qquad \overset{\shortparallel}{f(1}$

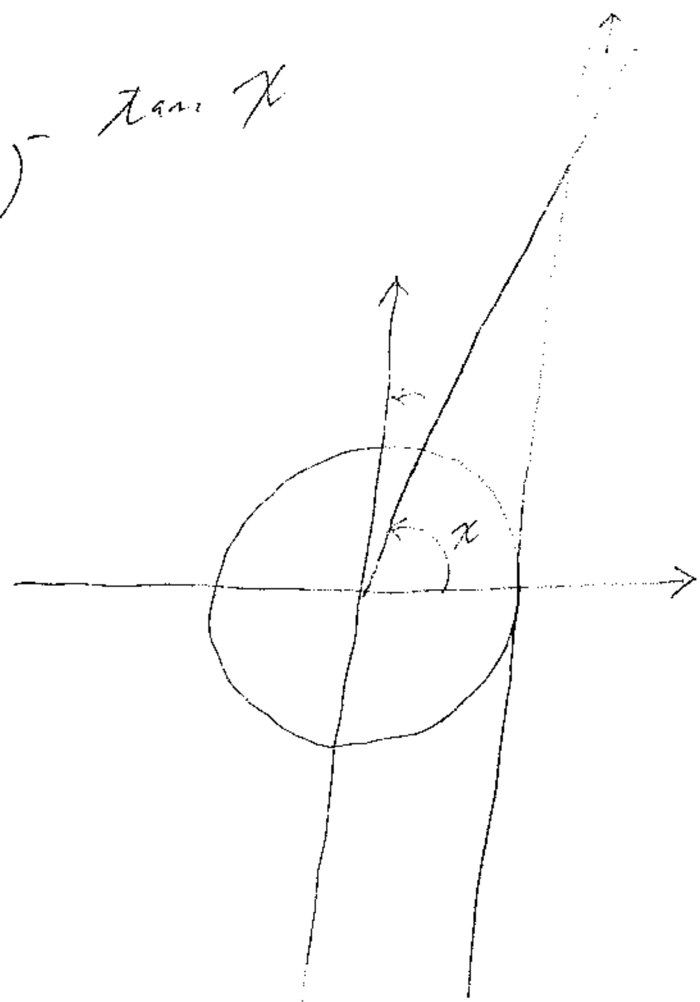
range $[\, 1 \;,\; 3 \,]$

5.1

(i) $$\lim_{x \to 5^-} \frac{x^2 - 3x - 5}{|x - 5|} = +\infty$$

(ii) $$\lim_{x \to \left(\frac{\pi}{2}\right)^-} \tan x$$

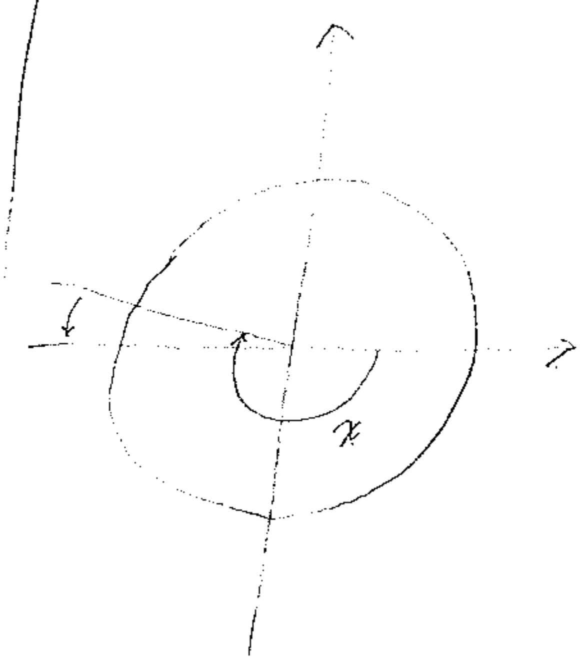
$$= +\infty$$



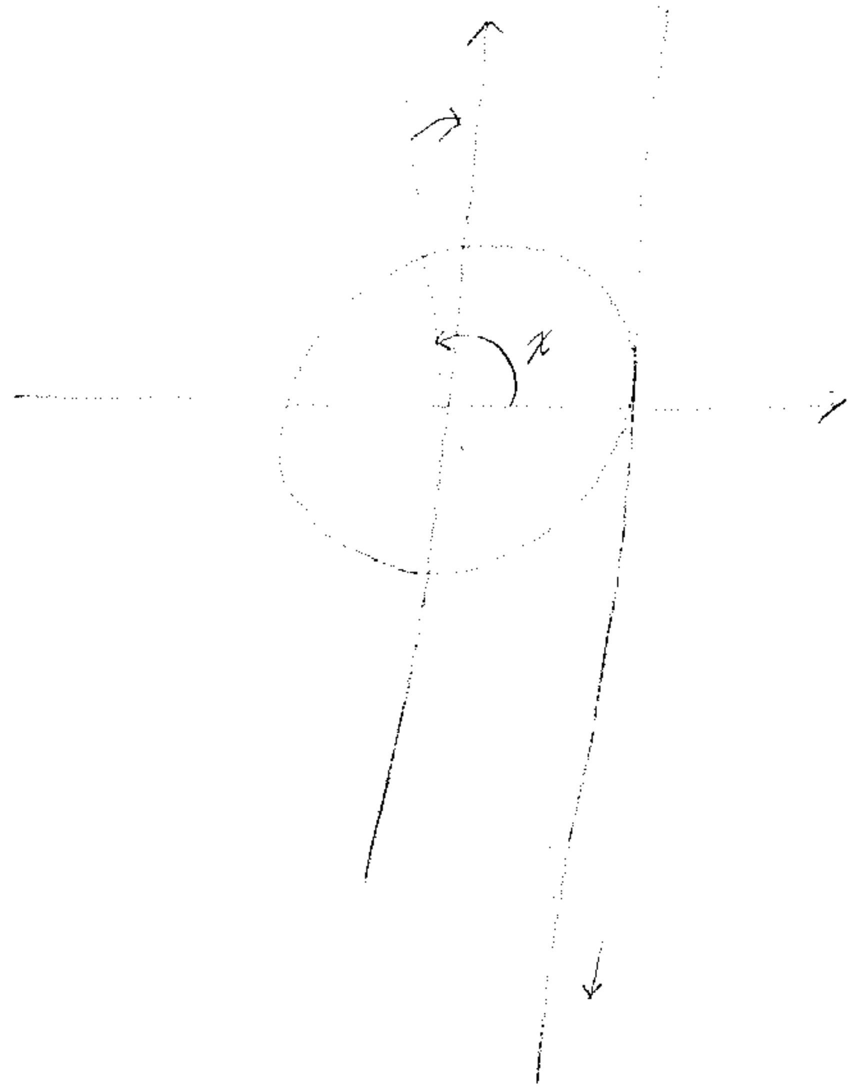
(iii)

$$\lim_{x \to (-\pi)^-} \cot x$$

$$= \lim_{x \to (-\pi)^-} \frac{\cos x}{\sin x} = -\infty$$



(iv) $\lim_{x \to \left(\frac{\pi}{2}\right)^+} e^{\tan x} = 0$ 22



(v) $\lim_{x \to 2^-} \frac{x^2 + 5x - 14}{x^2 - 6x + 8}$

$= \lim_{x \to 2^-} \frac{(x-2)(x+7)}{(x-2)(x-4)} = -\frac{9}{2}.$

(vi) $\lim_{x \to 0} \left(\frac{5}{x^2 - x} + \frac{5}{x} \right)$

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$= \lim_{x \to 0} \frac{5 + 5(x-1)}{x(x-1)}$

$= \lim_{x \to 0} \frac{5\cancel{x}}{\cancel{x}(x-1)} = -5$

