Answer Keys
for the example problems
in Study Guide for Exam I

1.1.

(i) $x^2 + y^2 + z^2 + 2x - 6y + 9 = 0$

$$x^2 + 2x + 1 + y^2 - 6y + 9 + z^2 = -9 + 1 + 9$$

$$(x+1)^2 + (y-3)^2 + z^2 = 1^2$$

center $C = (-1, 3, 0)$

radius $r = 1$.

distance from $P = (2, 1, -5)$

to $C = (-1, 3, 0)$

$$|\overrightarrow{PC}| = \sqrt{\{(-1)-2\}^2 + \{3-1\}^2 + \{0-(-5)\}^2}$$

$$= \sqrt{38}.$$

distance to the closest point on the sphere

$$= |\overrightarrow{PC}| - r$$

$$= \sqrt{38} - 1.$$

(ii) distance to the furthest point Q on the sphere

$$|\overrightarrow{PQ}| = |\overrightarrow{PC}| + r = \sqrt{38} + 1.$$

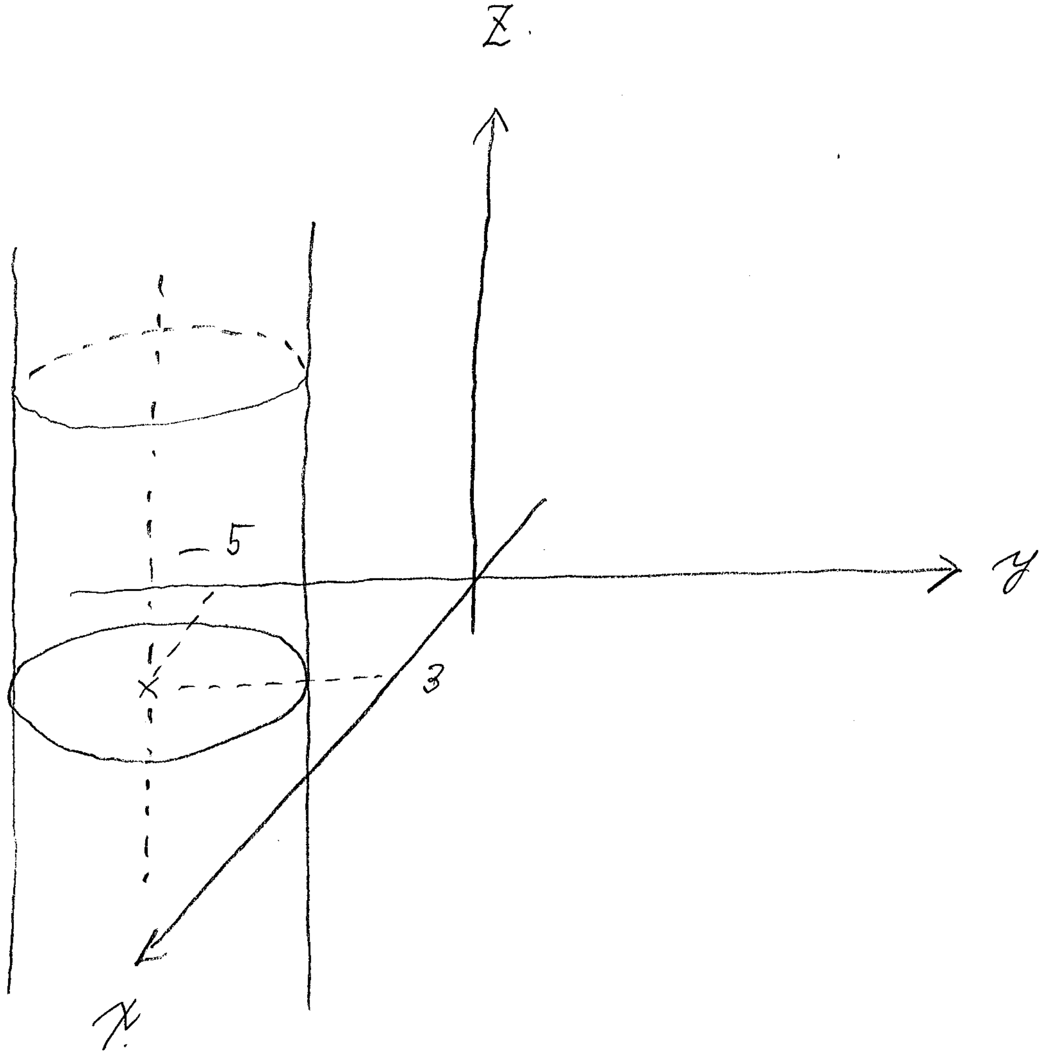
$\longrightarrow$

$$\overrightarrow{PQ} = |\overrightarrow{PQ}| \cdot \frac{\overrightarrow{PC}}{|\overrightarrow{PC}|}$$

$$= \frac{\sqrt{38}+1}{\sqrt{38}} \langle -3, 2, 5 \rangle$$

1、2

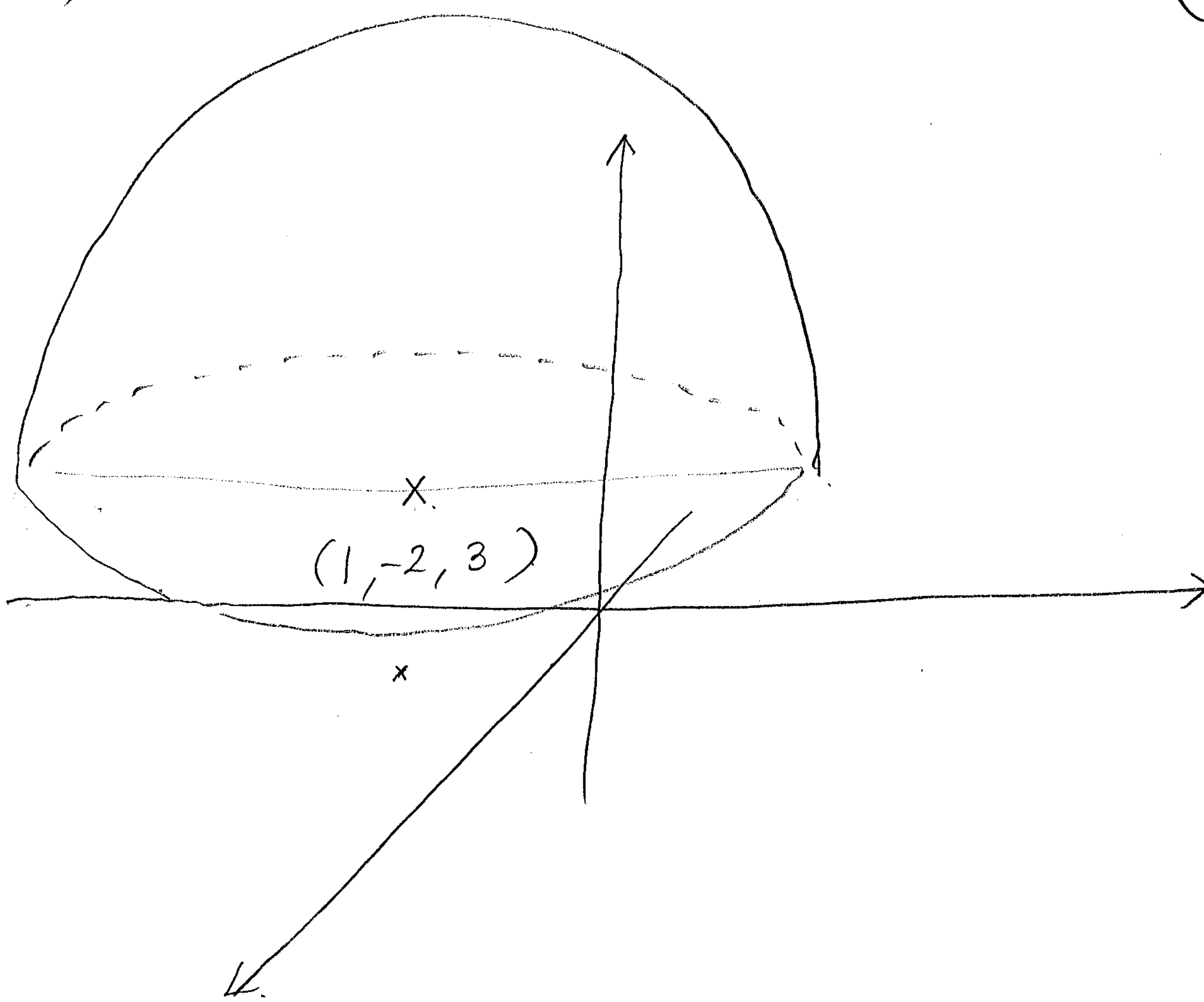
(i)



$$(x-3)^2+(y+5)^2 \leqq 2^2.$$

(ii)

④



$$(x-1)^2 + (y+2)^2 + (z-3)^2 \leqq 7^2$$

&

$$z \geqq 3$$

2.1. $P = (3, -1, 4)$ 5

$Q = (7, 2, -5)$

$$\overrightarrow{PQ} = \langle 4, 3, -9 \rangle$$

$$\vec{u} = \frac{1}{|\overrightarrow{PQ}|} \overrightarrow{PQ}$$

$$= \frac{1}{\sqrt{4^2 + 3^2 + (-9)^2}} \langle 4, 3, -9 \rangle$$

$$= \frac{1}{\sqrt{106}} \langle 4, 3, -9 \rangle$$

2.2.

