

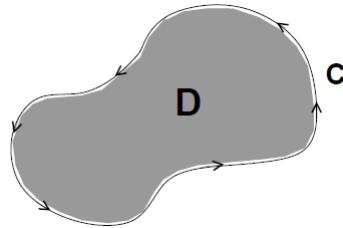
Study Guide # 3**You also need Study Guides # 1 and # 2 for the Final Exam**

- 1.** FUNDAMENTAL THEOREM OF CALCULUS FOR LINE INTEGRALS: $\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$:



- 2.** A vector field $\vec{F}(x, y) = P(x, y)\vec{i} + Q(x, y)\vec{j}$ is *conservative* (i.e. $\vec{F} = \nabla f$) if $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$; how to determine a potential function f if $\vec{F} = \nabla f$.

- 3.** GREEN'S THEOREM: $\int_C P(x, y) dx + Q(x, y) dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$ (C = boundary of D):



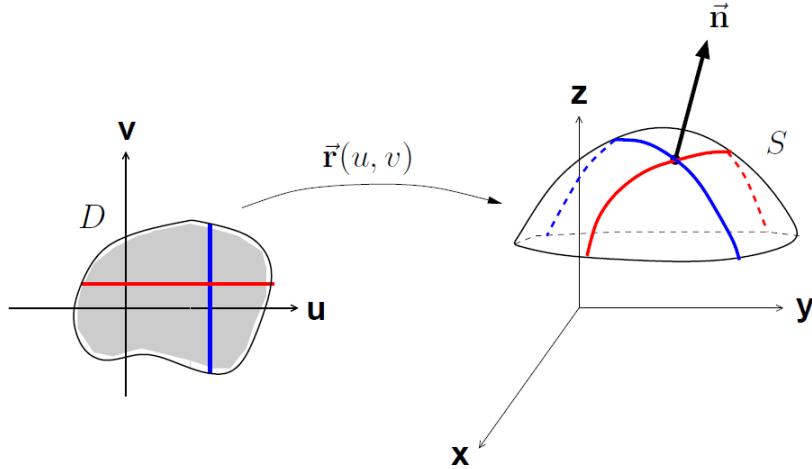
- 4.** DEL OPERATOR: $\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$; if $\vec{F}(x, y, z) = P(x, y, z)\vec{i} + Q(x, y, z)\vec{j} + R(x, y, z)\vec{k}$, then

$$\text{curl } \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} \quad \text{and} \quad \text{div } \vec{F} = \nabla \cdot \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

Properties of curl and divergence:

- (i) If $\text{curl } \vec{F} = \vec{0}$, then \vec{F} is a conservative vector field ($\vec{F} = \nabla f$) in a simply-connected domain.
- (ii) If $\text{curl } \vec{F} = \vec{0}$, then \vec{F} is *irrotational*; if $\text{div } \vec{F} = 0$, then \vec{F} is *incompressible*.
- (iii) *Laplace's Equation:* $\nabla^2 f = \text{div}(\nabla f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$.
- (iv) For functions with continuous partials, $\text{curl}(\nabla f) = \vec{0}$, and $\text{div}(\text{curl } \vec{F}) = 0$.

5. Parametric surface S : $\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$, where $(u, v) \in D$:



Normal vector to surface S : $\vec{n} = \vec{r}_u \times \vec{r}_v$; tangent planes and normal lines to parametric surfaces.

6. Surface area of a surface S :

$$(i) A(S) = \iint_D |\vec{r}_u \times \vec{r}_v| dA$$

$$(ii) \text{ If } S \text{ is the graph of } z = h(x, y) \text{ above } D, \text{ then } A(S) = \iint_D \sqrt{1 + (\partial h / \partial x)^2 + (\partial h / \partial y)^2} dA;$$

Remark: $dS = |\vec{r}_u \times \vec{r}_v| dA$ = differential of surface area; while $d\vec{S} = (\vec{r}_u \times \vec{r}_v) dA$

7. The surface integral of f over the surface S :

$$(i) \iint_S f(x, y, z) dS = \iint_D f(\vec{r}(u, v)) |\vec{r}_u \times \vec{r}_v| dA.$$

$$(ii) \text{ If } S \text{ is the graph of } z = h(x, y) \text{ above } D, \text{ then}$$

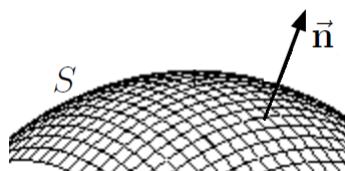
$$\iint_S f(x, y, z) dS = \iint_D f(x, y, h(x, y)) \sqrt{1 + (\partial h / \partial x)^2 + (\partial h / \partial y)^2} dA.$$