(vii) $\lim_{x \to \infty} \left(\sqrt{x^2+3x+1} - x \right)$

$= \lim_{x \to \infty} \frac{\left(\sqrt{x^2+3x+1} - x\right)\left(\sqrt{x^2+3x+1} + x\right)}{\sqrt{x^2+3x+1} + x}$

$= \lim_{x \to \infty} \frac{\{(x^2+3x+1) - x^2\} / x}{\{\sqrt{x^2+3x+1} + x\} / x}$

$= \lim_{x \to \infty} \frac{3 + \frac{1}{x}}{\sqrt{1 + 3\frac{1}{x} + \frac{1}{x^2}} + 1} = \frac{3}{2}.$

(viii) $$\lim_{x\to 0} \frac{|3x-4| - |5x+4|}{x}$$

$$= \lim_{x\to 0} \frac{-(3x-4) - (5x+4)}{x}$$

$$= \lim_{x\to 0} \frac{-8\cancel{x}}{\cancel{x}} = -8$$

(ix) $$\lim_{x\to 0} \frac{\sin(1/x)}{1/x} = \lim_{x\to 0} x \sin\left(\frac{1}{x}\right)$$

$$= 0$$

$\because$) $$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

$x > 0$

$$-x \leq x \sin\left(\frac{1}{x}\right) \leq x$$

($x < 0$

$$-x \geq x \sin\left(\frac{1}{x}\right) \geq x$$)

$x \to 0$ ↓ 0 Sq. Th. ↓ 0 ↓ 0.