6

$$\vec{u} = \langle 1, 2 \rangle$$
$$\vec{v} = \langle 3, -4 \rangle$$

$$\begin{aligned}\vec{w} &= \langle 1, 0 \rangle \\ &= \alpha \vec{u} + \beta \vec{v} \\ &= \alpha \langle 1, 2 \rangle + \beta \langle 3, -4 \rangle \\ &= \langle 1 \cdot \alpha + 3\beta, \ 2\alpha - 4\beta \rangle\end{aligned}$$

$$\begin{cases} 1\alpha + 3\beta = 1 \\ 2\alpha - 4\beta = 0 \end{cases}$$

$$\longrightarrow \quad \alpha = \frac{2}{5}, \quad \beta = \frac{1}{5}$$

3.1. $\vec{a} = \langle \frac{1}{3}, \frac{1}{3}, \alpha \rangle$ (7)

is a unit vector

$\iff$

$$1^2 = |\vec{a}|^2 = \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \alpha^2$$

$\iff$ $\alpha^2 = \frac{7}{9}$

$\iff$

$$\alpha = \pm \frac{\sqrt{7}}{3}$$

$\vec{a} \perp \vec{b}$

$\iff$

$$0 = \vec{a} \cdot \vec{b}$$

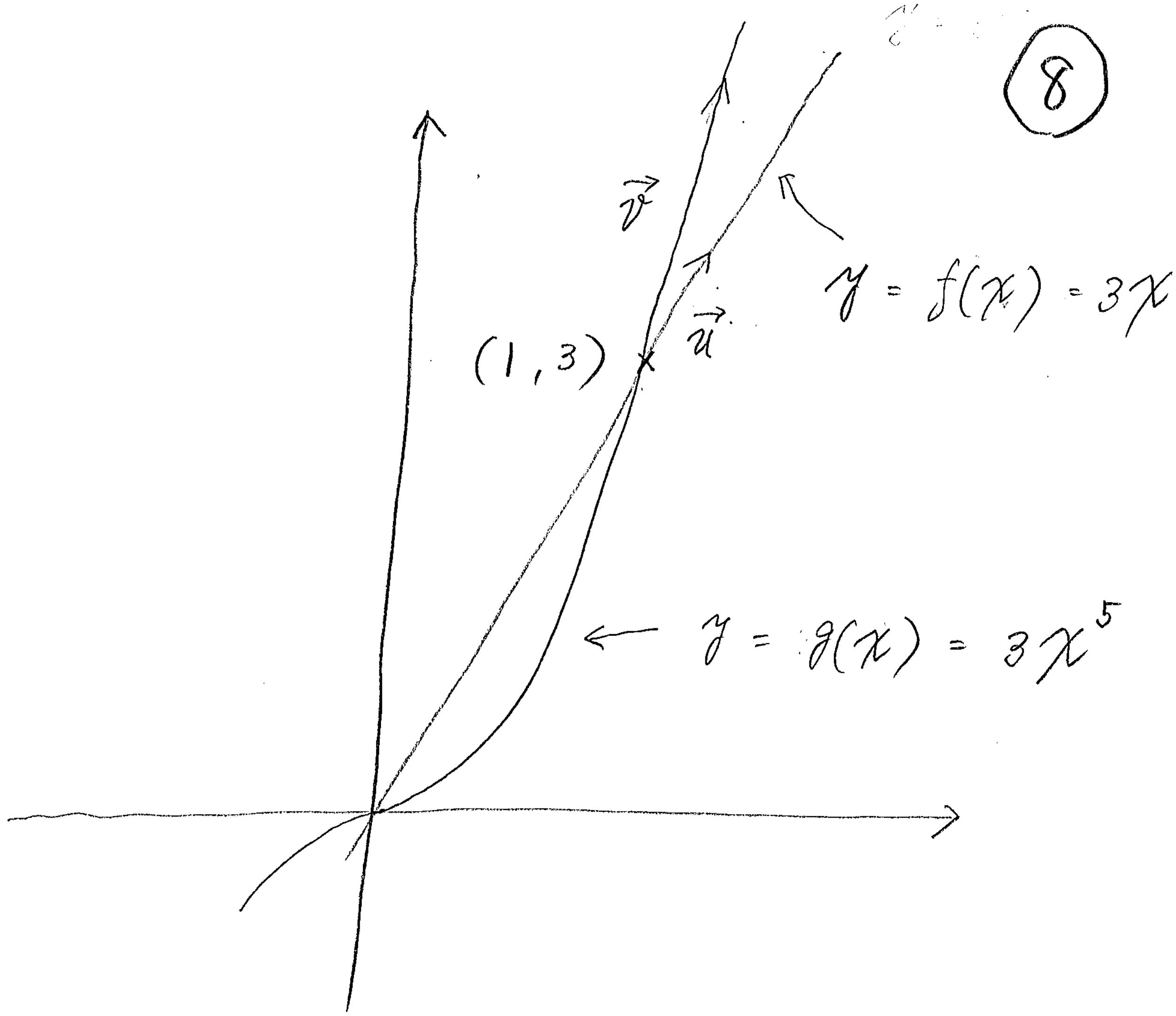
$$= \langle \frac{1}{3}, \frac{1}{3}, \alpha \rangle \cdot \langle \beta, 0, \sqrt{2} \rangle$$

$$= \frac{\beta}{3} + \sqrt{2}\alpha \quad \to \quad \beta = -3\sqrt{2}\,\alpha$$

Ans.

$$\begin{cases} \alpha = +\frac{\sqrt{7}}{3} \\ \beta = -\sqrt{14} \end{cases} \qquad \begin{cases} \alpha = -\frac{\sqrt{7}}{3} \\ \beta = +\sqrt{14} \end{cases}$$

3.2.



$y = f(x) = 3x$

$f'(x) = 3$

tangent vector $\vec{u}$ at $(1,3)$

$\vec{u} = \langle 1, f'(1) \rangle$

$= \langle 1, 3 \rangle$

$y = g(x) = 3x^5$

$g'(x) = 15x^4$

tangent vector $\vec{v}$ at $(1, 3)$

$$\vec{v} = \langle 1, g'(1) \rangle$$

$$= \langle 1, 15 \rangle$$

θ angle between the tangent lines

= angle between $\vec{u}$ & $\vec{v}$

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \frac{\langle 1, 3 \rangle \cdot \langle 1, 15 \rangle}{\sqrt{1^2+3^2}\sqrt{1^2+15^2}}$$

$$= \frac{1 \cdot 1 + 3 \cdot 15}{\sqrt{10}\sqrt{226}} = \frac{\cancel{46}\ 23}{\cancel{2}\sqrt{5}\sqrt{113}}$$

$$= \frac{23}{\sqrt{565}}$$

$$\theta = \cos^{-1}\left(\frac{23}{\sqrt{565}}\right)$$

4.1.

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$$\vec{a} = \langle 2, 1, -1 \rangle$$

$$\vec{b} = \langle 1, 3, 2 \rangle$$

$$\vec{a} \times \vec{b} =$$

$$\left\langle \begin{vmatrix} \cancel{2} & 1 & -1 \\ \cancel{1} & 3 & 2 \end{vmatrix}, -\begin{vmatrix} 2 & \cancel{1} & -1 \\ 1 & \cancel{3} & 2 \end{vmatrix}, \begin{vmatrix} 2 & 1 & \cancel{-1} \\ 1 & 3 & \cancel{2} \end{vmatrix} \right\rangle$$

$$= \left\langle \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix}, -\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \right\rangle$$

$$= \langle 5, -5, 5 \rangle$$

4.2

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$\vec{a} = \langle 3, -1, 5 \rangle$

$\vec{v}$ satisfies the property

(i) $\vec{v} \perp \vec{a}$

(ii) $\vec{a} \times \vec{v} = \langle -1, 2, 1 \rangle$

$\to \vec{v} \perp \langle -1, 2, 1 \rangle$

Therefore, $\vec{v}$ is a scalar product of

$\vec{a} \times \langle -1, 2, 1 \rangle$

i.e.

$$\vec{v} = t\, \vec{a} \times \langle -1, 2, 1 \rangle$$

$$= t \langle 3, -1, 5 \rangle \times \langle -1, 2, 1 \rangle$$

$$= t \langle -11, -8, 5 \rangle$$

for some $t \in \mathbb{R}$

Now since

(i) $v = t\, \vec{a} \times \langle -1, 2, 1 \rangle$

$\perp \vec{a}$

is automatically satisfied,

we have only to check

(ii) $\vec{a} \times \vec{v} = \langle -1, 2, 1 \rangle$

i.e.

$$\langle 3, -1, 5 \rangle \times t \langle -11, -8, 5 \rangle = \langle -1, 2, 1 \rangle$$

$$\| $$

$$t \underbrace{\langle 3, -1, 5 \rangle \times \langle -11, -8, 5 \rangle}_{\langle 35, -70, -35 \rangle}$$

Therefore, we conclude.

$$t = -\frac{1}{35}$$

$\longrightarrow$

$$\vec{v} = -\frac{1}{35} \langle -11, -8, 5 \rangle$$

$$= \left\langle \frac{11}{35}, \frac{8}{35}, -\frac{1}{7} \right\rangle$$

4.3.

$$\vec{a} = \langle 1, 4, k \rangle$$

$$\vec{b} = \langle 2, -1, 4 \rangle$$

$$c = \langle 0, -9, 18 \rangle$$

coplanar

$$\Longleftrightarrow$$

$$\begin{vmatrix} 1 & 4 & k \\ 2 & -1 & 4 \\ 0 & -9 & 18 \end{vmatrix} = 0$$

$$\parallel$$

$$1 \cdot \begin{vmatrix} -1 & 4 \\ -9 & 18 \end{vmatrix} - 4 \cdot \begin{vmatrix} 2 & 4 \\ 0 & 18 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ 0 & -9 \end{vmatrix}$$

$$\parallel$$

$$1 \cdot 18 - 4 \cdot 36 + k(-18)$$

$$\parallel$$

$$18(1 - 8 - k) = 0.$$

$$k = -7.$$

4.4.

