4.2

Another solution

Let $\vec{v} = \langle x, y, z \rangle$

(i) $\vec{v} \perp \vec{a}$

$\rightarrow \vec{v} \cdot \vec{a} = 0$

$3x - y + 5z = 0 \quad \cdots\cdots ①$

(ii) $\vec{a} \times \vec{v} = \langle -1, 2, 1 \rangle$

$$\left\langle \begin{vmatrix} -1 & 5 \\ y & z \end{vmatrix}, -\begin{vmatrix} 3 & 5 \\ x & z \end{vmatrix}, \begin{vmatrix} 3 & -1 \\ x & y \end{vmatrix} \right\rangle$$

$= \langle -z - 5y, -3z + 5x, 3y + x \rangle$

$\therefore$

$$\begin{cases} -5y - z = -1 & \cdots ② \\ 5x - 3z = 2 & \cdots ③ \\ x + 3y = 1 & \cdots ④ \end{cases}$$

② $\rightarrow$ $y = \dfrac{-z+1}{5}$

③ $\rightarrow$ $x = \dfrac{3z+2}{5}$

(Note that if you plug the above into x & y in ④, the equation ④ is satisfied)

Now plug these into ①

$$3\left(\frac{3z+2}{5}\right) - \left(\frac{-z+1}{5}\right) + 5z = 0$$

$\rightarrow$

$$(9z+6) - (-z+1) + 25z = 0$$

$\rightarrow$

$$35z = -5$$

$\rightarrow$

$$z = -\frac{5}{35} = -\frac{1}{7}$$

$$x = \frac{3z+2}{5} = \frac{11}{35}$$

$$y = \frac{-z+1}{5} = \frac{8}{35}$$

$\rightarrow$

$$\vec{v} = \left\langle \frac{11}{35}, \frac{8}{35}, -\frac{1}{7} \right\rangle.$$