8. The surface integral of \vec{F} over the surface S (recall, $d\vec{S} = (\vec{r}_u \times \vec{r}_v) dA$):

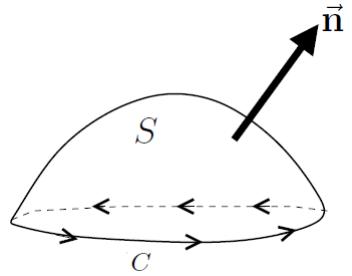
$$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \vec{F} \cdot (\vec{r}_u \times \vec{r}_v) dA = \iint_S (\vec{F} \cdot \vec{n}) dS = \text{flux of } \vec{F} \text{ across the surface } S.$$

If S is the graph of $z = h(x, y)$ above D , with \vec{n} oriented upward, and $\vec{F} = \langle P, Q, R \rangle$, then

$$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \left(-P \frac{\partial h}{\partial x} - Q \frac{\partial h}{\partial y} + R \right) dA$$

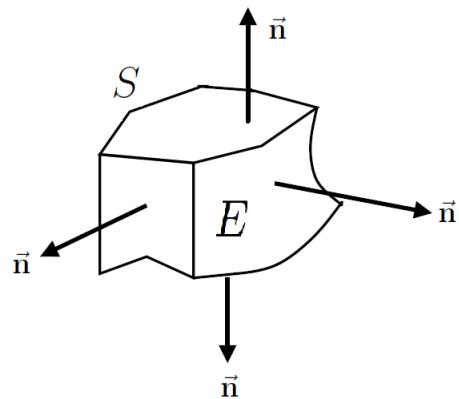


9. STOKES' THEOREM: $\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl } \vec{F} \cdot d\vec{S}$ (recall, $\text{curl } \vec{F} = \nabla \times \vec{F}$).



$$\int_C \vec{F} \cdot d\vec{r} = \text{circulation of } \vec{F} \text{ around } C.$$

10. THE DIVERGENCE THEOREM/GAUSS' THEOREM: $\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \text{div } \vec{F} \, dV$
 (recall, $\text{div } \vec{F} = \nabla \cdot \vec{F}$).

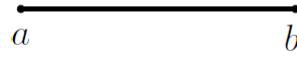


11. Summary of Line Integrals and Surface Integrals:

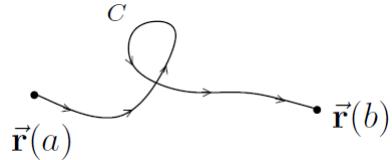
LINE INTEGRALS	SURFACE INTEGRALS
$C : \vec{r}(t)$, where $a \leq t \leq b$	$S : \vec{r}(u, v)$, where $(u, v) \in D$
$ds = \vec{r}'(t) dt =$ differential of arc length	$dS = \vec{r}_u \times \vec{r}_v dA =$ differential of surface area
$\int_C ds =$ length of C	$\iint_S dS =$ surface area of S
$\int_C f(x, y, z) ds = \int_a^b f(\vec{r}(t)) \vec{r}'(t) dt$ (independent of orientation of C)	$\iint_S f(x, y, z) dS = \iint_D f(\vec{r}(u, v)) \vec{r}_u \times \vec{r}_v dA$ (independent of normal vector \vec{n})
$d\vec{r} = \vec{r}'(t) dt$	$d\vec{S} = (\vec{r}_u \times \vec{r}_v) dA$
$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt$ (depends on orientation of C)	$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \vec{F}(\vec{r}(u, v)) \cdot (\vec{r}_u \times \vec{r}_v) dA$ (depends on normal vector \vec{n})
$\int_C \vec{F} \cdot d\vec{r} = \int_C (\vec{F} \cdot \vec{T}) ds$ The <i>circulation</i> of \vec{F} around C	$\iint_S \vec{F} \cdot d\vec{S} = \iint_S (\vec{F} \cdot \vec{n}) dS$ The <i>flux</i> of \vec{F} across S in direction \vec{n}

12. Integration Theorems:

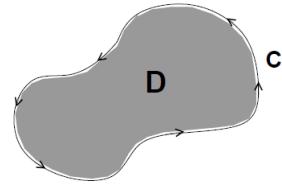
FUNDAMENTAL THEOREM OF CALCULUS: $\int_a^b F'(x) dx = F(b) - F(a)$



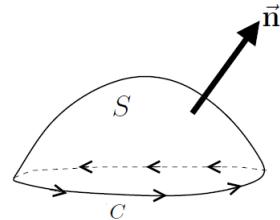
FUNDAMENTAL THEOREM OF CALCULUS FOR LINE INTEGRALS: $\int_a^b \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$



GREEN'S THEOREM: $\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_C P(x, y) dx + Q(x, y) dy$



STOKES' THEOREM: $\iint_S \text{curl } \vec{F} \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r}$



DIVERGENCE THEOREM: $\iiint_E \text{div } \vec{F} dV = \iint_S \vec{F} \cdot d\vec{S}$