14

$P = (1, 0, 1)$

$Q = (-2, 1, 3)$

$R = (4, 2, 5)$

$\overrightarrow{PQ} = \langle -3, 1, 2 \rangle$

$\overrightarrow{PR} = \langle 3, 2, 4 \rangle$

area of $\triangle PQR$

$= \frac{1}{2}$ of the area of the parallelogram formed by $\overrightarrow{PQ}$ & $\overrightarrow{PR}$

$= \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}|$.

$$\left(\overrightarrow{PQ} \times \overrightarrow{PR} = \left\langle \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}, -\begin{vmatrix} -3 & 2 \\ 3 & 4 \end{vmatrix}, \begin{vmatrix} -3 & 1 \\ 3 & 2 \end{vmatrix} \right\rangle = \langle 0, 18, -9 \rangle \right)$$

$= \frac{1}{2} \sqrt{0^2 + 18^2 + (-9)^2} = \frac{1}{2} 9\sqrt{5}$.

5.1 $\vec{a} = \langle -2, 3, 1 \rangle$

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$\vec{b} = \langle 1, 1, 2 \rangle$

$$\text{comp}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\sqrt{\vec{a} \cdot \vec{a}}}$$

$$= \frac{\langle -2, 3, 1 \rangle \cdot \langle 1, 1, 2 \rangle}{\sqrt{(-2)^2 + 3^2 + 1^2}}$$

$$= \frac{3}{\sqrt{14}}$$

$$\text{proj}_{\vec{a}} \vec{b} = \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}} \vec{a}$$

$$= \frac{3}{14} \langle -2, 3, 1 \rangle$$

6.1.

$\vec{a} = \langle 2, -1, 3 \rangle$

$\vec{e_1} = \langle 1, 0, 0 \rangle$

$\vec{e_2} = \langle 0, 1, 0 \rangle$

$\vec{e_3} = \langle 0, 0, 1 \rangle$

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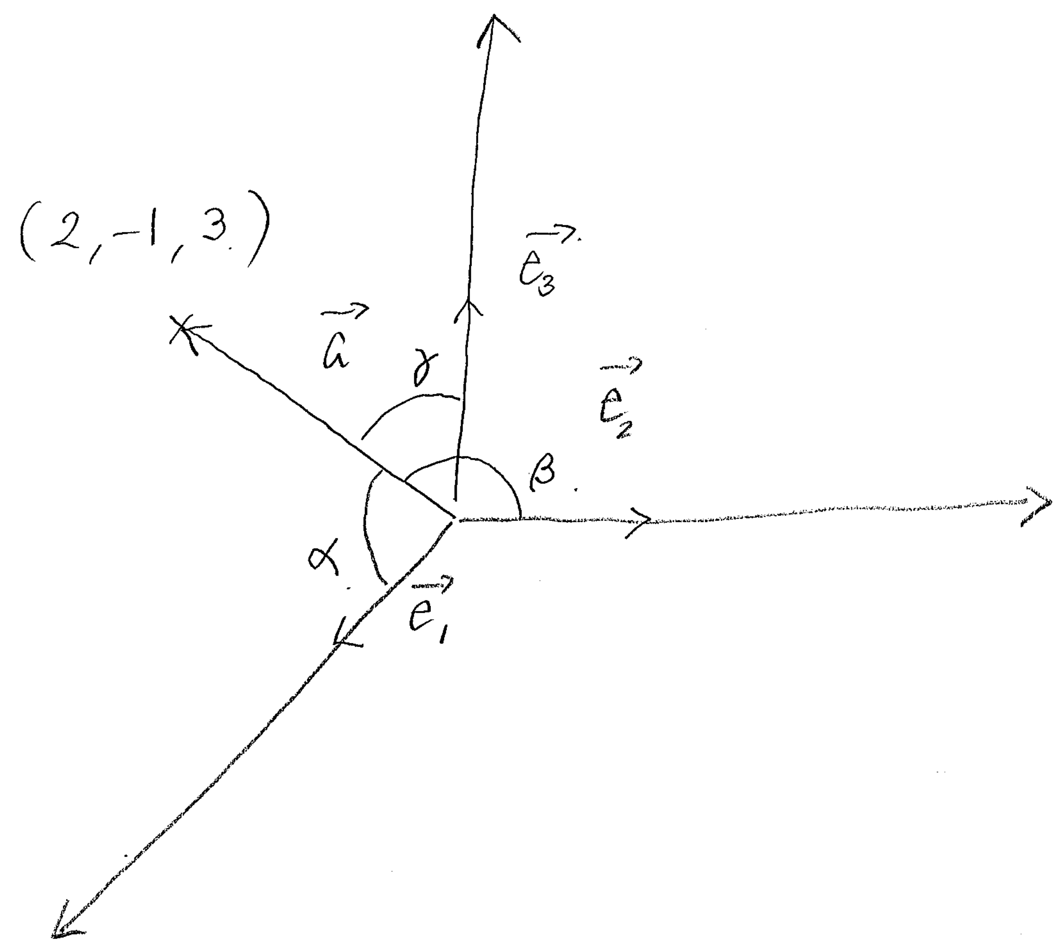


$$\cos\alpha = \frac{\vec{a}\cdot\vec{e_1}}{|\vec{a}||\vec{e_1}|} = \frac{2}{\sqrt{14}}, \quad \alpha = \cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$$

$$\cos\beta = \frac{\vec{a}\cdot\vec{e_2}}{|\vec{a}||\vec{e_2}|} = \frac{-1}{\sqrt{14}} \quad \beta = \cos^{-1}\left(\frac{-1}{\sqrt{14}}\right)$$

$$\cos\gamma = \frac{\vec{a}\cdot\vec{e_3}}{|\vec{a}||\vec{e_3}|} = \frac{3}{\sqrt{14}} \quad \gamma = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right)$$

6.2

We have the relation

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

(This is obvious, since

$$\vec{v} = \langle a, b, c \rangle$$

$$\begin{cases} \cos\alpha = \dfrac{a}{\sqrt{a^2+b^2+c^2}} \\ \cos\beta = \dfrac{b}{\sqrt{a^2+b^2+c^2}} \\ \cos\gamma = \dfrac{c}{\sqrt{a^2+b^2+c^2}}. \end{cases}$$
)

Now, since

$$\alpha = \frac{\pi}{4}, \quad \beta = \frac{\pi}{3},$$

we have

$$\cos^2\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{3}\right) + \cos^2\gamma = 1$$

i.e.

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$$

$$\cos^2\gamma = \frac{1}{4}$$

(18)

i.e.

$$\cos\gamma = \pm\frac{1}{2}$$

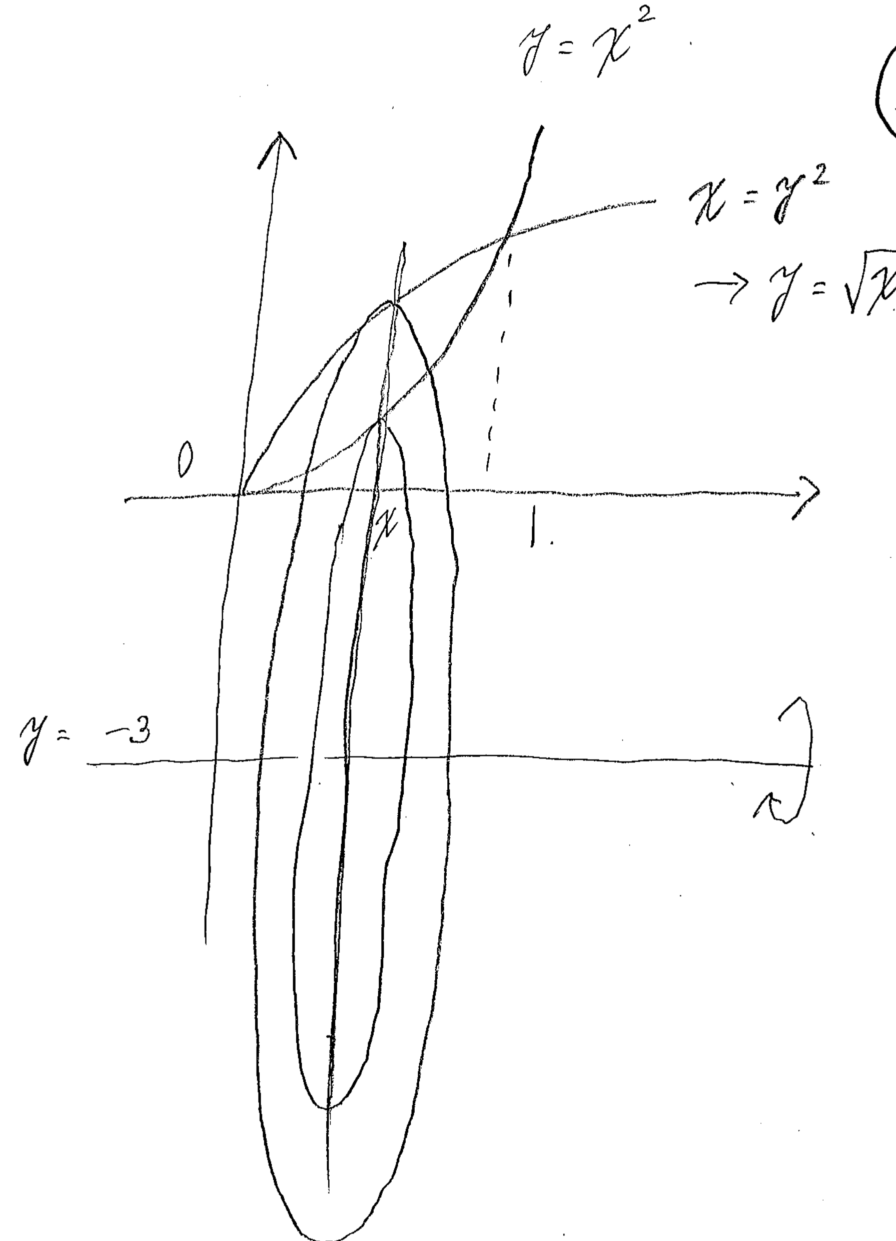
$$\gamma = \cos^{-1}\left(\frac{1}{2}\right) \text{ or } \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\gamma = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

7.1. Look at MyLab Math Lesson 5.

8.1.

