1. Constrained extreme values via **Lagrange Multipliers**: Max/min -ize \( f(v) \) subject to constraint \( g(v) = C \), solve the system \( \nabla f = \lambda \nabla g \) and \( g(v) = C \).

2. Double integrals; Double Riemann sums: \( \int \int_R f(x, y) dA \approx \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_i^*, y_j^*) \Delta A \); 

3. Type I region \( R \): \( \{ g_1(x) \leq y \leq g_2(x) \} \); Type II region \( R \): \( \{ h_1(y) \leq x \leq h_2(y) \} \); 

iterated integrals over Type I and II regions: 
\[
\int \int_R f(x, y) dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy \, dx \\
\int \int_R f(x, y) dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) dx \, dy,
\]
respectively; Reversing Order of Integration (regions that are both Type I and Type II); properties of double integrals.

4. Polar: \( r^2 = x^2 + y^2, x = r \cos \theta, y = r \sin \theta, \tan \theta = \frac{y}{x} \) (make sure \( \theta \) in correct quadrant). 

Change of Variables Formula in Polar Coordinates: if \( R \): \( \{ h_1(\theta) \leq r \leq h_2(\theta) \} \), then 
\[
\int \int_R f(x, y) dA = \int_{\alpha}^{\beta} \int_{h_1(\theta)}^{h_2(\theta)} f(r \cos \theta, r \sin \theta) r \, dr \, d\theta.
\]

5. Applications of double integrals:

(a) Area of region \( R \) is \( A(R) = \int \int_R dA \)

(b) Volume of solid under graph of \( z = f(x, y) \), where \( f(x, y) \geq 0 \), is \( V = \int \int_R f(x, y) dA \)

(c) Mass of \( R \) is \( m = \int \int_R \rho(x, y) dA \), where \( \rho(x, y) \) = density (per unit area).

(d) Moment about the \( x \)-axis \( M_x = \int \int_R y \rho(x, y) dA \); moment about the \( y \)-axis \( M_y = \int \int_R x \rho(x, y) dA \).

(e) Center of mass \((\bar{x}, \bar{y})\), where \( \bar{x} = \frac{M_y}{m} = \frac{\int \int_R x \rho(x, y) dA}{\int \int_R \rho(x, y) dA} \), \( \bar{y} = \frac{M_x}{m} = \frac{\int \int_R y \rho(x, y) dA}{\int \int_R \rho(x, y) dA} \).
6. Elementary solids $D \subset \mathbb{R}^3$ of Type 1, Type 2, Type 3; triple integrals over solids $D$:
\[
\iiint_D f(x, y, z) \, dV = \iint_R \int_{u(x,y)}^{v(x,y)} f(x, y, z) \, dz \, dA
\]
for $D = \{(x, y) \in R, \ u(x, y) \leq z \leq v(x, y)\}$; volume of solid $D$ is $V(D) = \iiint_D dV$; applications of triple integrals, mass of a solid, moments about the coordinate planes $M_{xy}$, $M_{xz}$, $M_{yz}$, center of mass of a solid $(\bar{x}, \bar{y}, \bar{z})$.

7. **Cylindrical Coordinates** $(r, \theta, z)$:

From CC to RC :
\[
\begin{align*}
x &= r \cos \theta \\
y &= r \sin \theta \\
z &= z
\end{align*}
\]

Going from RC to CC use $x^2 + y^2 = r^2$ and $\tan \theta = \frac{y}{x}$ (make sure $\theta$ is in correct quadrant).

8. **Spherical Coordinates** $(\rho, \theta, \phi)$, where $0 \leq \phi \leq \pi$:

From SC to RC :
\[
\begin{align*}
x &= (\rho \sin \phi) \cos \theta \\
y &= (\rho \sin \phi) \sin \theta \\
z &= \rho \cos \phi
\end{align*}
\]

Going from RC to SC use $x^2 + y^2 + z^2 = \rho^2$, $\tan \theta = \frac{y}{x}$ and $\cos \phi = \frac{z}{\rho}$.

9. **Triple integrals in Cylindrical Coordinates**:
\[
\iiint_D f(x, y, z) \, dV = \iiint_D f(r \cos \theta, r \sin \theta, z) \, r \, dz \, dr \, d\theta
\]
10. Triple integrals in Spherical Coordinates:

\[
\begin{align*}
  x &= \rho \sin \phi \cos \theta \\
  y &= \rho \sin \phi \sin \theta \\
  z &= \rho \cos \phi
\end{align*}
\]

\[dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta\]

\[
\int\int\int_D f(x, y, z) \, dV = \int\int\int_D f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta
\]

11. Vector fields on \(\mathbb{R}^2\) and \(\mathbb{R}^3\):

\(\mathbf{F}(x, y) = \langle P(x, y), Q(x, y) \rangle \)

\(\mathbf{F}(x, y, z) = \langle P(x, y, z), Q(x, y, z), R(x, y, z) \rangle\)

\(\mathbf{F}\) is a conservative vector field if \(\mathbf{F} = \nabla f\), for some real-valued function \(f\) (potential).

12. Line integral of a function \(f(x, y)\) along \(C\), parameterized by \(x = x(t), y = y(t)\) and \(a \leq t \leq b\), is

\[
\int_C f(x, y) \, ds = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt.
\]

Remarks:

(a) \(\int_C f(x, y) \, ds\) is sometimes called the “line integral of \(f\) with respect to arc length”

(b) \(\int_C f(x, y) \, dx = \int_a^b f(x(t), y(t)) \, x'(t) \, dt\)

(c) \(\int_C f(x, y) \, dy = \int_a^b f(x(t), y(t)) \, y'(t) \, dt\)