MA 16600
Study Guide - Exam 1

(1) Distance formula $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$; equation of a sphere with center (h, k, l) and radius r: $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$.

(2) Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$; displacement vectors $\overrightarrow{PQ}$; vector arithmetic; components; Standard basis vectors $\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}}$, hence $\vec{\mathbf{a}} = \langle a_1, a_2, a_3 \rangle = a_1\vec{\mathbf{i}} + a_2\vec{\mathbf{j}} + a_3\vec{\mathbf{k}}$; length (magnitude) of a vector $|\vec{\mathbf{a}}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$; dot (or inner) product of $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$: $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_1b_1 + a_2b_2 + a_3b_3$; properties of dot products.

(3) Angle between vectors: $\boxed{\cos\theta = \dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}|}}$;

Perpendicular (orthogonal) vectors; direction cosines: $\cos\alpha = \dfrac{a_1}{|\vec{\mathbf{a}}|}$, $\cos\beta = \dfrac{a_2}{|\vec{\mathbf{a}}|}$, $\cos\gamma = \dfrac{a_3}{|\vec{\mathbf{a}}|}$, direction angles α, β, γ.

(4) Vector projection of $\vec{\mathbf{b}}$ onto $\vec{\mathbf{a}}$: $\boxed{\text{proj}_{\vec{\mathbf{a}}}\,\vec{\mathbf{b}} = \left(\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|}\right)\dfrac{\vec{\mathbf{a}}}{|\vec{\mathbf{a}}|}}$;

Scalar projection of $\vec{\mathbf{b}}$ onto $\vec{\mathbf{a}}$: $\boxed{\text{comp}_{\vec{\mathbf{a}}}\,\vec{\mathbf{b}} = \left(\dfrac{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}{|\vec{\mathbf{a}}|}\right)}$;

Work done by constant force is $\boxed{W = \vec{\mathbf{F}} \cdot \vec{\mathbf{D}}.}$

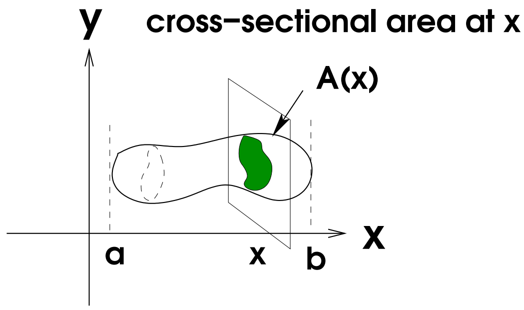
(5) Cross product: $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ (defined **only** for vectors in $\mathbb{R}^3$); $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \perp \vec{\mathbf{a}}$ and $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \perp \vec{\mathbf{b}}$; other properties of cross products; $|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = |\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}|\sin\theta$;
$A = |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| =$ area of parallelogram spanned by $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$;
$A = \frac{1}{2}|\vec{\mathbf{a}} \times \vec{\mathbf{b}}| =$ area of triangle spanned by $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$.
$V = |\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})| =$ volume of parallelepiped spanned by $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$:

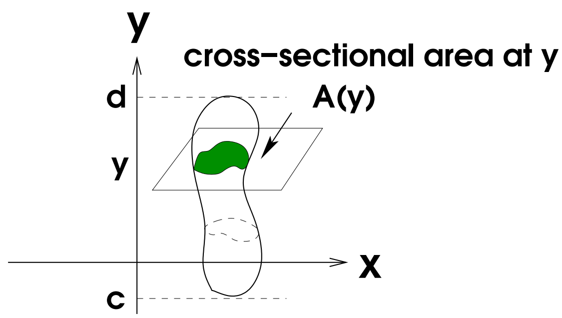
$$\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(6) APPLICATIONS OF INTEGRATION

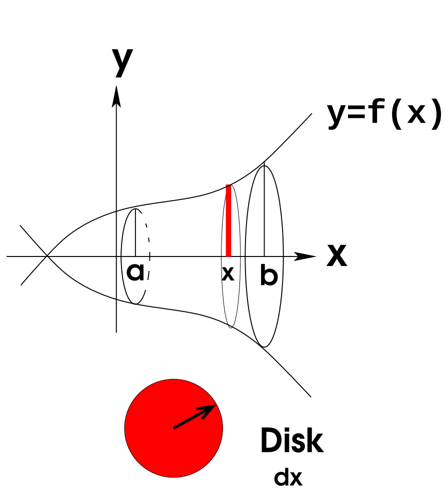
(a) Areas Between Curves: $A = \displaystyle\int_a^b \{f(x) - g(x)\}\,dx$ or $A = \displaystyle\int_c^d \{h(y) - k(y)\}\,dy$:
Note: If curves cross, you need to break up into several integrals.