(i) Washer



$$V = \int_0^1 \left(\pi r_{BIG}^2 - \pi r_{SMALL}^2 \right) dx$$

$$= \int_0^1 \left\{ \pi \left(\sqrt{x}\right)^2 - \pi \left(x^2\right)^2 \right\} dx$$

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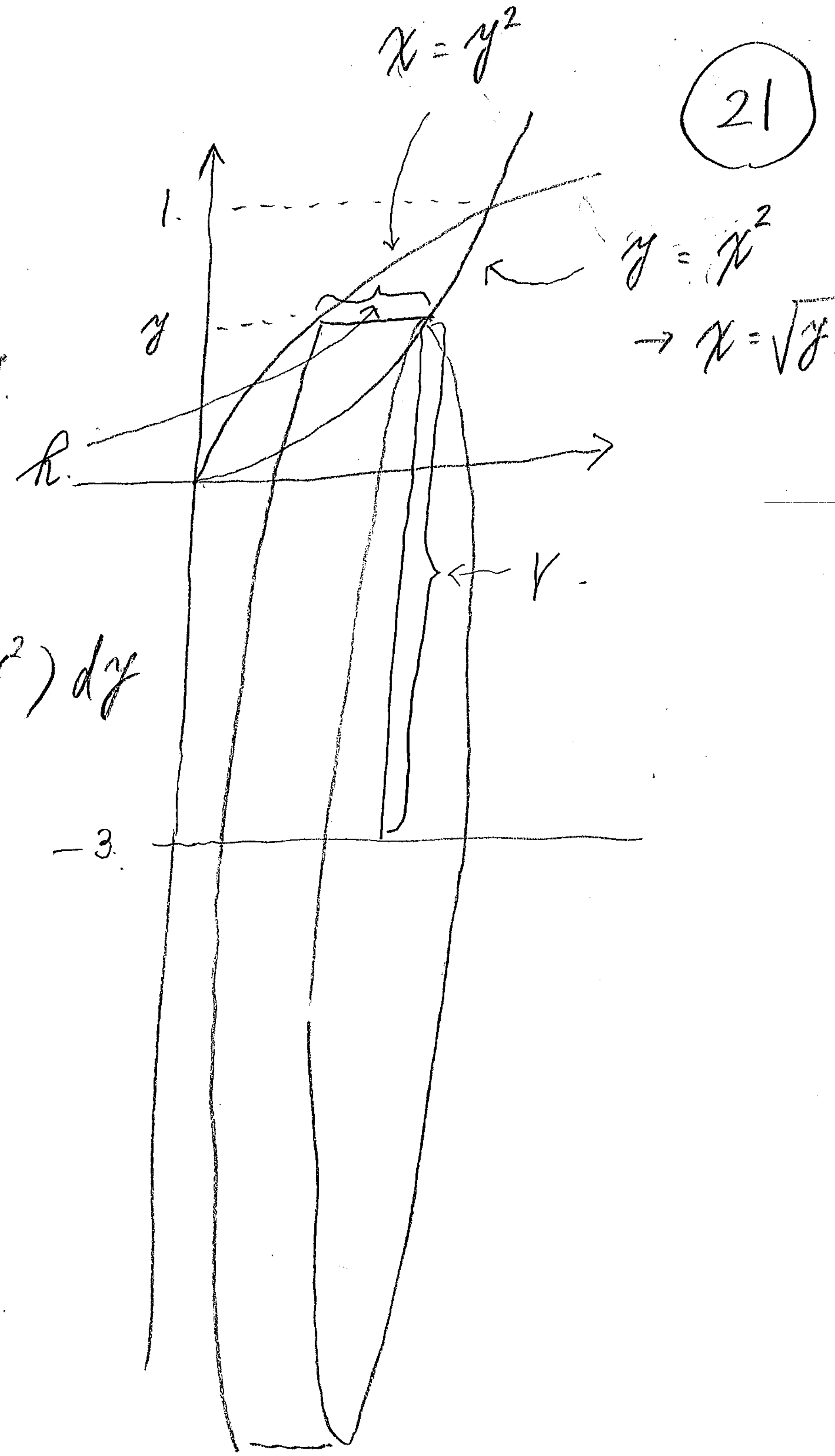
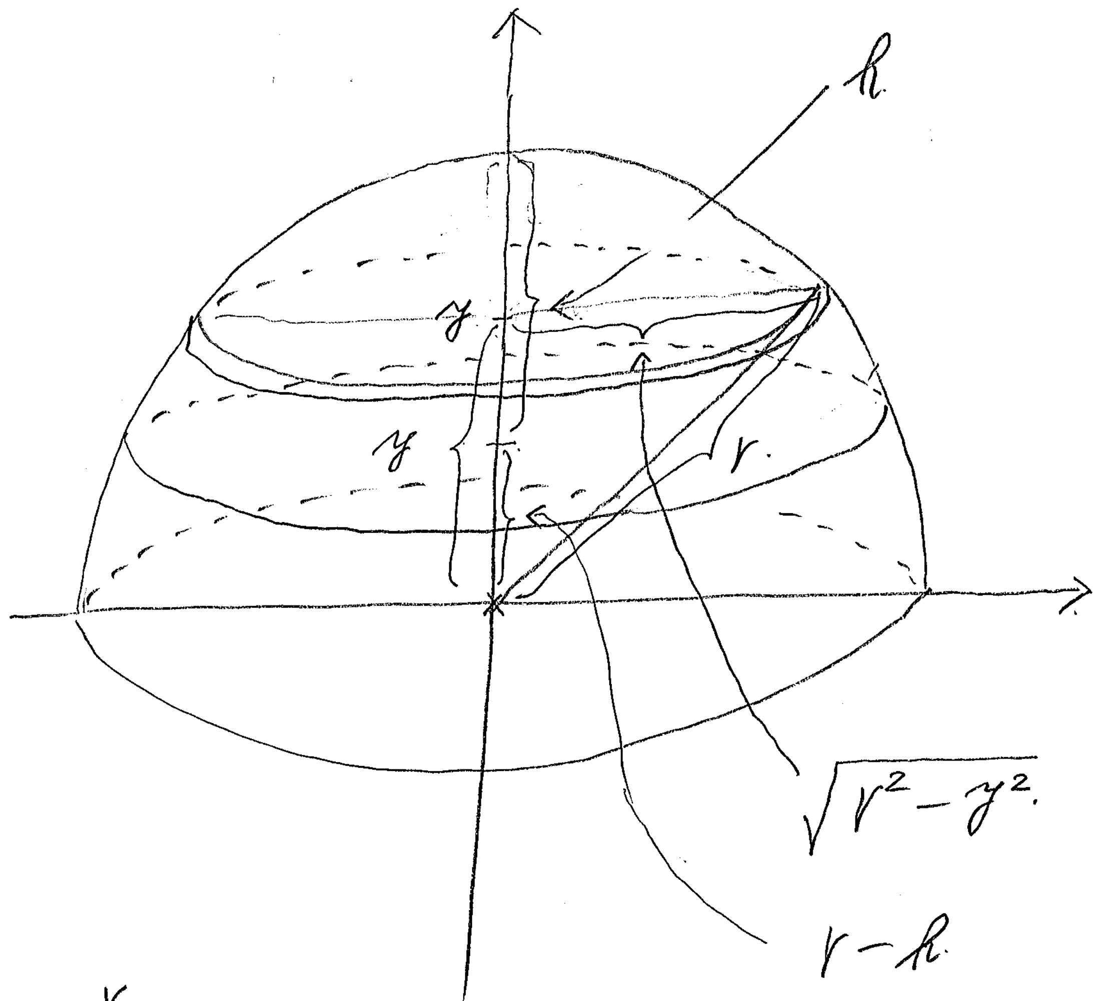
(ii)

$$V = \int_0^1 2\pi r \cdot h \, dy$$

$$= \int_0^1 2\pi (y+3)(\sqrt{y} - y^2)\, dy$$



8.2.