(x) $$\lim_{x\to 8} \frac{\sqrt[3]{x} - 2}{x - 8}$$

$$= \lim_{x\to 8} \frac{\sqrt[3]{x} - 2}{\cancel{(\sqrt[3]{x} - 2)}\{(\sqrt[3]{x})^2 + \sqrt[3]{x}\cdot 2 + 4\}}$$

$$= \lim_{x\to 8} \frac{1}{(\sqrt[3]{x})^2 + \sqrt[3]{x}\cdot 2 + 4} = \frac{1}{12}$$

(xi) $$\lim_{x\to\infty} \frac{(x^3 + 3x + 2)/x^3}{(2x^3 + \sqrt{9x^6 + 4x + 5})/x^3}$$

$$= \lim_{x\to\infty} \frac{1 + 3\cdot\frac{1}{x^2} + 2\cdot\frac{1}{x^3}}{2 + \frac{\sqrt{9x^6+4x+5}}{x^3}}$$

$$\frac{\sqrt{9x^6+4x+5}}{x^3} = \frac{\sqrt{9x^6+4x+5}}{\sqrt{x^6}} = \sqrt{9 + 4\cdot\frac{1}{x^5} + 5\cdot\frac{1}{x^6}}$$

$$= \frac{1}{5}.$$

(xii)

$$\lim_{x \to -\infty} \frac{(x^3 + 3x + 2)/x^3}{(2x^3 + \sqrt{9x^6 + 4x + 5})/x^3} \quad (26)$$

$$= \lim_{x \to -\infty} \frac{1 + 3 \cdot \frac{1}{x^2} + 2 \cdot \frac{1}{x^3}}{2 + \frac{\sqrt{9x^6 + 4x + 5}}{x^3}}$$

$$\parallel$$

$$\frac{\sqrt{9x^6 + 4x + 5}}{-\sqrt{x^6}}$$

$$\parallel$$

$$-\sqrt{9 + 4 \cdot \frac{1}{x^5} + 5 \cdot \frac{1}{x^6}}$$

$$= -1.$$

7.1

Set $f(x) = x^3 - 3x$

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x	0	1	2	3	4
$f(x)$	0	-2	2	18	52

$$2 < 5 < 18$$

$x^3 - 3x = 5$

has a root in $(2, 3)$

7.2.

Set $f(x) = \sin x - (x^3 + 1)$

x	-2	-1	0	
$f(x)$	$\sin(-2) + 7$	$\sin(-1)$	-1	

$$\begin{array}{ccc} \vee & & \wedge \\ 0 & & 0 \end{array}$$

$$f(-2) < 0 < f(0)$$

I.V.T.

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$\Rightarrow$ $\exists\ c \in (-2, 0)$

s.t. $f(c) = 0$

i.e. $\sin c - (c^3 + 1) = 0$.

i.e. $\sin c = c^3 + 1$.

$\Rightarrow$ $\sin x = x^3 + 1$

has a root $c \in (-2, 0)$.

$\square$

8.1.

(1) $g(x) = f(2x)$

$$g'(1) = \lim_{h \to 0} \frac{g(1+h) - g(1)}{h}$$

$$= \lim_{h \to 0} \frac{f(2(1+h)) - f(2 \cdot 1)}{h}$$

$$= \lim_{h \to 0} \underbrace{\frac{f(2+2h) - f(2)}{2h}}_{\downarrow \; f'(2)} \cdot \underbrace{\frac{2h}{h}}_{= \, 2}$$

$$= f'(2) \cdot 2 = 5 \cdot 2$$

$$= 10$$

(ii) $$\lim_{h \to 0} \frac{f(2+4h) - f(2)}{3h}$$

$$= \lim_{h \to 0} \underbrace{\frac{f(2+4h) - f(2)}{4h}}_{\downarrow \; f'(2)} \cdot \underbrace{\frac{4h}{3h}}_{= \frac{4}{3}}$$

$$= f'(2) \cdot \frac{4}{3} = 5 \cdot \frac{4}{3} = \frac{20}{3}$$

(iii) $$\lim_{h \to 0} \frac{f(2+4h) - f(2-5h)}{7h}$$

$$= \lim_{h \to 0} \underbrace{\frac{f(2+4h) - f(2-5h)}{\underbrace{(2+4h) - (2-5h)}_{9h}}}_{\downarrow \; f'(2)} \cdot \underbrace{\frac{9h}{7h}}_{= \frac{9}{7}}$$

$$= f'(2) \cdot \frac{9}{7} = 5 \cdot \frac{9}{7} = \frac{45}{7}$$

8.2.