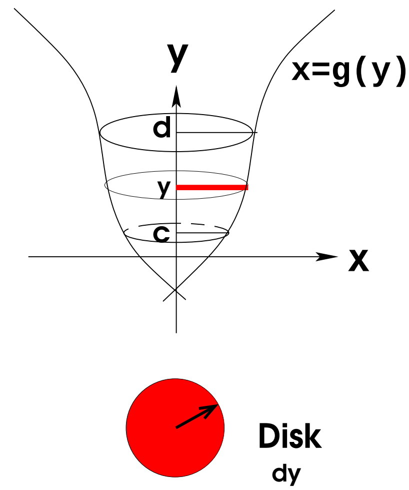
(b) Volumes of Solids by Cross-sectional Areas: $V = \displaystyle\int_a^b A(x)\,dx$, or $V = \displaystyle\int_c^d A(y)\,dy$, where
$A(x) =$ area of the cross-section of the solid with a plane $\perp$ x-axis at the point x, or
$A(y) =$ area of the cross-section of the solid with a plane $\perp$ y-axis at the point y:



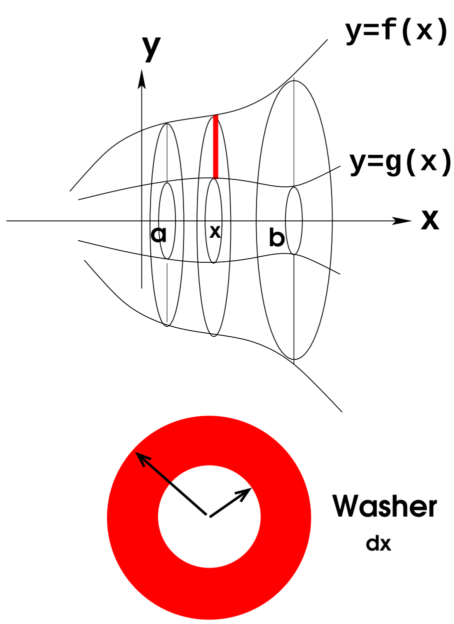


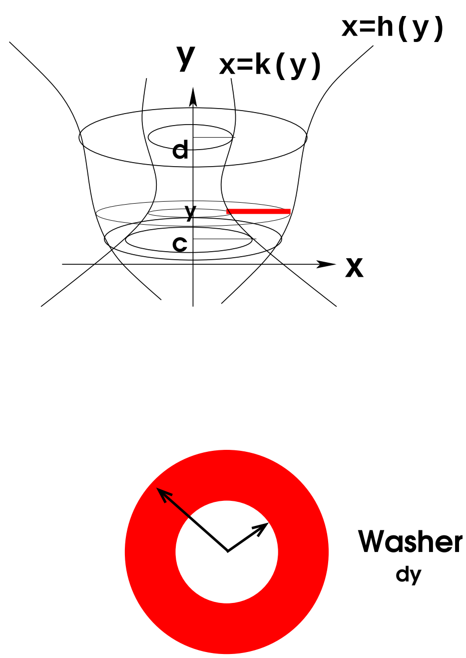
(c) Volumes of Solids of Revolution by Disk Method or Washer Method: Use Disk Method or Washer Method when slices of area are *perpendicular* to axis of rotation. *In either case, the cross-section is always a disk/washer* :





$$V = \int \pi \{\texttt{Radius}\}^2 \{\texttt{dx } or \texttt{ dy}\}$$



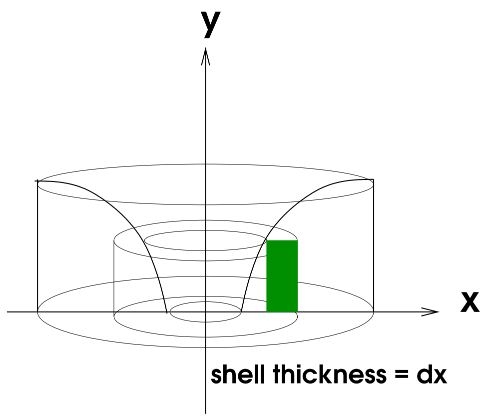


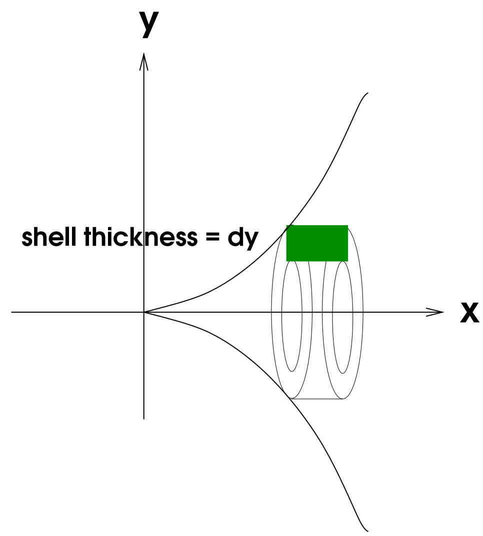
IMPORTANT - When to use dx or dy in Disk/Washer method?