$$V = \int_{r-h}^{r} \pi \left(\sqrt{r^2-y^2}\right)^2 dy$$

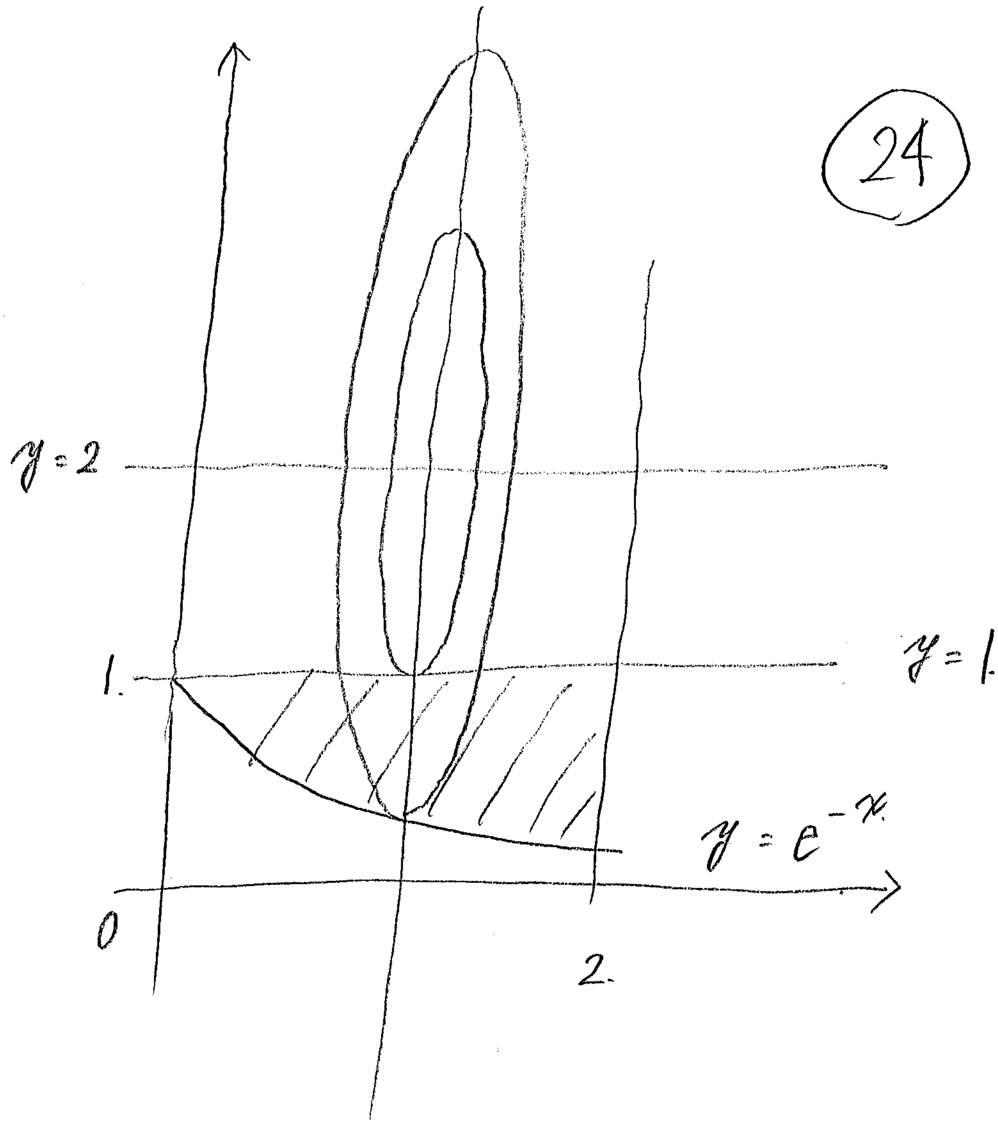
$$= \int_{r-h}^{r} \pi \left(r^2-y^2\right) dy$$

$$= \pi \int_{r-h}^{r} \left(r^2-y^2\right) dy$$

$$= \pi \left[r^2 y - \frac{y^3}{3} \right]_{r-h}^{r}$$

$$= \pi \left[r^2 h - \left\{ \frac{r^3}{3} - \frac{(r-h)^3}{3} \right\} \right]$$

8.3



$$V = \int_0^2 \left\{ \pi \cdot (2-e^{-x})^2 - \pi \cdot (2-1)^2 \right\} dx$$

$$= \pi \int_0^2 (3 - 4e^{-x} + e^{-2x})\, dx$$

$$= \pi \left[3x + 4e^{-x} - \frac{1}{2} e^{-2x} \right]_0^2$$

$$= \pi \left[\left(6 + 4e^{-2} - \frac{1}{2}e^{-4}\right) - \left(0 + 4 \cdot 1 - \frac{1}{2} \cdot 1\right) \right]$$

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$$= \pi \left(\frac{5}{2} + 4e^{-2} - \frac{1}{2}e^{-4} \right)$$

8.4.

$y = 3 + 2x - x^2$
$= -(x-1)^2 + 4$

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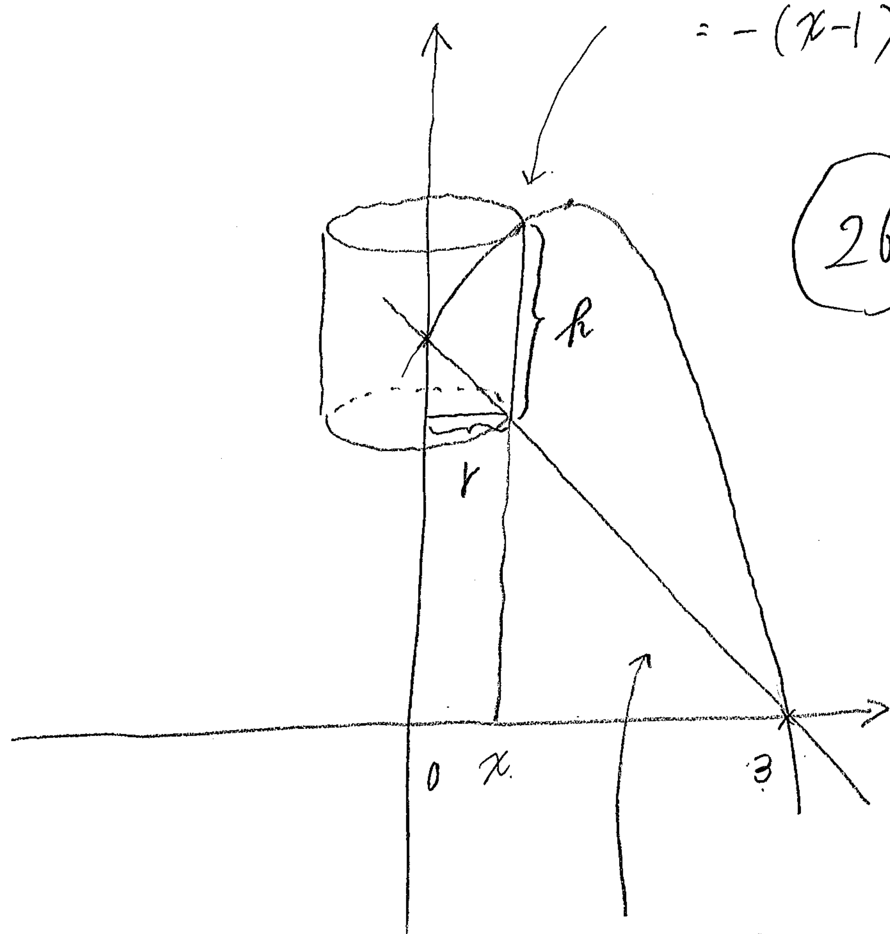

$x + y = 3$

$\rightarrow y = -x + 3$

$$V = \int_0^3 2\pi r \cdot h \, dx$$

$$= \int_0^3 2\pi x \{(3 + 2x - x^2) - (-x + 3)\} \, dx$$

$$= 2\pi \int_0^3 x(-x^2 + 3x) \, dx$$

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$$= 2\pi \int_0^3 (-x^3 + 3x^2)\,dx$$

$$= 2\pi \left[-\frac{x^4}{4} + 3 \cdot \frac{x^3}{3} \right]_0^3$$

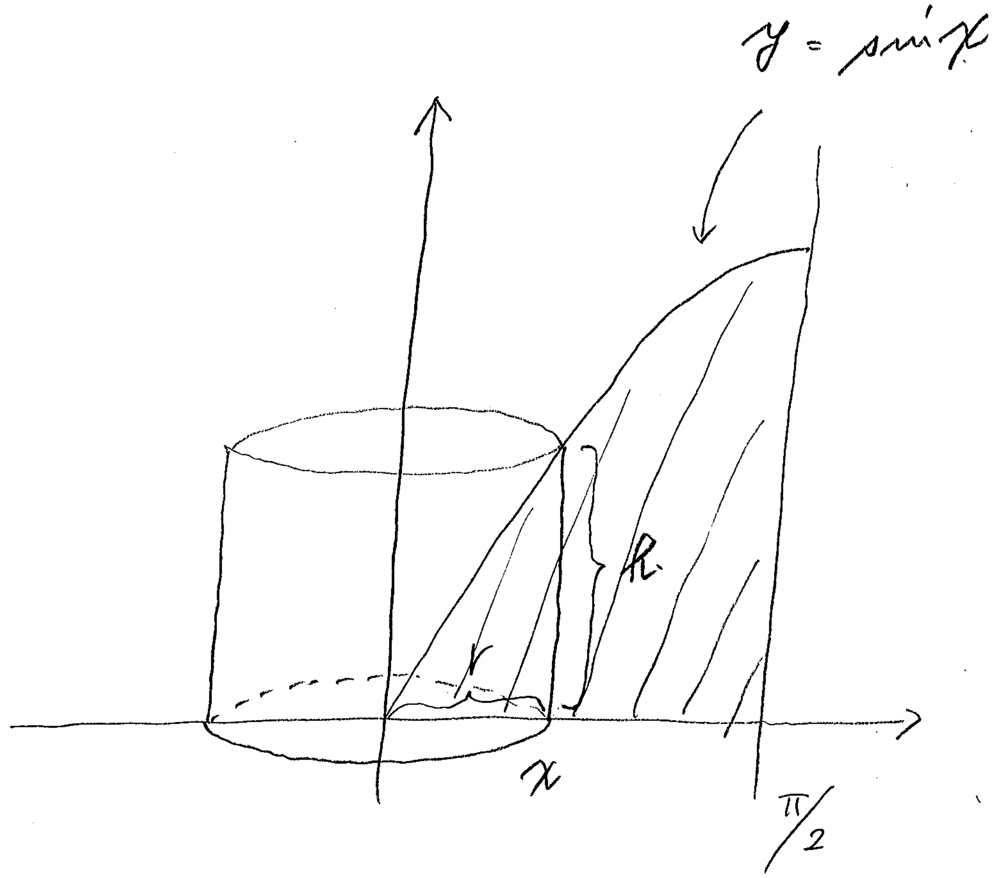
$$= 2\pi \left[\left(-\frac{81}{4} + 27 \right) - (0) \right]$$

$$= 2\pi \cdot 27 \left(-\frac{3}{4} + 1 \right)$$

$$= \frac{27\pi}{2}$$

8.5

(28)



$$V = \int_0^{\pi/2} 2\pi r \cdot h \, dx$$

$$= \int_0^{\pi/2} 2\pi x \cdot \sin x \, dx$$

$$= 2\pi \int_0^{\pi/2} x \cdot \sin x \, dx = 2\pi.$$

Note:

$$\int_0^{\pi/2} x \sin x \, dx = \int_0^{\pi/2} u \, dv$$

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$$u = x \qquad v = -\cos x$$
$$du = dx \qquad dv = \sin x \, dx$$

$$= [uv]_0^{\pi/2} - \int_0^{\pi/2} v \, du$$

$$= [x(-\cos x)]_0^{\pi/2} - \int_0^{\pi/2} (-\cos x) \, dx$$

$$= \int_0^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2} = 1$$

Warning: Since "Integration by Parts" is not covered by Exam I, no computation which requires I. by P. will show up in Exam I.