We want to compute

$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$\lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^-} \frac{\{-(h-1)^2+3\} - \{5\cdot 0 + 2\}}{h}$$

$$= \lim_{h\to 0^-} \frac{-h^2+2h}{h} = \lim_{h\to 0^-}(-h+2)$$

$$= 2.$$

$$\lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{\{5h+2.\} - \{5\cdot 0+2\}}{h} = \lim_{h\to 0^+} \frac{5h}{h}$$

$$= 5$$

$$\lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h} \neq \lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h}$$

$\therefore$

$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$ does NOT exist.

9.1.

$$f(x) = \begin{cases} x^2 - a & \text{if } x \le 1 \\ \dfrac{3x^2+12x-b}{x^2+2x-3} & \text{if } x > 1 \end{cases}$$

f cont. at $x = 1$

$\longrightarrow$

$\lim_{x \to 1^+} f(x)$ exists & $= L$

$\longrightarrow$

$$\lim_{x\to 1^+} (3x^2+12x-b) = \lim_{x \to 1^+} \underbrace{\frac{3x^2+12x+b}{x^2+2x-3}}_{\downarrow \, L} \cdot \underbrace{(x^2+2x-3)}_{\downarrow \, 0}$$

$$= L.$$

$$3 \cdot 1^2 + 12 \cdot 1 - b$$

$\shortparallel$

$$15 - b$$

$$\therefore \quad b = 15.$$

f cont. at $x = 1$

$\longrightarrow$

$$\underbrace{\lim_{x\to 1^-} f(x)}_{= 1^2 - a \,=\, 1 - a} = f(1) = \lim_{x\to 1^+} f(x)$$

$$= \lim_{x\to 1^+} \frac{3x^2 + 12x - \overset{15}{b}}{x^2 + 2x - 3}$$

$$= \lim_{x\to 1^+} \frac{3(x^2 + 4x - 5)}{x^2 + 2x - 3}$$

$$= \lim_{x\to 1^+} \frac{3(x+5)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}}$$

$$= \frac{3\cdot 6}{4} = \frac{9}{2}$$

$$\therefore\ 1 - a = \frac{9}{2}$$

$$a = 1 - \frac{9}{2} = -\frac{7}{2}$$

Answer. $a = -\dfrac{7}{2}$, $b = 15$

9. 2.

$$f(x) = \begin{cases} 3x - 2c & \text{if } x \le c \\ 5x^2 - 4 & \text{if } x > c. \end{cases}$$

f cont. at $x = c$.

$$\longrightarrow \quad \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$$

$$\underset{\overset{\|}{c}}{3c - 2c} \qquad 5c^2 - 4$$

$$\longrightarrow \quad c = 5c^2 - 4$$

i.e.

$$5c^2 - c - 4 = 0$$

$$= (5c + 4)(c - 1)$$

$$\therefore \quad c = -\frac{4}{5}, \ 1.$$

Answer. $c = -\frac{4}{5}, \ 1.$

10.1

(i) $y = f(x) = \dfrac{x^3 + 4x^2 + x - 6}{x(x^2 - 1)}$

$\lim_{x \to \infty} f(x) = 1.$

$\lim_{x \to -\infty} f(x) = 1.$

<u>hor. asymp. $y = 1.$</u>

$y = f(x) = \dfrac{(x-1)(x+2)(x+3)}{x(x+1)(x-1)}$

<u>vert. asymp. $x = 0, \ x = -1.$</u>

(ii) $y = f(x) = \dfrac{x^2 - x}{x^2 - 4x + 3}$

$\lim_{x \to \infty} f(x) = 1$

$\lim_{x \to \infty} f(x) = 1$

<u>hor. asymp. $y = 1.$</u>

$$y = f(x) = \frac{x(x-1)}{(x-1)(x-3)}$$

vert. asymp. $x = 3$

(iii) $$y = f(x) = \frac{3e^x}{e^x - 1}.$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3e^x / e^x}{(e^x - 1)/e^x}$$

$$= \lim_{x\to\infty} \frac{3}{1 - \frac{1}{e^x}} = 3$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{3e^x}{e^x - 1} = 0$$

hort. asymp. $y = 3$ & $y = 0$

$$f(x) = \frac{3e^x}{e^x - 1}$$

vert. asymp. $x = 0$

(iv) $y = f(x) = \dfrac{x-5}{\sqrt{x^2-5x}}$ 37

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x-5}{\sqrt{x^2-5x}} = 1.$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x-5}{\sqrt{x^2-5x}} = -1.$$

