Explanation of

Problem # 4

of My Lab Math

Lesson 8, Part 1

4. $y = \ln(x - \sqrt{x^2-1}) \quad 1 \leq x \leq \sqrt{37}$

$$e^y = x - \sqrt{x^2-1}$$

$$e^y - x = -\sqrt{x^2-1}$$

$$x - e^y = \sqrt{x^2-1}$$

$$(x-e^y)^2 = x^2 - 1$$

$$x^2 - 2xe^y + e^{2y}$$

$$e^{2y} + 1 = 2xe^y$$

$$\frac{e^{2y}+1}{2e^y} = x$$

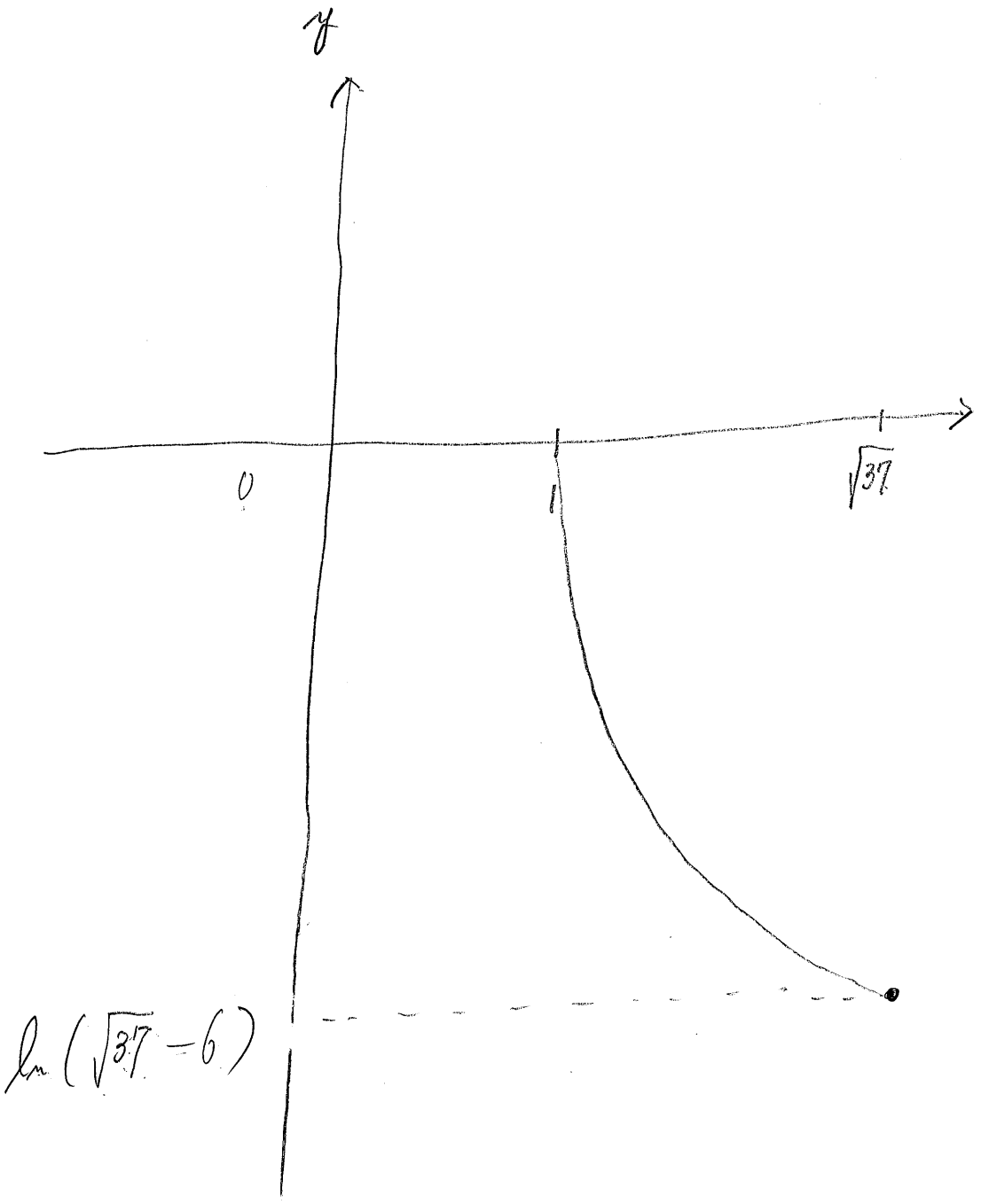
$$x = \frac{e^{2y}+1}{2e^y} = \frac{e^y + e^{-y}}{2}$$