9. 1.

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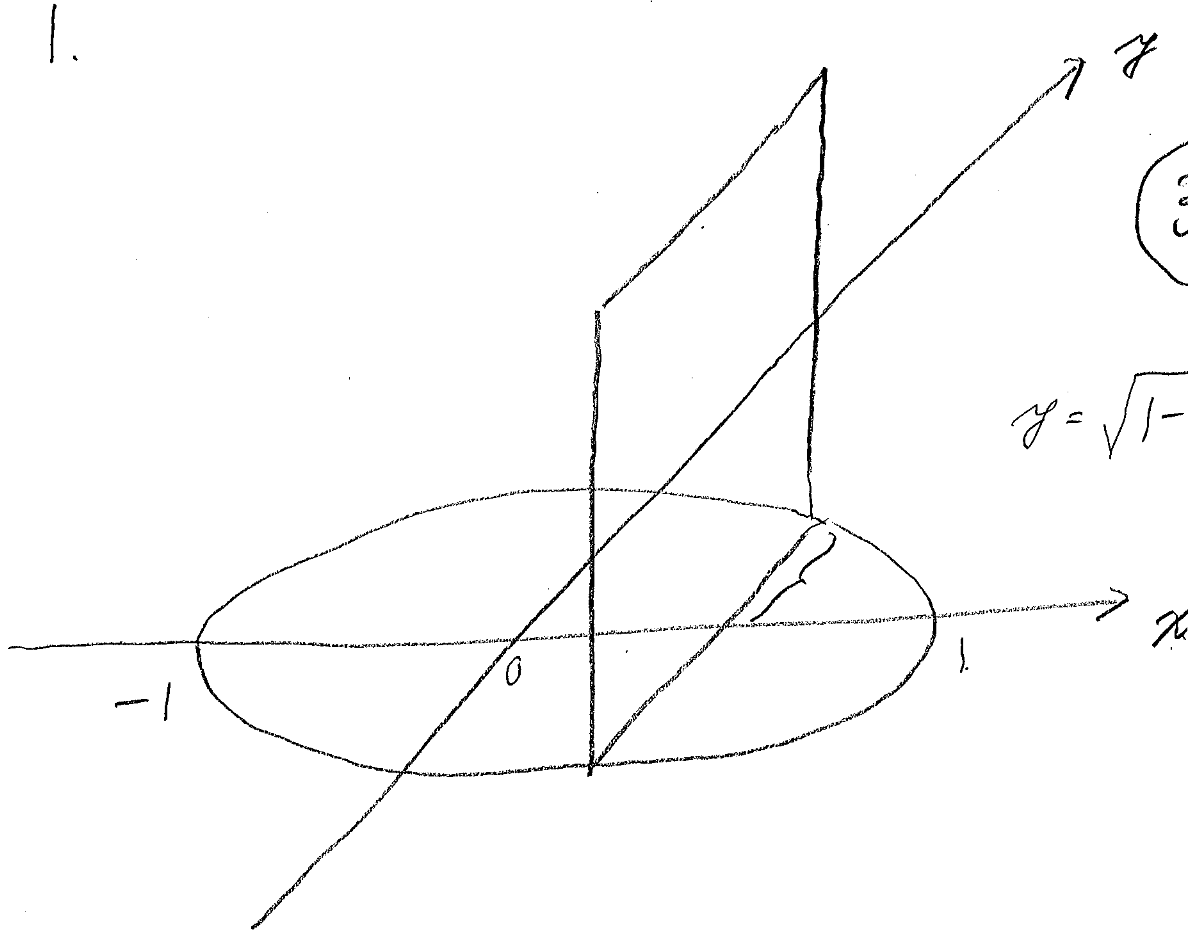

$y = \sqrt{1-x^2}$

$$V = \int_{-1}^{1} \left(2\sqrt{1-x^2}\right)^2 dx$$

$$= 4\int_{-1}^{1} (1-x^2)\,dx$$

$$= 4\left[x - \frac{x^3}{3}\right]_{-1}^{1} = \frac{16}{3}$$

9.2.

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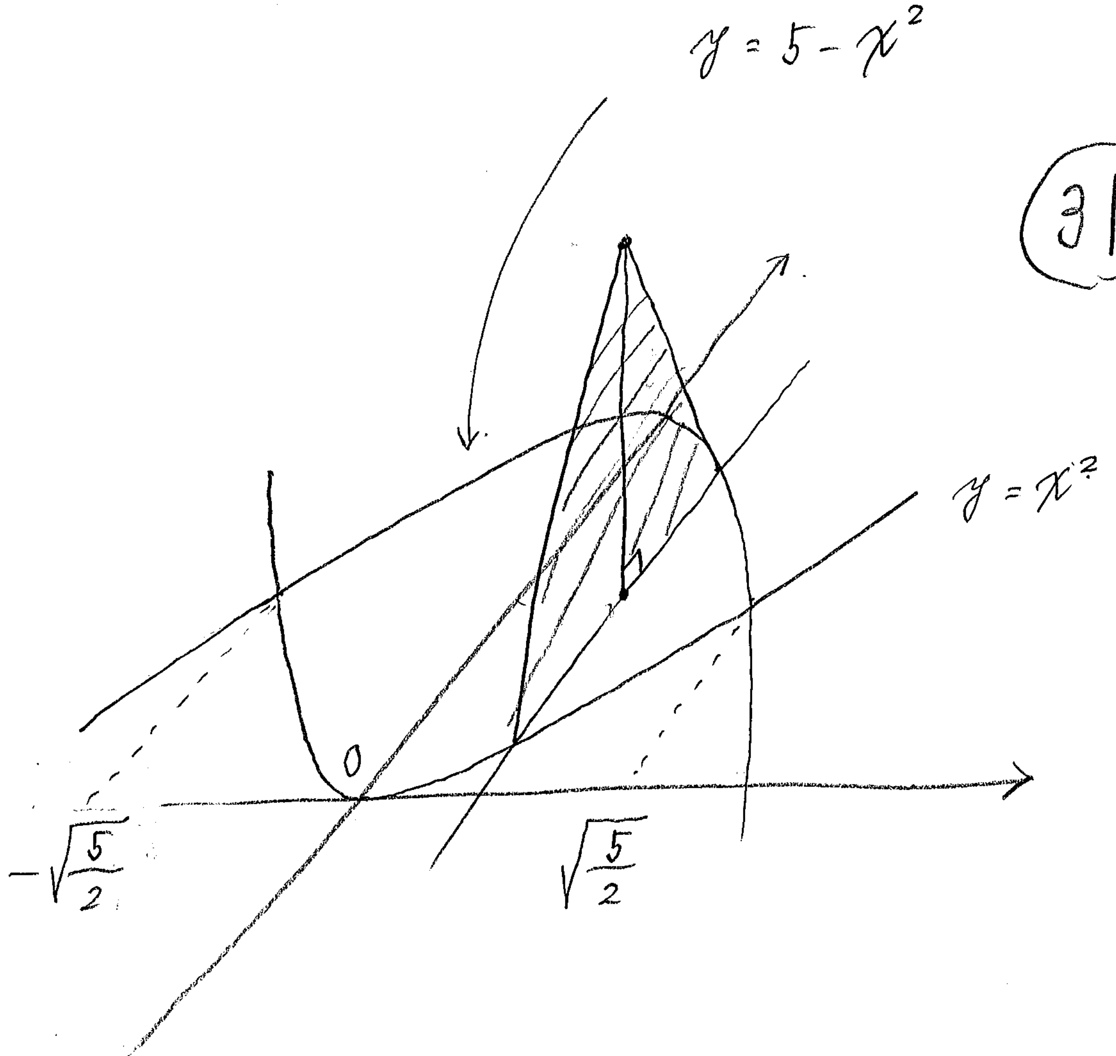

$$5-x^2 = x^2 \rightarrow 5 = 2x^2$$

$$\frac{5}{2} = x^2$$

$$\rightarrow x = \pm\sqrt{\frac{5}{2}}$$

$$V = \int_{-\sqrt{\frac{5}{2}}}^{\sqrt{\frac{5}{2}}} \frac{1}{2}\underbrace{\{(5-x^2)-x^2\}}_{\text{Base}} \cdot \underbrace{\frac{\sqrt{3}}{2}\{(5-x^2)-x^2\}}_{\text{Height}} \, dx$$

$\frac{1}{2}$ Base Height

$$= \frac{\sqrt{3}}{4} \int_{-\sqrt{\frac{5}{2}}}^{\sqrt{\frac{5}{2}}} (5 - 2x^2)^2 \, dx$$

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$$= \frac{\sqrt{3}}{4} \int_{-\sqrt{\frac{5}{2}}}^{\sqrt{\frac{5}{2}}} (25 - 20x^2 + 4x^4) \, dx$$

$$= \frac{\sqrt{3}}{4} \left[25x - 20\frac{x^3}{3} + 4 \cdot \frac{x^5}{5} \right]_{-\sqrt{\frac{5}{2}}}^{\sqrt{\frac{5}{2}}}$$

$$= \frac{10}{3}\sqrt{30}$$

9.3. Look at the textbook.

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10.1.