<u>hor. asymp.</u> $y = 1$ & $y = -1$.

$$y = f(x) = \frac{x-5}{\sqrt{x^2-5x}} = \frac{x-5}{\sqrt{x(x-5)}}$$

$$\lim_{x\to5^+} f(x) = \lim_{x\to5^+} \frac{x-5}{\sqrt{x(x-5)}}$$

$$= \lim_{x\to5^+} \frac{\sqrt{x-5}}{\sqrt{x}} = 0$$

$$\lim_{x\to5^-} f(x) = \lim_{x\to5^-} \frac{x-5}{\sqrt{x(x-5)}}$$

$$= \lim_{x\to5^-} -x\sqrt{x\cdot5} = 0.$$

vert. asymp. $x = 0$.

11.1 (38)

eq. tan to $y = x^3$

(i) parallel to $y = 3x$

(ii) perpendicular to $y = 3x$

(i) $y' = 3x^2 = 3$
$\uparrow$ parallel

$\rightarrow x^2 = 1$

$\rightarrow x = \pm 1$

point $(1, 1)$

or $(-1, -1)$

eq. of tan.

$y - 1 = 3(x - 1)$ through $(1, 1)$

or $y - (-1) = 3(x - (-1))$ through $(-1, -1)$

38'

11.1

(ii) $y' = 3x^2 = -\frac{1}{3}$

perpendicular

$\rightarrow \quad x^2 = -\frac{1}{9}$

No such x.

---

modify the problem to

(ii)' perpendicular to $y = -3x$

$y' = 3x^2 = \frac{1}{3}$

perpendicular

$\rightarrow \quad x^2 = \frac{1}{9}$

$\rightarrow \quad x = \pm \frac{1}{3}$

point $\left(\frac{1}{3}, \frac{1}{27}\right)$

or $\left(-\frac{1}{3}, -\frac{1}{27}\right)$

38''

eq. of tan

$$y - \frac{1}{27} = \frac{1}{3}\left(x - \frac{1}{3}\right)$$

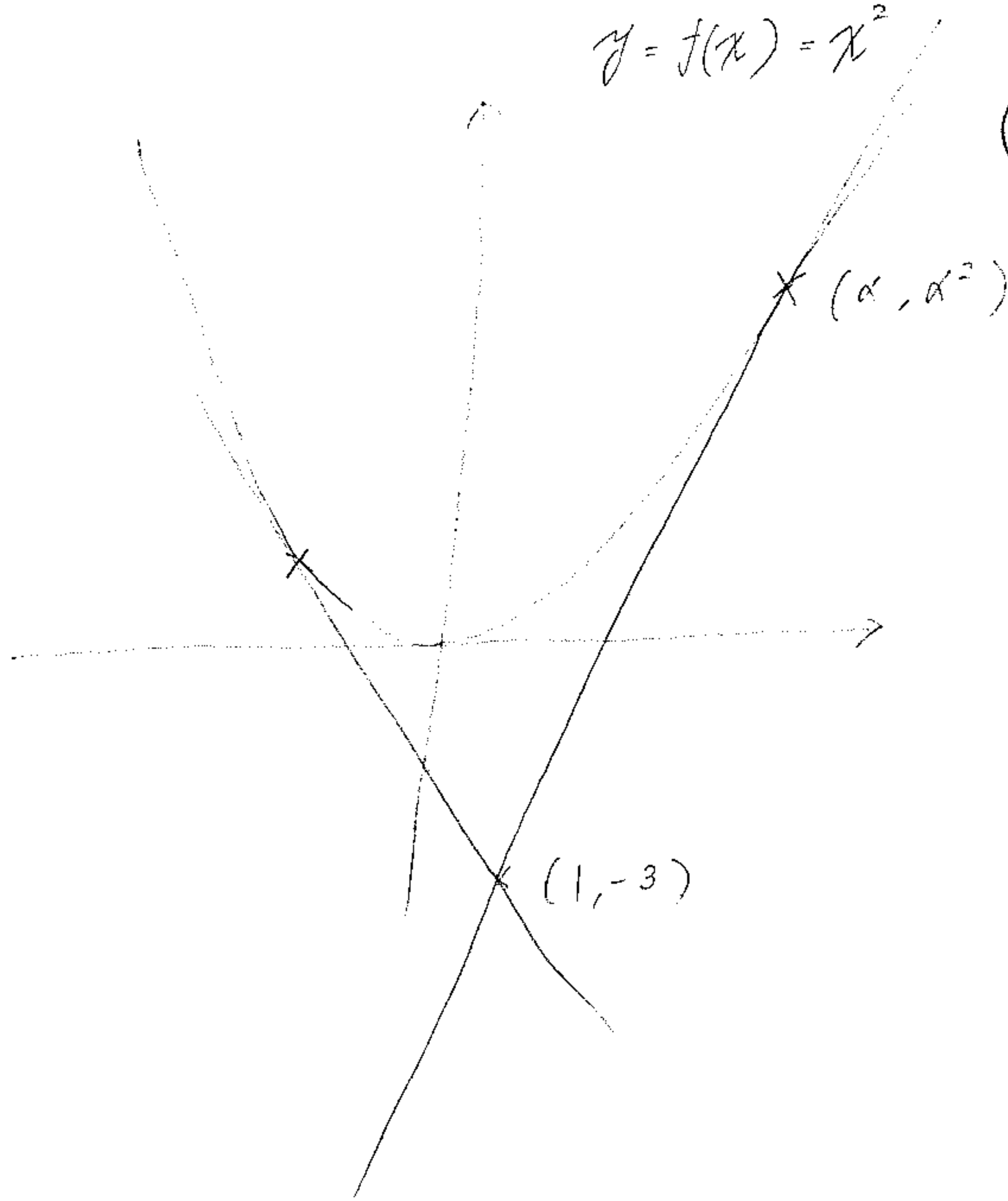
through $\left(\frac{1}{3}, \frac{1}{27}\right)$