$$1 \leq x \leq \sqrt{37}$$

$\sqrt{37} - 6$

$$0 \leq y = \ln(x - \sqrt{x^2-1})$$

$$\geq \ln(\underbrace{\sqrt{37} - 6}_{\wedge \; 1.})$$


y
0
1
$\sqrt{37}$
$\ln(\sqrt{37}-6)$

$$x = g(y) = \frac{e^y + e^{-y}}{2}$$

$$g'(y) = \frac{e^y - e^{-y}}{2}$$

$$L = \int_{\ln(\sqrt{37}-6)}^{0} \sqrt{1 + \{g'(y)\}^2}\, dy$$

$$= \int_{\ln(\sqrt{37}-6)}^{0} \sqrt{1 + \left(\frac{e^y - e^{-y}}{2}\right)^2}\, dy$$

$$= \int_{\ln(\sqrt{37}-6)}^{0} \sqrt{\left(\frac{e^y + e^{-y}}{2}\right)^2}\, dy$$

$$= \int_{\ln(\sqrt{37}-6)}^{0} \left(\frac{e^y + e^{-y}}{2}\right) dy$$

$$= \frac{1}{2}\left[e^{x} - e^{-x}\right]_{\ln(\sqrt{37}-6)}^{0}$$

$$= \frac{1}{2}\left[(1-1) - \left(\sqrt{37}-6 - \frac{1}{\sqrt{37}-6}\right)\right]$$

$$= \frac{1}{2}\left[0 - \frac{(\sqrt{37}-6)^{2}-1}{\sqrt{37}-6}\right]$$

$$= \frac{1}{2}\left[-\frac{37 - 2\sqrt{37}\cdot 6 + 6^{2} - 1}{\sqrt{37}-6}\right.$$

$$= \frac{1}{2}\left[-\frac{-2\sqrt{37}\cdot 6 + 2\cdot 6^{2}}{\sqrt{37}-6}\right]$$

$$= \frac{1}{2}\left[-\frac{-2\cdot 6(\sqrt{37}-6)}{\sqrt{37}-6}\right]$$

$$= \frac{1}{2}\cdot 2\cdot 6 = 6.$$

$$\boxed{6 \neq \sqrt{37}-1}$$

Note:

$$1 + \left(\frac{e^{y} - e^{-y}}{2}\right)^2$$

$$= 1 + \left(\frac{e^{y}}{2}\right)^2 - 2 \cdot \left(\frac{e^{y}}{2}\right)\left(\frac{e^{-y}}{2}\right) + \left(\frac{e^{-y}}{2}\right)^2$$

$$-2 \cdot \left(\frac{e^{y}}{2}\right)\left(\frac{e^{-y}}{2}\right) = -\frac{1}{2}$$

$$= \left(\frac{e^{y}}{2}\right)^2 + \frac{1}{2} + \left(\frac{e^{-y}}{2}\right)^2$$

$$= \left(\frac{e^{y}}{2}\right)^2 + 2\left(\frac{e^{y}}{2}\right)\left(\frac{e^{-y}}{2}\right) + \left(\frac{e^{-y}}{2}\right)^2$$

$$= \left\{\left(\frac{e^{y}}{2}\right) + \left(\frac{e^{-y}}{2}\right)\right\}^2$$

$$= \left(\frac{e^{y} + e^{-y}}{2}\right)^2$$