34

$$y = f(x) = \frac{e^x + e^{-x}}{2}$$

on $[-\ln 2, \ln 3]$

$$L = \int_{-\ln 2}^{\ln 3} \sqrt{1 + \{f'(x)\}^2}\, dx$$

$$= \int_{-\ln 2}^{\ln 3} \sqrt{1 + \left(\frac{e^x - e^{-x}}{2}\right)^2}\, dx$$

$$\left(\text{Note} : f'(x) = \frac{e^x - e^{-x}}{2} \right)$$

$$= \int_{-\ln 2}^{\ln 3} \sqrt{\underbrace{1 + \left(\frac{e^x}{2}\right)^2 + \left(\frac{e^{-x}}{2}\right)^2 \underbrace{-2\left(\frac{e^x}{2}\right)\left(\frac{e^{-x}}{2}\right)}_{-\frac{1}{2}}}_{\left(\frac{e^x}{2}\right)^2 + \left(\frac{e^{-x}}{2}\right)^2 + \underbrace{\frac{1}{2}}_{=\,2\cdot\left(\frac{e^x}{2}\right)\left(\frac{e^{-x}}{2}\right)}}}\, dx$$

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$$= \int_{-\ln 2}^{\ln 3} \sqrt{\left(\frac{e^{x}+e^{-x}}{2}\right)^{2}}\, dx$$

$$= \int_{-\ln 2}^{\ln 3} \left(\frac{e^{x}+e^{-x}}{2}\right) dx$$

$$= \frac{1}{2} \int_{-\ln 2}^{\ln 3} (e^{x}+e^{-x})\, dx$$

$$= \frac{1}{2} \left[e^{x} - e^{-x} \right]_{-\ln 2}^{\ln 3}$$

$$= \frac{1}{2} \left[(e^{\ln 3} - e^{-\ln 3}) - (e^{-\ln 2} - e^{\ln 2}) \right]$$

$$= \frac{1}{2} \left[\left(3 - \frac{1}{3}\right) - \left(\frac{1}{2} - 2\right) \right]$$

$$= \frac{25}{12}$$

10.2

$$y = f(x) = \frac{x^3}{3} + \frac{1}{4x} \text{ on } [1, 4]$$

$$L = \int_1^4 \sqrt{1 + \{f'(x)\}^2}\, dx$$

$$= \int_1^4 \sqrt{1 + \left(x^2 - \frac{1}{4x^2}\right)^2}\, dx$$

$$\left(\text{Note} : f'(x) = x^2 - \frac{1}{4x^2} \right)$$

$$= \int_1^4 \sqrt{\underbrace{1 + (x^2)^2 + \left(\frac{1}{4x^2}\right)^2 \underbrace{- 2(x^2)\left(\frac{1}{4x^2}\right)}_{-\frac{1}{2}}}_{(x^2)^2 + \left(\frac{1}{4x^2}\right)^2 + \underbrace{\frac{1}{2}}_{2(x^2)\left(\frac{1}{4x^2}\right)}}}\, dx$$

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$$= \int_1^4 \sqrt{\left(x^2 + \frac{1}{4x^2}\right)^2}\, dx$$

$$= \int_1^4 \left(x^2 + \frac{1}{4x^2}\right) dx$$

$$= \left[\frac{x^3}{3} - \frac{1}{4x}\right]_1^4$$

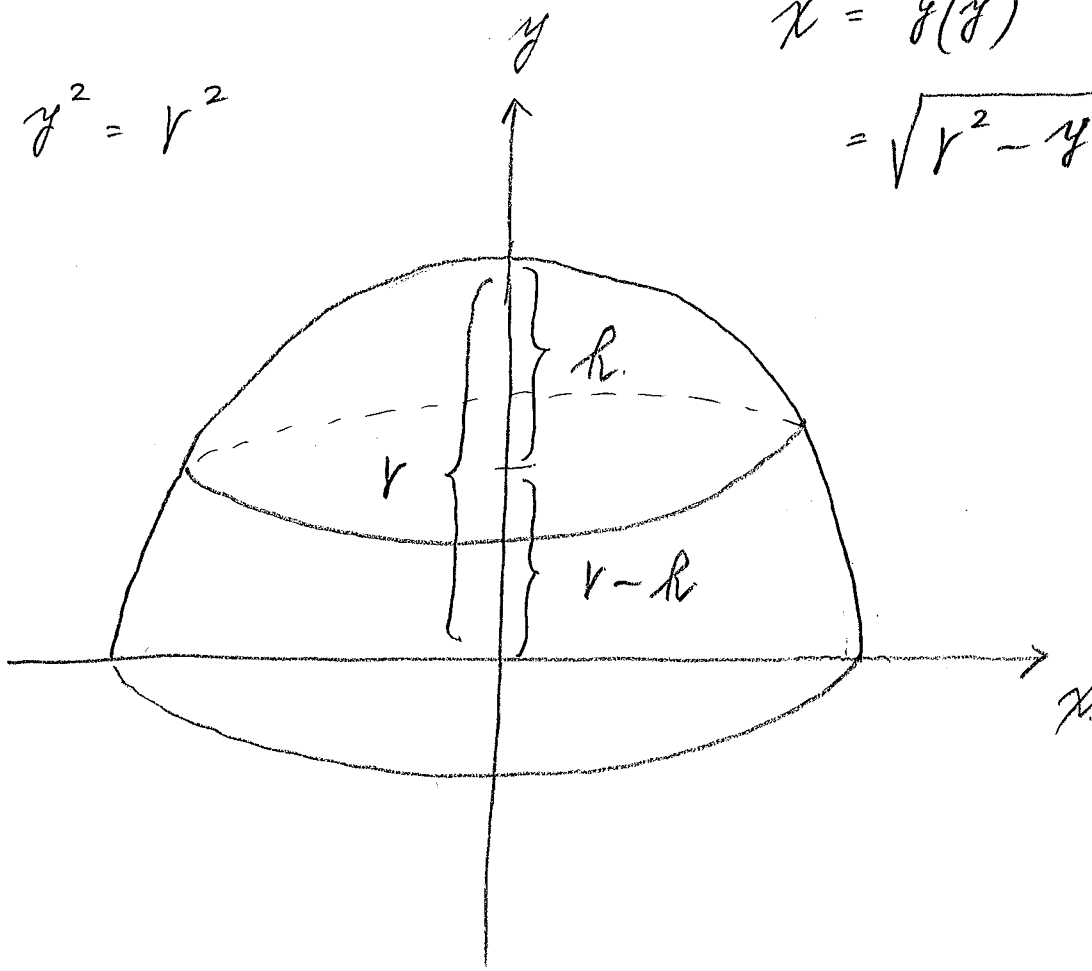
$$= \left[\left(\frac{64}{3} - \frac{1}{16}\right) - \left(\frac{1}{3} - \frac{1}{4}\right)\right]$$

$$= \frac{499}{16}.$$

11.1

$$x^2 + y^2 = r^2$$

$$x = g(y) = \sqrt{r^2 - y^2}$$



$$A = \int_{r-h}^{r} 2\pi x \sqrt{1 + \{g'(y)\}^2}\, dy$$

$$= \int_{r-h}^{r} 2\pi \sqrt{r^2 - y^2} \sqrt{1 + \left\{\frac{-2y}{2\sqrt{r^2 - y^2}}\right\}^2}\, dy$$

$$= \int_{r-h}^{r} 2\pi \sqrt{r^2 - y^2} \sqrt{\underbrace{1 + \frac{y^2}{r^2 - y^2}}_{\shortparallel}} \, dy$$

$$\frac{r^2 - \cancel{y^2} + \cancel{y^2}}{r^2 - y^2}$$

$$\shortparallel$$

$$\frac{r^2}{r^2 - y^2}$$

$$= \int_{r-h}^{r} 2\pi \cancel{\sqrt{r^2 - y^2}} \cdot \frac{r}{\cancel{\sqrt{r^2 - y^2}}} \, dy$$

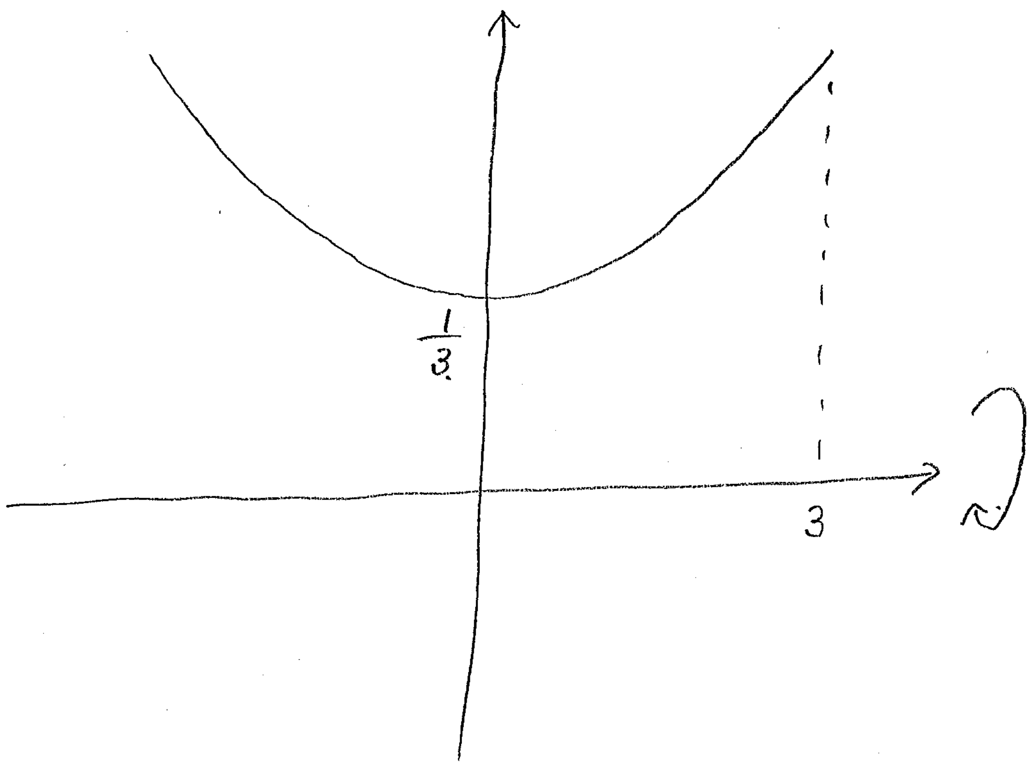
$$= 2\pi r \int_{r-h}^{r} dy = 2\pi r \, [y]_{r-h}^{r}$$

$$= 2\pi r \, [\, r - (r-h) \,] = 2\pi r h$$

11.2.