or

$$y - \left(-\frac{1}{3}\right) = \frac{1}{3}\left(x - \left(-\frac{1}{3}\right)\right)$$

through $\left(-\frac{1}{3}, -\frac{1}{27}\right)$

11.2.



(39)

Set the point of tangency (α, α^2)

slope of tangent $f'(\alpha) = 2\alpha$

eq. of tangent

$$y - \alpha^2 = 2\alpha(x - \alpha)$$

passing $(1, -3)$

$\rightarrow$

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$$(-3) - \alpha^2 = 2\alpha(1-\alpha)$$

$$\alpha^2 - 2\alpha - 3 = 0$$

$$\parallel$$

$$(\alpha - 3)(\alpha + 1)$$

$$\therefore \quad \alpha = -1, \; 3$$

When $\alpha = -1$,

eq. of tan.

$$y - (-1)^2 = 2(-1)(x - (-1))$$

When $\alpha = 3$

eq. of tan

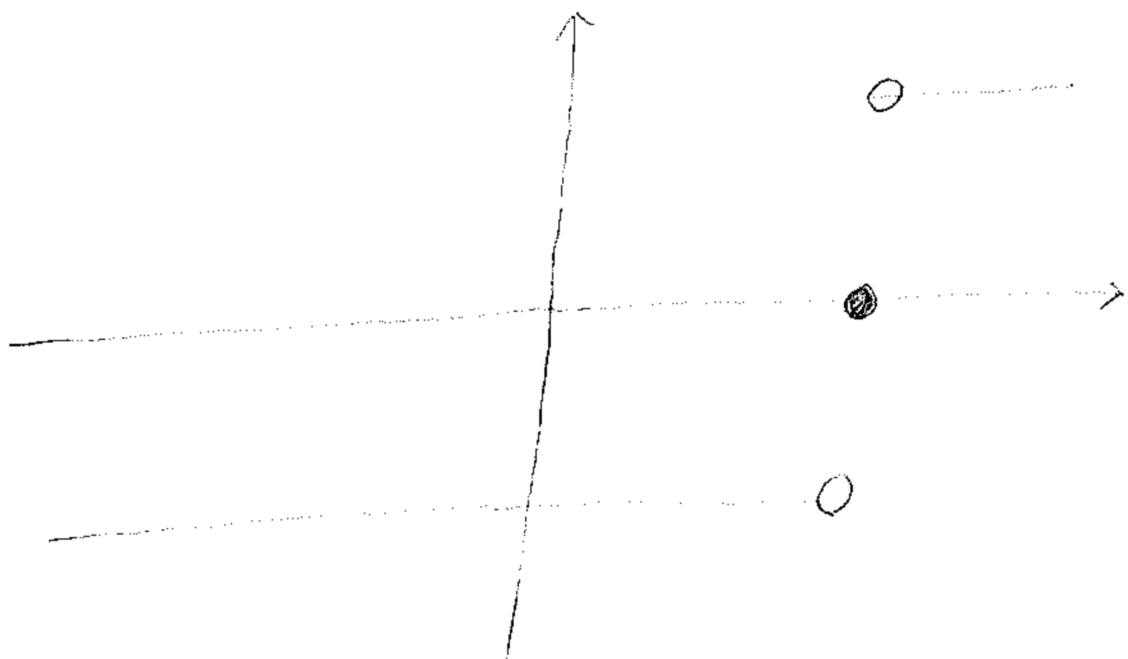
$$y - 3^2 = 2 \cdot 3\,(x - 3)$$

12.1.

(41)

(i)

$$f(x) = \begin{cases} \dfrac{x-2}{|x-2|} & x \neq 2 \\ 0 & x = 2 \end{cases}$$



$$\begin{cases} \text{cont.} & x \neq 2 \\ \text{not cont.} & x = 2 \end{cases}$$

$$\begin{cases} \text{diff.} & x \neq 2 \\ \text{not diff.} & x = 2 \end{cases}$$

(ii) $g(x) = x|x-2|$

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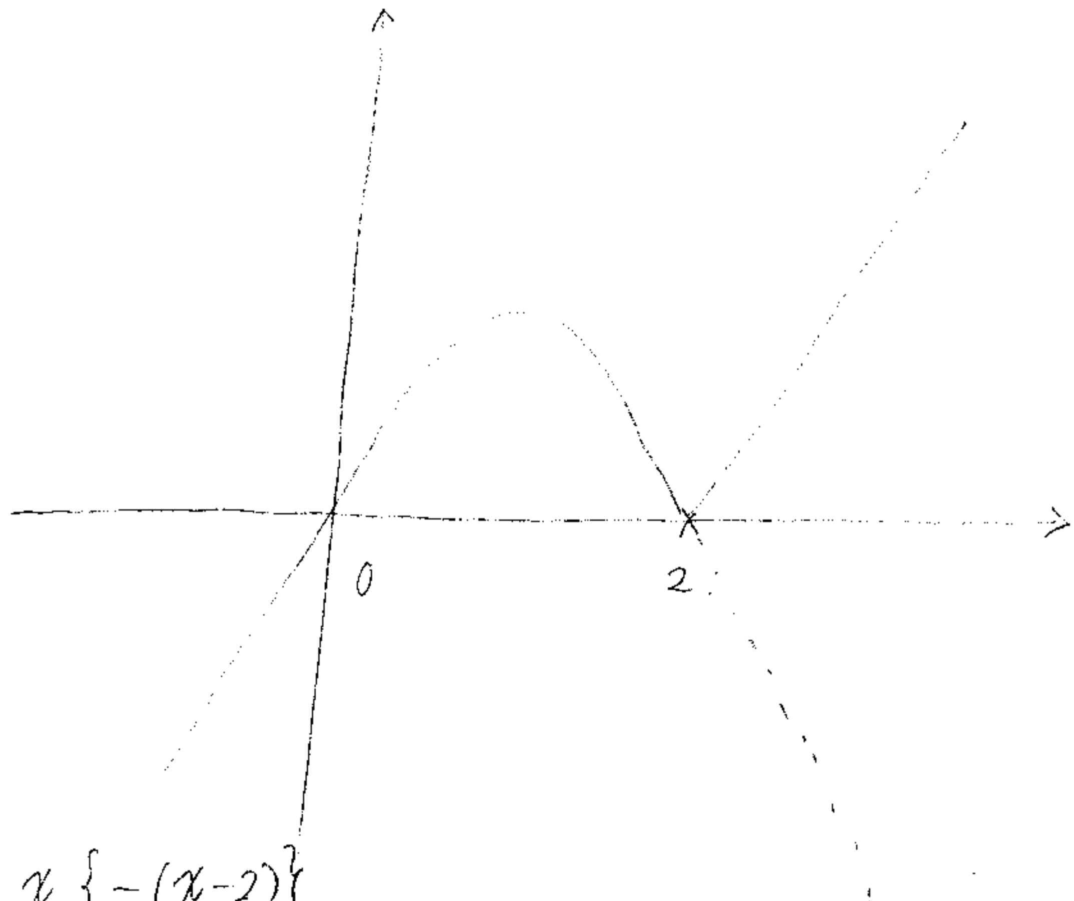
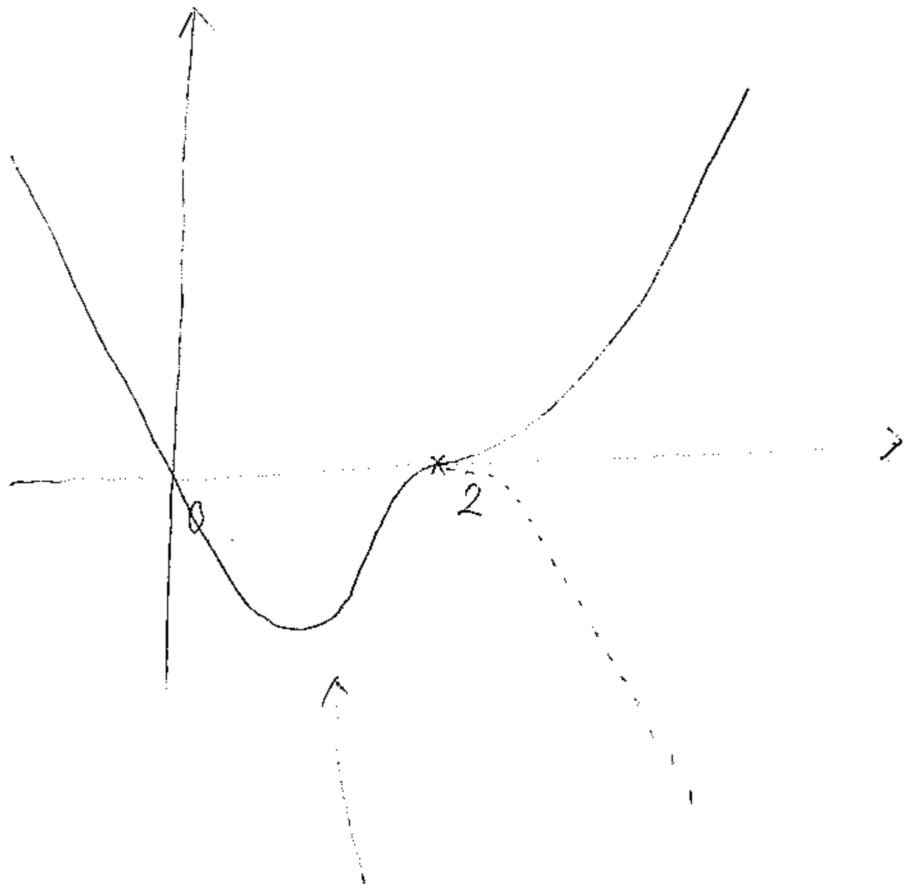