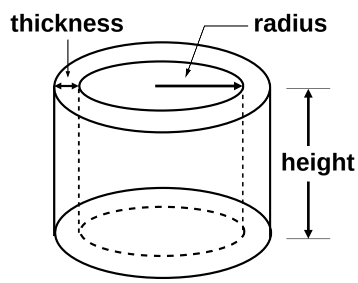
(i) If cross-sections are $\perp$ x - axis, use $\boxed{dx}$.

(ii) If cross-sections are $\perp$ y - axis, use $\boxed{dy}$.

(d) Volumes of Solids of Revolution by CYLINDRICAL SHELLS METHOD: Use Cyclindrical Shells Method when slices of area are *parallel* to axis of rotation. Shell thickness is always either dx or dy.







$$V = \int_a^b 2\pi \; \{\texttt{shell radius}\} \; \{\texttt{shell height}\} \; \{\texttt{shell thickness}\}$$

(e) If force F is constant and distance object moved along a line is d, then Work is $W = Fd$. Here are the English and Metric systems compared:

Quantity	*English System*	*Metric System*
Mass m	slug $(= \text{lb-sec}^2/\text{ft})$	kilogram kg
Force F	pounds (lbs)	Newtons N $(= \text{kg-m/sec}^2)$
Distance d	feet	meters m
Work W	ft-lbs	Joules J $(= \text{kg-m}^2/\text{sec}^2)$
g	32 ft/sec^2	9.8 m/sec^2

If the force is variable, say $f(x)$, then Work $W = \int_a^b f(x)\,dx$; Hooke's Law: $f_s(x) = kx$; work done compressing/stretching springs, emptying tanks, pulling up chains.

(f) Average of a function over an interval: $f_{ave} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$;

`Mean Value Thm for Integrals`: $f_{ave} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx = f(c),$ for some $a \le c \le b$.

(7) TECHNIQUES OF INTEGRATION

(a) **Simple Substitution**: $\int_a^b f(g(x))\, g'(x)\, dx = \int_{g(a)}^{g(b)} f(u)\, du$ (let $u = g(x)$)

(b) **Integration by Parts**: $\boxed{\int u\, dv = uv - \int v\, du}$

(c) **Trig Integrals**: Integrals of the type $\int \sin^m x \cos^n x\, dx$ and $\int \tan^m x \sec^n x\, dx$

Some useful trig identities:

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ and $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

(iii) $\sin 2\theta = 2\sin\theta\,\cos\theta$

(iv) $\tan^2\theta + 1 = \sec^2\theta$

Some useful trig integrals:

(i) $\int \tan u\ du = \ln|\sec u| + C$

(ii) $\int \sec u\ du = \ln|\sec u + \tan u| + C$

(d) **Trig integrals of the form**: $\int \sin mx\ \sin nx\ dx$, $\int \cos mx\ \cos nx\ dx$, $\int \sin mx\ \cos nx\ dx$, use these trig identities:

$$\sin A\ \sin B = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$$

$$\cos A\ \cos B = \frac{1}{2}\{\cos(A-B) + \cos(A+B)\}$$

$$\sin A\ \cos B = \frac{1}{2}\{\sin(A-B) + \sin(A+B)\}$$

(8) Suggested Exercises (at the end of the corresponding section):

Section 12.1: 17, 18, 19, 20, 31
Section 12.3: 15, 16, 23, 24, 27, 29, 30, 39, 40
Section 12.4: 4,5,6,27, 28, 35, 36, 27
Section 6.1: 13, 14, 17, 20, 22
Section 6.2: 1,3,4,6,8,10, 11, 12 Section 6.3: 3,4,9, 10, 15, 16
Section 6.4: 7, 8, 9, 20, 21
Section 7.1: 3,4,5, 11, 15, 51, 52
Section 7.2: 1, 2, 4, 5, 11, 16, 22, 25, 28, 29

Acknowledgement: These notes are due to Prof. Johnny Brown.