(40)

$$y = f(x) = \frac{1}{6}\left(e^{3x} + e^{-3x}\right) \text{ on } [0, 3]$$



$$A = \int_0^3 2\pi\, f(x) \sqrt{1 + \{f'(x)\}^2}\, dx$$

$$= \int_0^3 2\pi \cdot \frac{1}{6}\left(e^{3x} + e^{-3x}\right)$$

$$\sqrt{1 + \left\{\frac{1}{2}\left(e^{3x} - e^{-3x}\right)\right\}^2}\, dx$$

Note:

- $$f'(x) = \frac{1}{6}\left(3e^{3x} - 3e^{-3x}\right) = \frac{1}{2}\left(e^{3x} - e^{-3x}\right)$$

- $$\underbrace{1 + \underbrace{\left\{\frac{1}{2}\left(e^{3x} - e^{-3x}\right)\right\}^2}_{\left(\frac{e^{3x}}{2}\right)^2 + \left(\frac{e^{-3x}}{2}\right)^2 \underbrace{- 2\left(\frac{e^{3x}}{2}\right)\left(\frac{e^{-3x}}{2}\right)}_{-\frac{1}{2}}}}_{\underbrace{\left(\frac{e^{3x}}{2}\right)^2 + \left(\frac{e^{-3x}}{2}\right)^2 + \frac{1}{2}}_{\left(\frac{e^{3x} + e^{-3x}}{2}\right)^2}}$$

continuing the computation of A :

$$= \int_0^3 2\pi \cdot \frac{1}{6}\left(e^{3x}+e^{-3x}\right)\sqrt{\left(\frac{e^{3x}+e^{-3x}}{2}\right)^2}\,dx$$

$$= \int_0^3 2\pi \cdot \frac{1}{6}\left(e^{3x}+e^{-3x}\right)\frac{e^{3x}+e^{-3x}}{2}\,dx$$

$$= \frac{\pi}{6}\int_0^3 \left(e^{3x}+e^{-3x}\right)^2 dx$$

$$= \frac{\pi}{6}\int_0^3 \left(e^{6x}+e^{-6x}+2\right) dx$$

$$= \frac{\pi}{6}\left[\frac{1}{6}e^{6x}-\frac{1}{6}e^{-6x}+2x\right]_0^3$$

$$= \frac{\pi}{6}\left[\left(\frac{1}{6}e^{18}-\frac{1}{6}e^{-18}+6\right)-\left(\frac{1}{6}-\frac{1}{6}+2\cdot 0\right)\right]$$

$$= \frac{\pi}{36} e^{18} - \frac{\pi}{36} e^{-18} + \pi.$$

12.1. Took at MyLabMath Lesson 9.

12.2.

mass of the piece

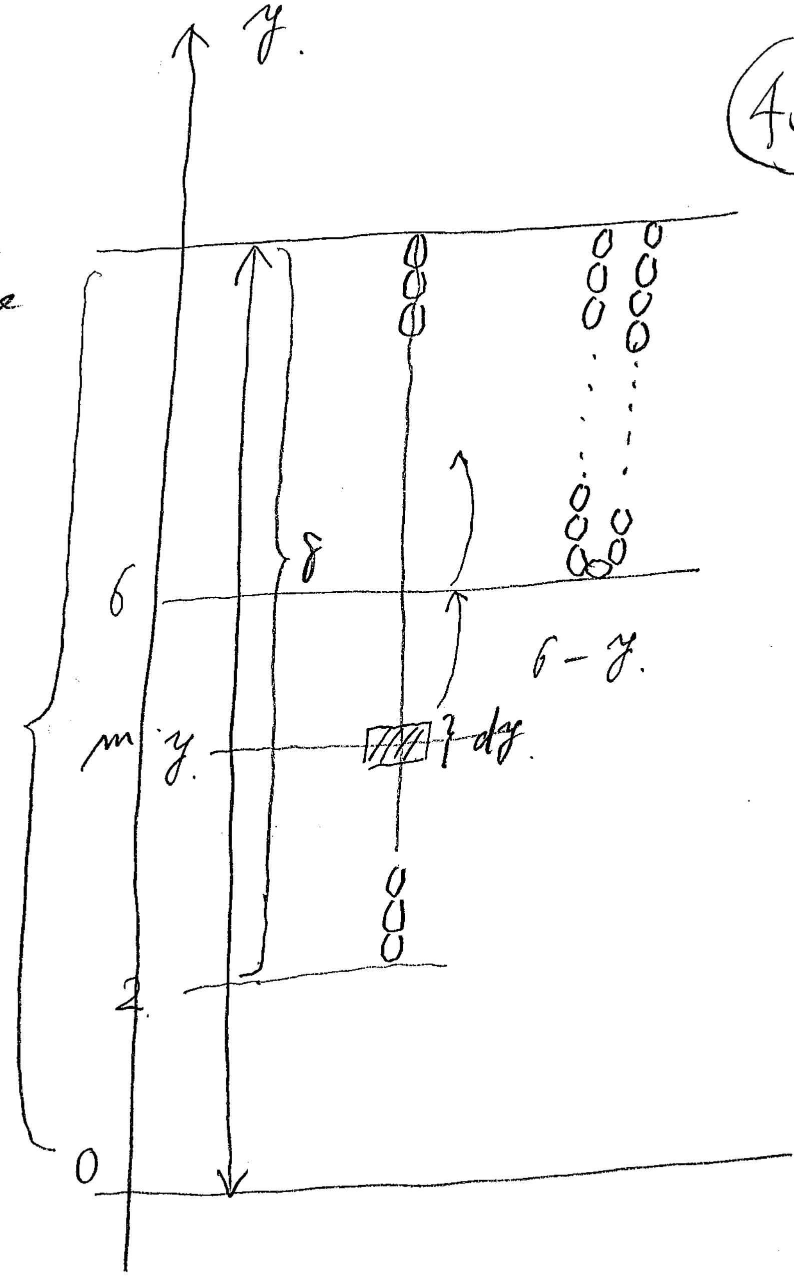
1.5 dy.

weight of the piece

1.5 g dy

the distance to lift the piece

$2(6-y)$



Work to be done to lift the piece

$$2(6-y)\,1.5\,g\,dy$$

Total work

$$W = \int_2^6 2(6-y)\,1.5\,g\,dy$$

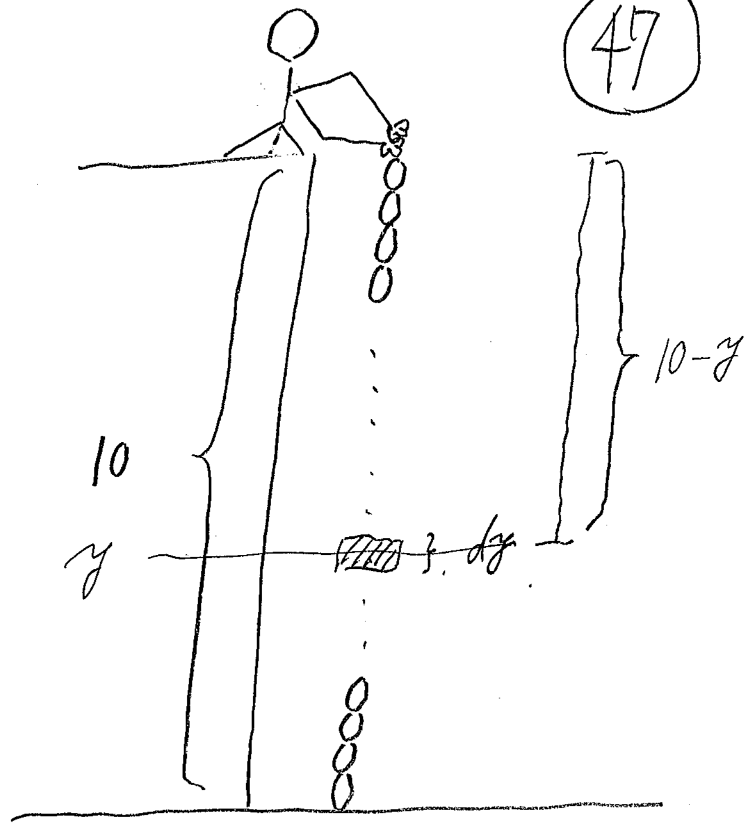
$$W = 3g\int_2^6 (6-y)\,dy$$

$$= 3g\left[6y - \frac{y^2}{2}\right]_2^6$$

$$= 3g\left[\left(36 - \frac{36}{2}\right) - \left(12 - \frac{4}{2}\right)\right]$$

$$= 24g.$$

12.3.



density

$100/10 \ kg/m$

$= 10 \ kg/m.$

mass of the piece

$10 \, dy$

weight of the piece

$10 g \, dy$

distance to lift the piece

$10 - y$

work to be done to lift the piece to the top

$(10-y) \cdot 10 g \, dy.$

Total work

48

$$W = \int_0^{10} (10-y)\, 10g\, dy$$

$$= 10g \int_0^{10} (10-y)\, dy$$

$$= 10g \left[10y - \frac{y^2}{2} \right]_0^{10}$$

$$= 10g \left[100 - \frac{100}{2} \right]$$

$$= 500g.$$

12.4. Look at the textbook.

12.5. Look at MyLabMath Lesson 10

Part II.