$y = x\{-(x-2)\}$

$\{$ cont. everywhere.

$\begin{cases} \text{diff.} & x \neq 2 \\ \text{not diff.} & x = 2. \end{cases}$

(iii)



$y = x(x-2)\{-(x-2)\}.$

{ cont. everywhere

{ diff everywhere

12.2

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

∘ continuity

$$\lim_{x \to 0} g(x)$$

$$= \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0 = g(0)$$

$\because$) $\quad -1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

Multiply x^2 (> 0, when $x \neq 0$)

$$\begin{array}{cccccc} & -x^2 & \leq & x^2 \sin\left(\frac{1}{x}\right) & \leq & x^2 \\ x \to 0 & \downarrow & & \downarrow \text{ by Sq. Th.} & & \downarrow \\ & 0 & & 0 & & 0 \end{array}$$

$\therefore$ g cont. at $x = 0$.

- differentiability

Compute

$$\lim_{h\to 0} \frac{g(0+h)-g(0)}{h}$$

$$= \lim_{h\to 0} \frac{h^2 \sin\left(\frac{1}{h}\right) - 0}{h}$$

$$= \lim_{h\to 0} h \sin\left(\frac{1}{h}\right) = 0.$$

$\because$) $\quad -1 \leqq \sin\left(\frac{1}{h}\right) \leqq 1$

When $h > 0$

$$-h \leqq h \sin\left(\frac{1}{h}\right) \leqq h$$

(When $h < 0$,

$$-h \geqq h \sin\left(\frac{1}{h}\right) \geqq h \quad)$$

$$\begin{array}{ccccc} \downarrow & & \downarrow \text{ by Sq. Th.} & & \downarrow \\ 0 & & 0 & & 0. \end{array}$$

$\therefore$ g diff. at $x = 0$.